**Autoregressive Processes Basic Concepts**

In a simple linear regression model, the predicted dependent variable is modeled as a linear function of the independent variable plus a random error term.

[image048z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image048z.png)

A **first-order** **autoregressive process**, denoted **AR(1)**, takes the form

[image049z](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image049z.png)

Thinking of the subscripts i as representing time, we see that the value of y at time i+1 is a linear function of y at time i plus a fixed constant and a random error term. Similar to the ordinary linear regression model, we assume that the error terms are independently distributed based on a normal distribution with zero mean and a constant variance σ2 and that the error terms are independent of the y values. Thus

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[image050z](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image050z.png)

Similarly, a **second-order autoregressive process**, denoted **AR(2)**, takes the form

[image051z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image051z.png)

and a **p-order autoregressive process**, **AR(p)**, takes the form

[image052z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image052z.png)

**Property 1**: The mean of the yi in a stationary AR(p) process is

[Mean of AR(p) process](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image053z.png)

Proof:

Since the process is stationary, for any k, E[yi] = E[yi-k], a value which we will denote μ. Since E[εi] = 0,  E[φ0] = φ0 and

[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image052z.png?resize=298%2C22](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image052z.png)

it follows that

[Mean of AR(p) process](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image012d.png)

Solving for μ yields the desired result.

**Property 2**: The variance of the yi in a stationary AR(1) process is

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Proof:

Since the yi and εi are independent, by basic properties of variance, it follows that

[https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image014d.png?resize=602%2C40](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image014d.png)

[https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image015d.png?resize=362%2C20](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image015d.png)

Since the process is stationary, yi = yi-1, and so

[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image016d.png?resize=166%2C20](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image016d.png)

Solving for var(yi) yields the desired result.

**Property 3**: The lag h autocorrelation in a stationary AR(1) process is

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Proof:

First note that for any constant a, cov(a+x, a+y) = cov(x,y). Thus, cov(yi,yj) has the same value even if we assume that φ0 = 0, and similarly for var(yi) = cov(yi,y*i*). Thus, it suffices to prove the property when φ0 = 0. In this case, by Property 1, μ = 0, and so cov(yi,yj) = E[yiyj].

Thus

[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image018d.png?resize=602%2C40](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image018d.png)

since by the stationary property, E[yi-1,y*i-k*] = γi-1. Now, by induction on k, it is easy to see that

[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image019d.png?resize=66%2C21](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image019d.png)

Hence

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image020d.png?resize=137%2C42](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2017/12/image020d.png)

**Property 4**: For any stationary AR(p) process. The autocovariance at lag k > 0 can be calculated as

[image065z](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image065z.png)

Similarly the autocorrelation at lag k > 0 can be calculated as

[image066z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/03/image066z.png)

Here we assume that γh = γ-h and ρh = ρ-h if h < 0, and ρ0 = 1.

These are known as the **Yule-Walker equations**.