Interval Estimation

Let,

$x_1, x_2, \ldots, x_n$ be a random sample of n observations selected from a population ~~p~~ having p.d.f $f(x, \theta)$, $\theta \in S$; where S is the parametric space. The parameter θ or its any function ~~θ~~ $\phi(\theta)$ is unknown. The problem is to estimate the parameter or any parametric function.

Let, $T = t(x_1, x_2, \ldots, x_n)$ be any function of sample observations and $g(T, \theta)$ is the distribution of T. It is mentioned that T, the function of sample statistic is a point estimator of θ. The estimator T may be the best one to estimate θ. However their is discrepancy between T ~~and~~ and θ. Let, this discrepancy is $T - \theta = \epsilon$, where ϵ is a very small quantity. Therefore ~~$P(|T-\theta| < \epsilon)$~~ $P\{(T-\theta) < \epsilon\}$ can be used as a measure of precision of the estimator T. The estimator T becomes more precise if the error $T - \theta$ does not exceed a pre-determined value of ϵ, i.e., $P((T-\theta) < \epsilon)$ should be more. The lower bound of ϵ can be ~~assert~~ ascertained in such a way that

$P\{|T-\theta| < \epsilon\} > 1 - \alpha$, ~~for~~ $\forall \theta \in S$, where α is a small positive quantity.

This probability can be expressed as,

$$P(T-\epsilon \leq \theta \leq T+\epsilon) \geq 1-\alpha \quad \forall \theta \in S.$$

This probability indicates that $T-\epsilon$ and $T+\epsilon$ will contain θ with probability $1-\alpha$.

Here, $T \pm \epsilon$ is the interval based on sample observations which will contain θ with probability $1-\alpha$. The estimation of such an interval of $T \pm \epsilon$ is known as Interval Estimation.

It is observed that in interval estimation the basic point is to decide a statistic T and then decision is needed about its suitability as an estimator of θ. Since it is assumed that T follows the distribution $g(T, \theta)$, for any calculated value of T, the probability of error $|T-\theta|$ can be found out, where the probability for small quantity α, is $P(T-\epsilon \leq \theta \leq T+\epsilon) \geq 1-\alpha$. $\forall \theta \in S$. Here $[T-\epsilon, T+\epsilon]$ are the confidence interval, $T-\epsilon$, $T+\epsilon$ are the confidence limit and $1-\alpha$ is the confidence coefficient. In practice $T-\epsilon$ and $T+\epsilon$ are the functions of sample observations.

Let, $T-\epsilon = l(x_1, x_2, \dots x_n)$ and $T+\epsilon = U(x_1, x_2, \dots, x_n)$ where $l(\cdot)$ is the lower limit of confidence interval and $U(\cdot)$ is the upper limit of confidence interval. The confidence interval $[l, U]$ is correct upto $100 \times (1-\alpha)\%$.

In practice, T is used as an estimator θ and all probable intervals with probability $1-\alpha$ are found out. Out of all probable intervals the smallest one is excepted as an interval ~~ester~~ estimator of θ. If a smallest interval is available it is called 'Tight'. If the distribution of T is symmetric about θ, the confidence interval using T becomes smallest, if the interval is constructed with equal tails, i.e., ϵ is to be determined in such a way that $P(|T-\theta| \leq \epsilon) = 1-\alpha$. This is possible if $P\{(T-\theta) \geq \epsilon\} = \alpha/2$ and $P\{(T-\theta) \leq -\epsilon\} = \alpha/2$. The confidence interval what has been ~~dis~~ discussed above is two sided. In practice one sided confidence interval can also be constructed. The interval is given by $P(T_1 < \phi(\theta)) = 1-\alpha$ and $P(T_2 > \phi(\theta)) = 1-\alpha$. The function T_1 is called one sided upper confidence interval of $\phi(\theta)$.