G ACCEPTANCE SAMPLING PLANS

LEARNING GOALS

After reading this supplement, you should be able to:

1. Distinguish between single-sampling, double-sampling, and sequential-sampling plans and describe the unique characteristics of each.
2. Develop an operating characteristic curve for a single-sampling plan and estimate the probability of accepting a lot with a given proportion defective.
3. Construct a single-sampling plan.
4. Compute the average outgoing quality for a single-sampling plan.

Acceptance sampling is an inspection procedure used to determine whether to accept or reject a specific quantity of material. As more firms initiate total quality management (TQM) programs and work closely with suppliers to ensure high levels of quality, the need for acceptance sampling will decrease. The TQM concept is that no defects should be passed from a producer to a customer, whether the customer is an external or internal customer. However, in reality, many firms must still rely on checking their materials inputs. The basic procedure is straightforward.

1. A random sample is taken from a large quantity of items and tested or measured relative to the quality characteristic of interest.
2. If the sample passes the test, the entire quantity of items is accepted.
3. If the sample fails the test, either (a) the entire quantity of items is subjected to 100 percent inspection and all defective items repaired or replaced or (b) the entire quantity is returned to the supplier.

We first discuss the decisions involved in setting up acceptance sampling plans. We then address several attribute sampling plans.

myomlab and the Companion Website at **www.pearsonhighered.com** contain many tools, activities, and resources designed for this supplement.

Acceptance Sampling Plan Decisions

acceptance sampling

An inspection procedure used to determine whether to accept or reject a specific quantity of materials.

Acceptance sampling involves both the producer (or supplier) of materials and the consumer (or buyer). Consumers need acceptance sampling to limit the risk of rejecting good-quality materials or accepting bad-quality materials. Consequently, the consumer, sometimes in conjunction with the producer through contractual agreements, specifies the parameters of the plan. Any company can be both a producer of goods purchased by another company and a consumer of goods or raw materials supplied by another company.

Quality and Risk Decisions

acceptable quality level (AQL)

The quality level desired by the consumer.

producer's risk (α)

The risk that the sampling plan will fail to verify an acceptable lot's quality and, thus, reject it (a type I error).

Two levels of quality are considered in the design of an acceptance sampling plan. The first is the **acceptable quality level (AQL)**, or the quality level desired by the *consumer*. The producer of the item strives to achieve the AQL, which typically is written into a contract or purchase order. For example, a contract might call for a quality level not to exceed one defective unit in 10,000, or an AQL of 0.0001. The **producer's risk (α)** is the risk that the sampling plan will fail to verify an acceptable lot's quality and, thus, reject it—a type I error. Most often the producer's risk is set at 0.05, or 5 percent.

Although producers are interested in low risk, they often have no control over the consumer's acceptance sampling plan. Fortunately, the consumer also is interested in a low producer's risk because sending good materials back to the producer (1) disrupts the consumer's production process and increases the likelihood of shortages in materials, (2) adds unnecessarily to the lead time for finished products or services, and (3) creates poor relations with the producer.

lot tolerance proportion defective (LTPD)

The worst level of quality that the consumer can tolerate.

consumer's risk (β)

The probability of accepting a lot with LTPD quality (a type II error).

The second level of quality is the **lot tolerance proportion defective (LTPD)**, or the worst level of quality that the consumer can tolerate. The LTPD is a definition of bad quality that the consumer would like to reject. Recognizing the high cost of defects, operations managers have become more cautious about accepting materials of poor quality from suppliers. Thus, sampling plans have lower LTPD values than in the past. The probability of accepting a lot with LTPD quality is the **consumer's risk (β)**, or the type II error of the plan. A common value for the consumer's risk is 0.10, or 10 percent.

Sampling Plans

All sampling plans are devised to provide a specified producer's and consumer's risk. However, it is in the consumer's best interest to keep the average number of items inspected (ANI) to a minimum because that keeps the cost of inspection low. Sampling plans differ with respect to ANI. Three often-used attribute sampling plans are the single-sampling plan, the double-sampling plan, and the sequential-sampling plan. Analogous plans also have been devised for variable measures of quality.

single-sampling plan

A decision to accept or reject a lot based on the results of one random sample from the lot.

Single-Sampling Plan The **single-sampling plan** is a decision rule to accept or reject a lot based on the results of one random sample from the lot. The procedure is to take a random sample of size (n) and inspect each item. If the number of defects does not exceed a specified acceptance number (c), the consumer accepts the entire lot. Any defects found in the sample are either repaired or returned to the producer. If the number of defects in the sample is greater than c, the consumer subjects the entire lot to 100 percent inspection or rejects the entire lot and returns it to the producer. The single-sampling plan is easy to use but usually results in a larger ANI than the other plans. After briefly describing the other sampling plans, we focus our discussion on this plan.

double-sampling plan

A plan in which management specifies two sample sizes and two acceptance numbers; if the quality of the lot is very good or very bad, the consumer can make a decision to accept or reject the lot on the basis of the first sample, which is smaller than in the single-sampling plan.

Double-Sampling Plan In a **double-sampling plan**, management specifies two sample sizes (n_1 and n_2) and two acceptance numbers (c_1 and c_2). If the quality of the lot is very good or very bad, the consumer can make a decision to accept or reject the lot on the basis of the first sample, which is smaller than in the single-sampling plan. To use the plan, the consumer takes a random sample of size n_1. If the number of defects is less than or equal to (c_1), the consumer accepts the lot. If the number of defects is greater than (c_2), the consumer rejects the lot. If the number of defects is between c_1 and c_2, the consumer takes a second sample of size n_2. If the combined number of defects in the two samples is less than or equal to c_2, the consumer accepts the lot. Otherwise, it is rejected. A double-sampling plan can significantly reduce the costs of inspection relative to a single-sampling plan for lots with a very low or very high proportion defective because a decision can be made after taking the first sample. However, if the decision requires two samples, the sampling costs can be greater than those for the single-sampling plan.

sequential-sampling plan

A plan in which the consumer randomly selects items from the lot and inspects them one by one.

Sequential-Sampling Plan A further refinement of the double-sampling plan is the **sequential-sampling plan**, in which the consumer randomly selects items from the lot and inspects them one by one. Each time an item is inspected, a decision is made to (1) reject the lot,

(2) accept the lot, or (3) continue sampling, based on the cumulative results so far. The analyst plots the total number of defectives against the cumulative sample size, and if the number of defectives is less than a certain acceptance number (c_1), the consumer accepts the lot. If the number is greater than another acceptance number (c_2), the consumer rejects the lot. If the number is somewhere between the two, another item is inspected. Figure G.1 illustrates a decision to reject a lot after examining the 40th unit. Such charts can be easily designed with the help of statistical tables that specify the accept or reject cut-off values c_1 and c_2 as a function of the cumulative sample size.

The ANI is generally lower for the sequential-sampling plan than for any other form of acceptance sampling, resulting in lower inspection costs. For very low or very high values of the proportion defective, sequential sampling provides a lower ANI than any comparable sampling plan. However, if the proportion of defective units falls between the AQL and the LTPD, a sequential-sampling plan could have a larger ANI than a comparable single- or double-sampling plan (although that is unlikely). In general, the sequential-sampling plan may reduce the ANI to 50 percent of that required by a comparable single-sampling plan and, consequently, save substantial inspection costs.

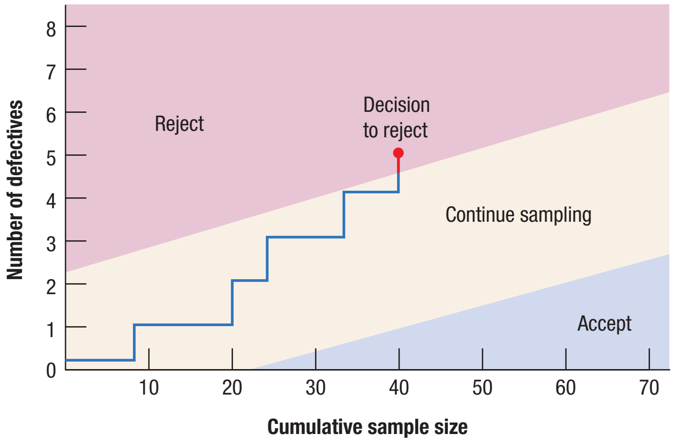


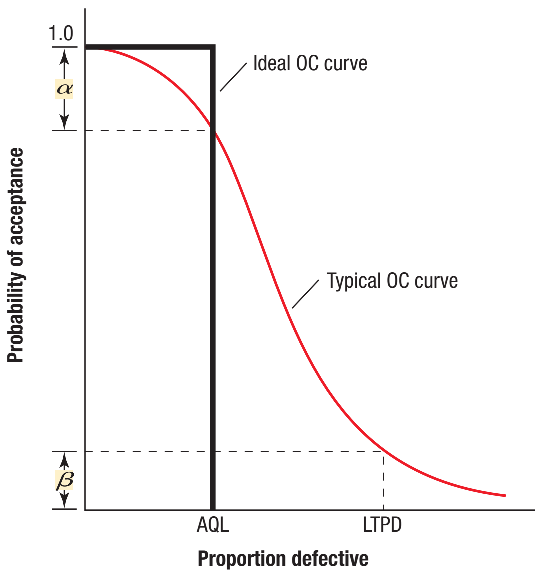
FIGURE G.1
Sequential-Sampling Chart

Operating Characteristic Curves

Analysts create a graphic display of the performance of a sampling plan by plotting the probability of accepting the lot for a range of proportions of defective units. This graph, called an **operating characteristic (OC) curve**, describes how well a sampling plan discriminates between good and bad lots. Undoubtedly, every manager wants a plan that accepts lots with a quality level better than the AQL 100 percent of the time and accepts lots with a quality level worse than the AQL 0 percent of the time. This ideal OC curve for a single-sampling plan is shown in Figure G.2. However, such performance can be achieved only with 100 percent inspection. A typical OC curve for a single-sampling plan, plotted in red, shows the probability a of rejecting a good lot (producer's risk) and the probability β of accepting a bad lot (consumer's risk). Consequently, managers are left with choosing a sample size n and an acceptance number c to achieve the level of performance specified by the AQL, α, LTPD, and β.

operating characteristic (OC) curve
A graph that describes how well a sampling plan discriminates between good and bad lots.

FIGURE G.2
Operating Characteristic Curves



Drawing the OC Curve

The sampling distribution for the single-sampling plan is the binomial distribution because each item inspected is either defective (a failure) or not (a success). The probability of accepting the lot equals the probability of taking a sample of size n from a lot with a proportion defective of p and finding c or fewer defective items. However, if n is greater than 20 and p is less than 0.05, the Poisson distribution can be used as an approximation to the binomial to take advantage of tables prepared for the purpose of drawing OC curves (see Table G.1 on pp. G.9–G.11). To draw the OC curve, look up the probability of accepting the lot for a range of values of p. For each value of p,

1. multiply p by the sample size n.
2. find the value of np in the left column of the table.
3. move to the right until you find the column for c.
4. record the value for the probability of acceptance, P_a.

When p = AQL, the producer's risk, α, is 1 minus the probability of acceptance. When (p = LTPD), the consumer's risk, β, equals the probability of acceptance.

EXAMPLE G.1 Constructing an OC Curve

PEARSON myomlab

Tutor G.1 in myomlab provides a new example for constructing an OC curve.

The Noise King Muffler Shop, a high-volume installer of replacement exhaust muffler systems, just received a shipment of 1,000 mufflers. The sampling plan for inspecting these mufflers calls for a sample size $n = 60$ and an acceptance number $c = 1$. The contract with the muffler manufacturer calls for an AQL of 1 defective muffler per 100 and an LTPD of 6 defective mufflers per 100. Calculate the OC curve for this plan, and determine the producer's risk and the consumer's risk for the plan.

SOLUTION

Let $p = 0.01$. Then multiply n by p to get 60(0.01) = 0.60. Locate 0.60 in Table G.1 (pp. G.9–G.11). Move to the right until you reach the column for $c = 1$. Read the probability of acceptance: 0.878. Repeat this process for a range of p values. The following table contains the remaining values for the OC curve.

Values for the Operating Characteristic Curve with $n = 60$ and $c = 1$

Proportion Defective (p)	np	Probability of c or Less Defects (P_a)	Comments
0.01 (AQL)	0.6	0.878	$\alpha = 1.000 - 0.878 = 0.122$
0.02	1.2	0.663	
0.03	1.8	0.463	
0.04	2.4	0.308	
0.05	3.0	0.199	
0.06 (LTPD)	3.6	0.126	$\beta = 0.126$
0.07	4.2	0.078	
0.08	4.8	0.048	
0.09	5.4	0.029	
0.10	6.0	0.017	

DECISION POINT

Note that the plan provides a producer's risk of 12.2 percent and a consumer's risk of 12.6 percent. Both values are higher than the values usually acceptable for plans of this type (5 and 10 percent, respectively). Figure G.3 shows the OC curve and the producer's and consumer's risks. Management can adjust the risks by changing the sample size.

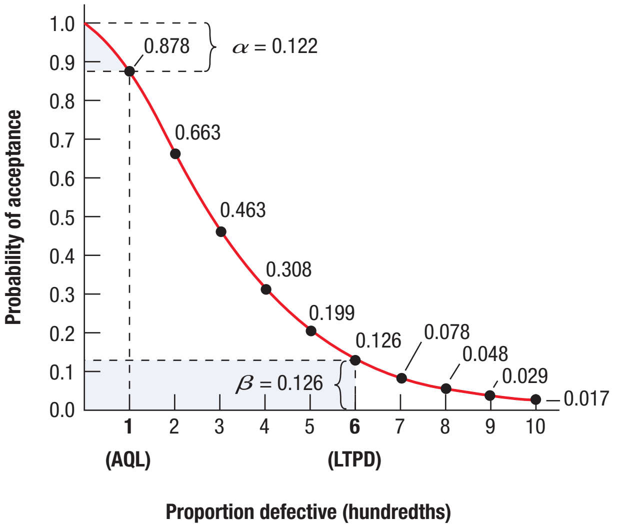


FIGURE G.3 The OC Curve for Single-Sampling Plan with $n = 60$ and $c = 1$

Explaining Changes in the OC Curve

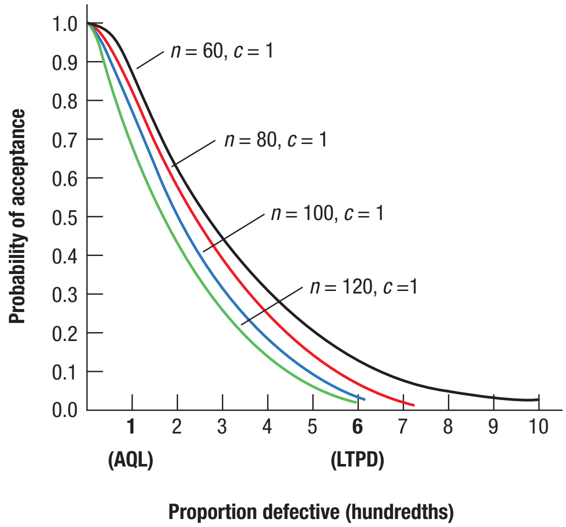
Example G.1 raises the question: How can management change the sampling plan to reduce the probability of rejecting good lots and accepting bad lots? To answer this question, let us see how n and c affect the shape of the OC curve. In the Noise King example, a better single-sampling plan would have a lower producer's risk and a lower consumer's risk.

Sample Size Effect What would happen if we increased the sample size to 80 and left the acceptance level, c, unchanged at 1? We can use Table G.1 (pp. G.9–G.11). If the proportion defective of the lot is $p = \text{AQL} = 0.01$, then $np = 0.8$ and the probability of acceptance of the lot is only 0.809. Thus, the producer's risk is 0.191. Similarly, if $p = \text{LTPD} = 0.06$, the probability of acceptance is 0.048. Other values of the producer's and consumer's risks are shown in the following table:

n	Producer's Risk (p = AQL)	Consumer's Risk (p = LTPD)
60	0.122	0.126
80	0.191	0.048
100	0.264	0.017
120	0.332	0.006

These results, shown in Figure G.4, yield the following principle: *Increasing* n *while holding* c *constant increases the producer's risk and reduces the consumer's risk*. For the producer of the mufflers, keeping $c = 1$ and increasing the sample size makes getting a lot accepted by the customer tougher—only two bad mufflers will get the lot rejected. And the likelihood of finding those 2 defects is greater in a sample of 120 than in a sample of 60. Consequently, the producer's risk increases. For the management of Noise King, the consumer's risk goes down because a random sample of 120 mufflers from a lot with 6 percent defectives is less likely to have only 1 or fewer defective mufflers.

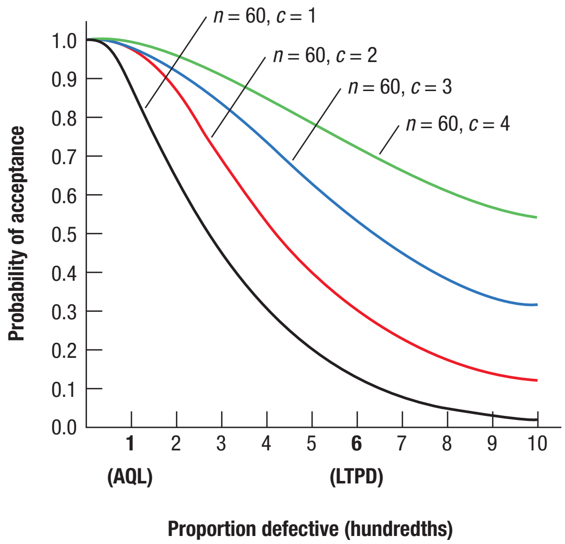
▼ **FIGURE G.4**
Effects of Increasing Sample Size While Holding Acceptance Number Constant



Acceptance Level Effect Suppose that we keep the sample size constant at 60 but change the acceptance level. Again, we use Table G.1 (pp. G.9–G.11).

c	Producer's Risk (p = AQL)	Consumer's Risk (p = LTPD)
1	0.122	0.126
2	0.023	0.303
3	0.003	0.515
4	0.000	0.706

The results are plotted in Figure G.5. They demonstrate the following principle: *Increasing* c *while holding* n *constant decreases the producer's risk and increases the consumer's risk*. The producer of the mufflers would welcome an increase in the acceptance number because it makes getting the lot accepted by the consumer easier. If the lot has only 1 percent defectives (the AQL) with a sample size of 60, we would expect only $0.01(60) = 0.6$ defect in the sample. An increase in the acceptance number from one to two lowers the probability of finding more than two defects and, consequently, lowers the producer's risk. However, raising the acceptance number for a given sample size increases the risk of accepting a bad lot. Suppose that the lot has 6 percent defectives (the LTPD). We would expect to have $0.6(60) = 3.6$ defectives in the sample. An increase in the acceptance number from one to two increases the probability of getting a sample with two or fewer defects and, therefore, increases the consumer's risk.



▲ **FIGURE G.5**
Effects of Increasing Acceptance Number While Holding Sample Size Constant

Thus, to improve Noise King's single-sampling acceptance plan, management should increase the sample size, which reduces the consumer's risk, *and* increase the acceptance number, which reduces the producer's risk. An improved combination can be found by trial and error using Table G.1 (pp. G.9–G.11). Alternatively, a computer can be used to find the best combination. For any acceptance number, the computer determines the sample size needed to achieve the desired producer's risk and compares it to the sample size needed to meet the consumer's risk. It selects the smallest sample size that will meet both the producer's risk and the consumer's risk. The following table shows that a sample size of 111 and an acceptance number of 3 are best. This combination actually yields a producer's risk of 0.026 and a consumer's risk of 0.10 (not shown). The risks are not exact because c and n must be integers.

Acceptance Sampling Plan Data

	AQL Based		LTPD Based	
Acceptance Number	Expected Defectives	Sample Size	Expected Defectives	Sample Size
0	0.0509	5	2.2996	38
1	0.3552	36	3.8875	65
2	0.8112	81	5.3217	89
3	1.3675	137	6.6697	111
4	1.9680	197	7.9894	133
5	2.6256	263	9.2647	154
6	3.2838	328	10.5139	175
7	3.9794	398	11.7726	196
8	4.6936	469	12.9903	217
9	5.4237	542	14.2042	237
10	6.1635	616	15.4036	257

Average Outgoing Quality

We have shown how to choose the sample size and acceptance number for a single-sampling plan, given AQL, α, LTPD, and β parameters. To check whether the performance of the plan is what we want, we can calculate the plan's **average outgoing quality (AOQ)**, which is the expected proportion of defects that the plan will allow to pass. We assume that all defective items in the lot will be replaced with good items if the lot is rejected and that any defective items in the sample will be replaced if the lot is accepted. This approach is called **rectified inspection**. The equation for AOQ is

average outgoing quality (AOQ)

The expressed proportion of defects that the plan will allow to pass.

rectified inspection

The assumption that all defective items in the lot will be replaced with good items if the lot is rejected and that any defective items in the sample will be replaced if the lot is accepted.

$$\text{AOQ} = \frac{p(P_a)(N - n)}{N}$$

where

p = true proportion defective of the lot
P_a = probability of accepting the lot
N = lot size
n = sample size

The analyst can calculate AOQ to estimate the performance of the plan over a range of possible proportion defectives in order to judge whether the plan will provide an acceptable degree of protection. The maximum value of the average outgoing quality over all possible values of the proportion defective is called the **average outgoing quality limit (AOQL)**. If the AOQL seems too high, the parameters of the plan must be modified until an acceptable AOQL is achieved.

average outgoing quality limit (AOQL)

The maximum value of the average outgoing quality over all possible values of the proportion defective.

EXAMPLE G.2 Calculating the AOQL

Suppose that Noise King is using rectified inspection for its single-sampling plan. Calculate the average outgoing quality limit for a plan with $n = 110$, $c = 3$, and $N = 1{,}000$. Use Table G.1 (pp. G.9–G.11) to estimate the probabilities of acceptance for values of the proportion defective from 0.01 to 0.08 in steps of 0.01.

PEARSON myomlab

Tutor G.2 in myomlab provides a new example for calculating the AOQL.

SOLUTION

Use the following steps to estimate the AOQL for this sampling plan:

Step 1: Determine the probabilities of acceptance for the desired values of *p*. These are shown in the following table. However, the values for $p = 0.03$, 0.05, and 0.07 had to be interpolated because the table does not have them. For example, P_a for $p = 0.03$ was estimated by averaging the P_a values for $np = 3.2$ and $np = 3.4$, or $(0.603 + 0.558)/2 = 0.580$.

Proportion Defective (*p*)	*np*	Probability of Acceptance (P_a)
0.01	1.10	0.974
0.02	2.20	0.819
0.03	3.30	$0.581 = (0.603 + 0.558)/2$
0.04	4.40	0.359
0.05	5.50	$0.202 = (0.213 + 0.191)/2$
0.06	6.60	0.105
0.07	7.70	$0.052 = (0.055 + 0.048)/2$
0.08	8.80	0.024

Step 2: Calculate the AOQ for each value of *p*.

For $p = 0.01$: $0.01(0.974)(1000 - 110)/1000 = 0.0087$
For $p = 0.02$: $0.02(0.819)(1000 - 110)/1000 = 0.0146$
For $p = 0.03$: $0.03(0.581)(1000 - 110)/1000 = 0.0155$
For $p = 0.04$: $0.04(0.359)(1000 - 110)/1000 = 0.0128$
For $p = 0.05$: $0.05(0.202)(1000 - 110)/1000 = 0.0090$
For $p = 0.06$: $0.06(0.105)(1000 - 110)/1000 = 0.0056$
For $p = 0.07$: $0.07(0.052)(1000 - 110)/1000 = 0.0032$
For $p = 0.08$: $0.08(0.024)(1000 - 110)/1000 = 0.0017$

The plot of the AOQ values is shown in Figure G.6.

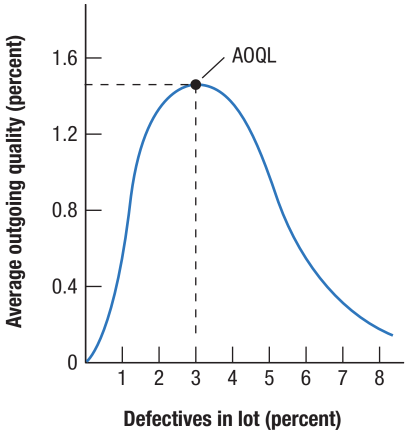


FIGURE G.6
Average Outgoing Quality Curve for the Noise King Muffler Service

Step 3: Identify the largest AOQ value, which is the estimate of the AOQL. In this example, the AOQL is 0.0155 at $p = 0.03$.

Key Equation

$$\text{Average outgoing quality: AOQ} = \frac{p(P_a)(N - n)}{N}$$

Solved Problem

An inspection station has been installed between two production processes. The feeder process, when operating correctly, has an acceptable quality level of 3 percent. The consuming process, which is expensive, has a specified lot tolerance proportion defective of 8 percent. The feeding process produces in batch sizes; if a batch is rejected by the inspector, the entire batch must be checked and the defective items reworked. Consequently, management wants no more than a 5 percent producer's risk and, because of the expensive process that follows, no more than a 10 percent chance of accepting a lot with 8 percent defectives or worse.

a. Determine the appropriate sample size, n, and the acceptable number of defective items in the sample, c.

b. Calculate values and draw the OC curve for this inspection station.

c. What is the probability that a lot with 5 percent defectives will be rejected?

SOLUTION

a. For AQL = 3 percent, LTPD = 8 percent, $\alpha = 5$ *percent*, and $\beta = 10$ percent, use Table G.1 (pp. G.9–G.11) and trial and error to arrive at a sampling plan. If $n = 180$ and $c = 9$,

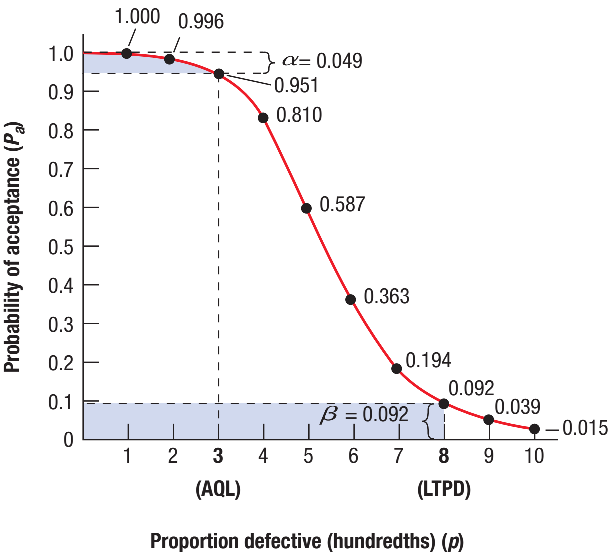
$$np = 180(0.03) = 5.4$$
$$\alpha = 0.049$$
$$np = 180(0.08) = 14.4$$
$$\beta = 0.092$$

Sampling plans that would also work are $n = 200$, $c = 10$; $n = 220$, $c = 11$; and $n = 240$, $c = 12$.

b. The following table contains the data for the OC curve. Table G.1 (pp. G.9–G.11) was used to estimate the probability of acceptance. Figure G.7 shows the OC curve.

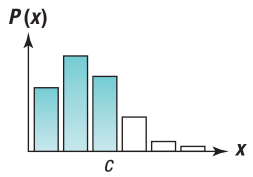
c. According to the table, the probability of accepting a lot with 5 percent defectives is 0.587. Therefore, the probability that a lot with 5 percent defects will be rejected is 0.413, or 1.00 − 0.587.

▼ FIGURE G.7



Proportion Defective (p)	np	Probability of c or Less Defects (P_a)	Comments
0.01	1.8	1.000	
0.02	3.6	0.996	
0.03 (AQL)	5.4	0.951	$\alpha = 1 - 0.951 = 0.049$
0.04	7.2	0.810	
0.05	9.0	0.587	
0.06	10.8	0.363	
0.07	12.6	0.194	
0.08 (LTPD)	14.4	0.092	$\beta = 0.092$
0.09	16.2	0.039	
0.10	18.0	0.015	

TABLE G.1 CUMULATIVE POISSON PROBABILITIES



$$P(x \le c) = \sum_{x=0}^{x=c} \frac{\lambda^x e^{-\lambda}}{x!}$$

	c													
np	0	1	2	3	4	5	6	7	8	9	10	11	12	13
.05	.951	.999	1.000											
.10	.905	.995	1.000											
.15	.861	.990	.999	1.000										
.20	.819	.982	.999	1.000										
.25	.779	.974	.998	1.000										
.30	.741	.963	.996	1.000										
.35	.705	.951	.994	1.000										
.40	.670	.938	.992	.999	1.000									
.45	.638	.925	.989	.999	1.000									
.50	.607	.910	.986	.998	1.000									
.55	.577	.894	.982	.998	1.000									
.60	.549	.878	.977	.997	1.000									
.65	.522	.861	.972	.996	.999	1.000								
.70	.497	.844	.966	.994	.999	1.000								
.75	.472	.827	.959	.993	.999	1.000								
.80	.449	.809	.953	.991	.999	1.000								
.85	.427	.791	.945	.989	.998	1.000								
.90	.407	.772	.937	.987	.998	1.000								
.95	.387	.754	.929	.984	.997	1.000								
1.0	.368	.736	.920	.981	.996	.999	1.000							
1.1	.333	.699	.900	.974	.995	.999	1.000							
1.2	.301	.663	.879	.966	.992	.998	1.000							
1.3	.273	.627	.857	.957	.989	.998	1.000							
1.4	.247	.592	.833	.946	.986	.997	.999	1.000						
1.5	.223	.558	.809	.934	.981	.996	.999	1.000						
1.6	.202	.525	.783	.921	.976	.994	.999	1.000						
1.7	.183	.493	.757	.907	.970	.992	.998	1.000						
1.8	.165	.463	.731	.891	.964	.990	.997	.999	1.000					
1.9	.150	.434	.704	.875	.956	.987	.997	.999	1.000					
2.0	.135	.406	.677	.857	.947	.983	.995	.999	1.000					
2.2	.111	.355	.623	.819	.928	.975	.993	.998	1.000					
2.4	.091	.308	.570	.779	.904	.964	.988	.997	.999	1.000				
2.6	.074	.267	.518	.736	.877	.951	.983	.995	.999	1.000				
2.8	.061	.231	.469	.692	.848	.935	.976	.992	.998	.999	1.000			
3.0	.050	.199	.423	.647	.815	.916	.966	.988	.996	.999	1.000			
3.2	.041	.171	.380	.603	.781	.895	.955	.983	.994	.998	1.000			
3.4	.033	.147	.340	.558	.744	.871	.942	.977	.992	.997	.999	1.000		
3.6	.027	.126	.303	.515	.706	.844	.927	.969	.988	.996	.999	1.000		
3.8	.022	.107	.269	.473	.668	.816	.909	.960	.984	.994	.998	.999	1.000	
4.0	.018	.092	.238	.433	.629	.785	.889	.949	.979	.992	.997	.999	1.000	

(continued)

TABLE G.1 (CONT.)

	c													
np	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
4.2	.015	.078	.210	.395	.590	.753	.867	.936	.972	.989	.996	.999	1.000	
4.4	.012	.066	.185	.359	.551	.720	.844	.921	.964	.985	.994	.998	.999	1.000
4.6	.010	.056	.163	.326	.513	.686	.818	.905	.955	.980	.992	.997	.999	1.000
4.8	.008	.048	.143	.294	.476	.651	.791	.887	.944	.975	.990	.996	.999	1.000
5.0	.007	.040	.125	.265	.440	.616	.762	.867	.932	.968	.986	.995	.998	.999
5.2	.006	.034	.109	.238	.406	.581	.732	.845	.918	.960	.982	.993	.997	.999
5.4	.005	.029	.095	.213	.373	.546	.702	.822	.903	.951	.977	.990	.996	.999
5.6	.004	.024	.082	.191	.342	.512	.670	.797	.886	.941	.972	.988	.995	.998
5.8	.003	.021	.072	.170	.313	.478	.638	.771	.867	.929	.965	.984	.993	.997
6.0	.002	.017	.062	.151	.285	.446	.606	.744	.847	.916	.957	.980	.991	.996
6.2	.002	.015	.054	.134	.259	.414	.574	.716	.826	.902	.949	.975	.989	.995
6.4	.002	.012	.046	.119	.235	.384	.542	.687	.803	.886	.939	.969	.986	.994
6.6	.001	.010	.040	.105	.213	.355	.511	.658	.780	.869	.927	.963	.982	.992
6.8	.001	.009	.034	.093	.192	.327	.480	.628	.755	.850	.915	.955	.978	.990
7.0	.001	.007	.030	.082	.173	.301	.450	.599	.729	.830	.901	.947	.973	.987
7.2	.001	.006	.025	.072	.156	.276	.420	.569	.703	.810	.887	.937	.967	.984
7.4	.001	.005	.022	.063	.140	.253	.392	.539	.676	.788	.871	.926	.961	.980
7.6	.001	.004	.019	.055	.125	.231	.365	.510	.648	.765	.854	.915	.954	.976
7.8	.000	.004	.016	.048	.112	.210	.338	.481	.620	.741	.835	.902	.945	.971
8.0	.000	.003	.014	.042	.100	.191	.313	.453	.593	.717	.816	.888	.936	.966
8.2	.000	.003	.012	.037	.089	.174	.290	.425	.565	.692	.796	.873	.926	.960
8.4	.000	.002	.010	.032	.079	.157	.267	.399	.537	.666	.774	.857	.915	.952
8.6	.000	.002	.009	.028	.070	.142	.246	.373	.509	.640	.752	.840	.903	.945
8.8	.000	.001	.007	.024	.062	.128	.226	.348	.482	.614	.729	.822	.890	.936
9.0	.000	.001	.006	.021	.055	.116	.207	.324	.456	.587	.706	.803	.876	.926
9.2	.000	.001	.005	.018	.049	.104	.189	.301	.430	.561	.682	.783	.861	.916
9.4	.000	.001	.005	.016	.043	.093	.173	.279	.404	.535	.658	.763	.845	.904
9.6	.000	.001	.004	.014	.038	.084	.157	.258	.380	.509	.633	.741	.828	.892
9.8	.000	.001	.003	.012	.033	.075	.143	.239	.356	.483	.608	.719	.810	.879
10.0	0	.000	.003	.010	.029	.067	.130	.220	.333	.458	.583	.697	.792	.864
10.2	0	.000	.002	.009	.026	.060	.118	.203	.311	.433	.558	.674	.772	.849
10.4	0	.000	.002	.008	.023	.053	.107	.186	.290	.409	.533	.650	.752	.834
10.6	0	.000	.002	.007	.020	.048	.097	.171	.269	.385	.508	.627	.732	.817
10.8	0	.000	.001	.006	.017	.042	.087	.157	.250	.363	.484	.603	.710	.799
11.0	0	.000	.001	.005	.015	.038	.079	.143	.232	.341	.460	.579	.689	.781
11.2	0	.000	.001	.004	.013	.033	.071	.131	.215	.319	.436	.555	.667	.762
11.4	0	.000	.001	.004	.012	.029	.064	.119	.198	.299	.413	.532	.644	.743
11.6	0	.000	.001	.003	.010	.026	.057	.108	.183	.279	.391	.508	.622	.723
11.8	0	.000	.001	.003	.009	.023	.051	.099	.169	.260	.369	.485	.599	.702
12.0	0	.000	.001	.002	.008	.020	.046	.090	.155	.242	.347	.462	.576	.682

(continued)

TABLE G.1 (CONT.)

	c													
np	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
12.2	0	0	0.000	0.002	0.007	0.018	0.041	0.081	0.142	0.225	0.327	0.439	0.553	0.660
12.4	0	0	0.000	0.002	0.006	0.016	0.037	0.073	0.131	0.209	0.307	0.417	0.530	0.639
12.6	0	0	0.000	0.001	0.005	0.014	0.033	0.066	0.120	0.194	0.288	0.395	0.508	0.617
12.8	0	0	0.000	0.001	0.004	0.012	0.029	0.060	0.109	0.179	0.269	0.374	0.485	0.595
13.0	0	0	0.000	0.001	0.004	0.011	0.026	0.054	0.100	0.166	0.252	0.353	0.463	0.573
13.2	0	0	.000	.001	.003	.009	.023	.049	.091	.153	.235	.333	.441	.551
13.4	0	0	.000	.001	.003	.008	.020	.044	.083	.141	.219	.314	.420	.529
13.6	0	0	.000	.001	.002	.007	.018	.039	.075	.130	.204	.295	.399	.507
13.8	0	0	.000	.001	.002	.006	.016	.035	.068	.119	.189	.277	.378	.486
14.0	0	0	0	.000	.002	.006	.014	.032	.062	.109	.176	.260	.358	.464
14.2	0	0	0	.000	.002	.005	.013	.028	.056	.100	.163	.244	.339	.443
14.4	0	0	0	.000	.001	.004	.011	.025	.051	.092	.151	.228	.320	.423
14.6	0	0	0	.000	.001	.004	.010	.023	.046	.084	.139	.213	.302	.402
14.8	0	0	0	.000	.001	.003	.009	.020	.042	.077	.129	.198	.285	.383
15.0	0	0	0	.000	.001	.003	.008	.018	.037	.070	.118	.185	.268	.363
15.2	0	0	0	.000	.001	.002	.007	.016	.034	.064	.109	.172	.251	.344
15.4	0	0	0	.000	.001	.002	.006	.014	.030	.058	.100	.160	.236	.326
15.6	0	0	0	.000	.001	.002	.005	.013	.027	.053	.092	.148	.221	.308
15.8	0	0	0	0	.000	.002	.005	.011	.025	.048	.084	.137	.207	.291
16.0	0	0	0	0	.000	.001	.004	.010	.022	.043	.077	.127	.193	.275
16.2	0	0	0	0	.000	.001	.004	.009	.020	.039	.071	.117	.180	.259
16.4	0	0	0	0	.000	.001	.003	.008	.018	.035	.065	.108	.168	.243
16.6	0	0	0	0	.000	.001	.003	.007	.016	.032	.059	.100	.156	.228
16.8	0	0	0	0	.000	.001	.002	.006	.014	.029	.054	.092	.145	.214
17.0	0	0	0	0	.000	.001	.002	.005	.013	.026	.049	.085	.135	.201
17.2	0	0	0	0	.000	.001	.002	.005	.011	.024	.045	.078	.125	.188
17.4	0	0	0	0	.000	.001	.002	.004	.010	.021	.041	.071	.116	.176
17.6	0	0	0	0	0	.000	.001	.004	.009	.019	.037	.065	.107	.164
17.8	0	0	0	0	0	.000	.001	.003	.008	.017	.033	.060	.099	.153
18.0	0	0	0	0	0	.000	.001	.003	.007	.015	.030	.055	.092	.143
18.2	0	0	0	0	0	.000	.001	.003	.006	.014	.027	.050	.085	.133
18.4	0	0	0	0	0	.000	.001	.002	.006	.012	.025	.046	.078	.123
18.6	0	0	0	0	0	.000	.001	.002	.005	.011	.022	.042	.072	.115
18.8	0	0	0	0	0	.000	.001	.002	.004	.010	.020	.038	.066	.106
19.0	0	0	0	0	0	.000	.001	.002	.004	.009	.018	.035	.061	.098
19.2	0	0	0	0	0	0	.000	.001	.003	.008	.017	.032	.056	.091
19.4	0	0	0	0	0	0	.000	.001	.003	.007	.015	.029	.051	.084
19.6	0	0	0	0	0	0	.000	.001	.003	.006	.013	.026	.047	.078
19.8	0	0	0	0	0	0	.000	.001	.002	.006	.012	.024	.043	.072
20.0	0	0	0	0	0	0	.000	.001	.002	.005	.011	.021	.039	.066

Problems

1. For $n = 200$, $c = 4$, AQL $= 0.5$ percent, and LTPD $= 4$ percent, find α and β.

2. You are responsible for purchasing bearings for the maintenance department of a large airline. The bearings are under contract from a local supplier, and you must devise an appropriate acceptance sampling plan for them. Management has stated in the contract that the acceptable quality level is 1 percent defective. In addition, the lot tolerance proportion defective is 4 percent, the producer's risk is 5 percent, and the consumer's risk is 10 percent.
   - **a.** Specify an appropriate acceptance sampling plan that meets all these criteria.
   - **b.** Draw the OC curve for your plan. What is the resultant producer's risk?
   - **c.** Determine the AOQL for your plan. Assume a lot size of 3,000.

3. The Sunshine Shampoo Company purchases the label that is pasted on each bottle of shampoo it sells. The label contains the company logo, the name of the product, and directions for the product's use. Sometimes the printing on the label is blurred or the colors are not right. The company wants to design an acceptance sampling plan for the purchased item. The acceptable quality level is 5 defectives per 500 labels, and the lot tolerance proportion defective is 5 percent. Management wants to limit the producer's risk to 5 percent or less and the consumer's risk to 10 percent or less.
   - **a.** Specify a plan that satisfies those desires.
   - **b.** What is the probability that a shipment with 3 percent defectives will be rejected by the plan?
   - **c.** Determine the AOQL for your plan. Assume that the lot size is 2,000 labels.

4. Your company supplies sterile syringes to a distributor of hospital supplies. The contract states that quality should be no worse than 0.1 percent defective, or 10 parts in 10,000. During negotiations, you learned that the distributor will use an acceptance sampling plan with $n = 350$ to test quality.
   - **a.** If the producer's risk is to be no greater than 5 percent, what is the lowest acceptance number, c, that should be used?
   - **b.** The syringe production process averages 17 defective parts in 10,000. With $n = 350$ and the acceptance level suggested in part (a), what is the probability that a shipment will be returned to you?
   - **c.** Suppose that you want a less than 5 percent chance that your shipment will be returned to you. For the data in part (b), what acceptance number, c, should you have suggested in part (a)? What is the producer's risk for that plan?

5. A buyer of electronic components has a lot tolerance proportion defective of 20 parts in 5,000, with a consumer's risk of 15 percent. If the buyer will sample 1,500 of the components received in each shipment, what acceptance number, c, would the buyer want? What is the producer's risk if the AQL is 10 parts per 5,000?

6. Consider a certain raw material for which a single-sampling attribute plan is needed. The AQL is 1 percent, and the LTPD is 4 percent. Two plans have been proposed. Under plan 1, $n = 150$ and $c = 4$; under plan 2, $n = 300$ and $c = 8$. Are the two plans equivalent? Substantiate your response by determining the producer's risk and the consumer's risk for each plan.

7. You currently have an acceptance sampling plan in which $n = 40$ and $c = 1$, but you are unsatisfied with its performance. The AQL is 1 percent, and the LTPD is 5 percent.
   - **a.** What are the producer's and consumer's risks for this plan?
   - **b.** While maintaining the same 1:40 ratio of $c:n$ (called the *acceptance proportion*), increase c and n to find a sampling plan that will decrease the producer's risk to 5 percent or less *and* the consumer's risk to 10 percent or less. What producer's and consumer's risks are associated with this new plan?
   - **c.** Compare the AOQLs for your plan and the current plan. Assume a lot size of 1,000 units.

8. For AQL $= 1$ percent, LTPD $= 4$ percent, and $n = 400$, what value(s) of the acceptance number, c, would result in the producer's risk and the consumer's risk *both* being under 5 percent?

9. For AQL $= 1$ percent and $c = 2$, what is the largest value of n that will result in a producer's risk of 5 percent? Using that sample size, determine the consumer's risk when LTPD $= 2$ percent.

10. For $c = 10$ and LTPD $= 5$ percent, what value of n results in a 5 percent consumer's risk?

11. Design a sampling plan for AQL $= 0.1$ percent, LTPD $= 0.5$ percent, producer's risk ≤ 5 percent, and consumer's risk ≤ 10 percent.

12. Design a sampling plan for AQL $= 0.01$ percent (100 parts per million), LTPD $= 0.05$ percent (500 ppm), producer's risk ≤ 5 percent, and consumer's risk ≤ 10 percent. Observe the similarity of this problem to Problem 11. As AQL decreases by a factor of K, what is the effect on the sample size, n?

13. Suppose that AQL $= 0.5$ percent, $\alpha = 5$, LTPD $= 2$ percent, $\beta = 6$ percent, and $N = 1{,}000$.
   - **a.** Find the AOQL for the single-sampling plan that best fits the given parameter values.
   - **b.** For each of the following experiments, find the AOQL for the best single-sampling plan. Change only the parameter indicated, holding all others at their original values.
     - **i.** Change N to 2,000.
     - **ii.** Change AQL to 0.8 percent.
     - **iii.** Change LTPD to 6 percent.
   - **c.** Discuss the effects of changes in the design parameters on plan performance, based on the three experiments in part (b).

14. Peter Lamb is the quality assurance manager at an engine plant. The summer intern assigned to Lamb is a student in operations management at a local university. The intern's first task is to calculate the following parameters, based on the SPC information at the engine plant:

$$\text{AQL} = 0.02 \text{ percent}, \beta = 1 \text{ percent}, \alpha = 2 \text{ percent},$$
$$N = 1000, \text{LTPD} = 2.5 \text{ percent}$$

a. Find the AOQL for the single-sampling plan that best fits the given parameter values.

b. For each of the following experiments, find the AOQL for the best single-sampling plan. Change only the parameter indicated, holding all others at their original values.

i. Change N to 2,000.

ii. Change AQL to 0.3 percent.

iii. Change LTPD to 4 percent.

c. Discuss the effects of changes in the design parameters on plan performance, based on the three experiments in part (b).

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