χ^2- test of Homogeneity of Correlation coefficient : *

Let $r_1, r_2, \cdots r_k$ be k independent estimates of correlation coefficients from independent samples of sizes $n_1, n_2 \cdots n_k$ respectively.

We want to test the null hypothesis

H_0: the correlations coefficients are homogeneous i.e. the sample correlation coefficients are the estimates of the same correlation coefficient ρ from a bivariate normal population.

against H_1: the correlation coefficients are not homogeneous.

Let us consider the transformation

$$z_i = \frac{1}{2}\log_e \frac{1+r_i}{1-r_i} \quad , i=1,2,\cdots k.$$

This transformation is known as Fishers' z transformation. These z_i's are normally distributed about a common mean $\xi = \frac{1}{2}\log_e \frac{1+\rho}{1-\rho}$ and variance $= \frac{1}{n_i-3}$

The minimum variance estimate $\bar{z}$ of the common mean ξ of the z_i's is obtained by weighting the values z_i's inversely with their respectively variances. The estimate of ξ is, therefo.

$$\bar{z} = \frac{\sum_1 z_i(n_i-3)}{\sum_1 (n_i-3)}$$

Hence $(z_i-\bar{z})\sqrt{n_i-3}$; $i=1,2,\cdots k$. are independent standard normal variates ~~Hence~~ and $\sum_1^k (n_i-3)(z_i-\bar{z})^2$ is a χ^2- variated with $k-1$ df.

If χ^2 value thus obtained is greater than α. value of χ^2 for $(k-1)$ df. the hypothesis of homogeneity of correlation coefficients is rejected. If not, the correlat coefficients are supposed, to be homogeneous in which case we combine the sample correlation coefficients to find the estimate $\hat{\rho}$ of the population correlation coefficient ρ.

We have $\bar{z} = \frac{1}{2}\log\left(\frac{1+\hat{\rho}}{1-\hat{\rho}}\right)$

$$\Rightarrow (1+\hat{\rho}) = (1-\hat{\rho})e^{2\bar{z}}$$

$$\Rightarrow \hat{\rho} = \frac{e^{2\bar{z}}-1}{e^{2\bar{z}}+1} = \tanh \bar{z}$$