ASSIGNMENT FOR III- Semester (H)

Q1. Let $X_1, X_2, \ldots X_{21}$ be a random sample from a distribution having the variance 5. Let $\bar{X} = \frac{1}{21}\sum_{i=1}^{21} X_i$ and $S = \sum_{i=1}^{21} (X_i - \bar{X})^2$. Find $E(S)$.

Q2. Let X and Y be independent standard normal random variables. Then find the distribution of $U = \left(\frac{X-Y}{X+Y}\right)^2$

Q3. Let, X_1 and X_2 be two independent random variables having the same mean θ. Suppose that $E(X_1-\theta)^2 = 1$ and $E(X_2-\theta)^2 = 2$. For estimating θ. Consider the estimators $T_\alpha(X_1,X_2) = \alpha X_1 + (1-\alpha)X_2$, for $\alpha \in [0,1]$. Find the value of $\alpha \in [0,1]$, for which the variance of $T_\alpha(X_1,X_2)$ is minimum.

Q4. Let, $x_1 = 3$, $x_2 = 4$, $x_3 = 3.5$, $x_4 = 2.5$ be the observed values of a random sample from the p.d.f.

$$f(x;\theta) = \frac{1}{3}\left[\frac{1}{\theta}e^{-x/\theta} + \frac{1}{\theta^2}e^{-x/\theta^2} + e^{-x}\right],$$

where, $x > 0$, $\theta > 0$.

Find the estimator of θ by method of moments.

Q5. Let, $x_1 = -2$, $x_2 = 1$, $x_3 = 3$, $x_4 = -4$, be the Observed values of a random sample from the distribution having p.d.f.

$$f(x;\theta) = \frac{e^{-x}}{e^{\theta} - e^{-\theta}} \quad ; -\theta \le x \le \theta ; \; \theta > 0$$

Find the maximum likelihood estimate of θ.

Q6. Let, $X_1, X_2, \cdots X_n$ be a random sample from a $N(2\theta, \theta^2)$ population, $\theta > 0$. Check the Consistency of the following estimators

(A) $\frac{1}{n} \sum_{i=1}^{n} X_i$ (C) $\left(\frac{1}{5n} \sum_{i=1}^{n} X_i^2 \right)^{1/2}$

(B) $\left(\frac{5}{n} \sum_{i=1}^{n} X_i^2 \right)^{1/2}$ $\cancel{(D) \left(\frac{1}{5n} \sum_{i=1}^{n} X_i^2 \right)^{1/2}}$

(D) $\frac{1}{5n} \sum_{i=1}^{n} X_i^2$

Q7. Let, $X_1, X_2, \cdots X_{100}$ be a random sample from a $N(2,4)$ population. Let, $\bar{X} = \frac{1}{99} \sum_{i=1}^{99} X_i$

$S = \sqrt{\frac{1}{98} \sum_{i=1}^{99} (X_i - \bar{X})^2}$ and $W = \frac{X_{100} - 2}{S}$. Then find the distribution of W.

Q8. Let, $X_1, X_2, \cdots X_n, X_{n+1}$ be a random sample from a $N(\mu, 1)$ population. If $\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$ and $T = \frac{1}{2} (\bar{X}_n + X_{n+1})$, then for estimating μ Check the Consistency and Unbiasedness of T.

Q9. Let, $X_1, X_2, \dots X_n$ be a random sample of size 'n' from $N(\mu, 16)$ population. If a 95% Confidence interval for μ is $[\bar{X} - 0.98, \bar{X} + 0.98]$. Find the value of 'n'.

Q.10. Let, X_1, X_2, X_3 be i.i.d $N(0, \theta^2)$ random variables, $\theta > 0$. Then, find the value of 'K' for which the estimator $\left(K \sum_{i=1}^{3} |X_i|\right)$ is an unbiased estimator of θ.