3 - 9 control limits

Let θ be the quality characteristic of an item manufactured. Let $x_1, x_2, \dots x_n$ be a random sample of size n drawn from the given process and let T be the corresponding statistics computed from the sample such that $E(T) = \mu_T$ and $V(T) = \sigma_T^2$. If the process is under control, T will remain more or less same from sample to sample. and hence the variations in the values of T from sample to sample is due to chance causes

If T is normally distributed with mean μ_T and var. σ_T^2 then

$$P(|T-\mu_T| \leq 3\sigma_T) = 0.9973$$

or $$P(\mu_T - 3\sigma_T \leq T \leq \mu_T + 3\sigma_T) = 0.9973$$

Even when T is non-normal we have from Tchebychef's inequality

$$P(\mu_T - 3\sigma_T \leq T \leq \mu_T + 3\sigma_T) \geq \frac{8}{9}$$

Thus, if the observe T_i lies betn the limits $\mu_T - 3\sigma_T$ and $\mu_T + 3\sigma_T$, it is taken to be a fairly good indication of the non-existence of assignable causes of variation at the time when the ith sample was taken. If the observe T_i falls outside the limits, one suspects the existence of assignable causes and the processed is supposed to be out of control.

$\mu_T + 3\sigma_T$ is called the upper control limit and $\mu_T - 3\sigma_T$ is called the lower control limit.

Types of control Chart :-

Control charts can be classified into two categories –

(i) Control chart for variables :-

Here variables are quality characteristic that can be measured an expressed in numbers. There are 3 commonly used control charts for variables.

(a) Mean chart or $\bar{x}$-chart.

(b) Range chart or R-chart.

(c) Standard deviation chart or σ-chart.

(ii) Control Charts for attributes :-

Attributes usually refer to the classification, of a quality characteristic into any one of two classes, either confirming or not-confirming to specifications as in exceptance or rejection by sapling plans.

There are 3-types of control chart for attributes -

(a) Control chart for fraction defective (or proportion of defectives) or P-chart.

(b) Control chart for the no. of defectives or np chart.

(c) Control chart for the no of defects per unit or C-chart.

Control chart for mean or $\bar{x}$-chart :-

A control chart for mean or $\bar{x}$-chart is the control chart on which the means of the samples drawn from a production process at equal intervals of times are plotted as points. It indicates the fluxuation of the means of the samples about the mean of the process. $\bar{x}$-chart is used to determine whether this fluxuation are due to chance causes or assignable causes.

The construction of $\bar{x}$-chart involves the following steps -

(i) Measurement :- The first step in the construction of $\bar{x}$-chart is to take measurement on the sample unit selected at random from the product of the process. Attempts should be made to minimize the errors in measurements.

(ii) Selection of Samples or subgroup :-

The sample observations are taken at regular intervals of time and are classified into K-subgroups having $n_1, n_2, n_3 --- n_K$ obs. respectively. The subgroups are called rational subgroups. If possible the rational subgroups should be of equal size. In practice 25 samples of size 4 each or 20 samples of size 5 each are taken.

(iii) Calculation of mean range and SD for each sample :-

Let $\bar{x}_i$, R_i and S_i denote respectively the mean, range and SD of the i-th sample. We find $\bar{\bar{x}}$, $\bar{R}$ and $\bar{s}$ as follows

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{K}$$

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_K}{K}$$

$$\bar{s} = \frac{S_1 + S_2 + \cdots + S_K}{K}$$

(iv) Computation of control limits :- We assume that the quality characteristics $X \sim N(\mu, \sigma^2)$. Then μ is the process mean and σ is the process SD. The sample mean $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$ i.e, $E(\bar{x}) = \mu$ and $S.E.(\bar{x}) = \frac{\sigma}{\sqrt{n}}$

Hence the control chart for $\bar{x}$ will be given by

$$LCL = \mu - \frac{3\sigma}{\sqrt{n}} = \mu - A\sigma$$

$$CL = \mu$$

$$UCL = \mu + 3\frac{\sigma}{\sqrt{n}} = \mu + A\sigma$$

where $A = \frac{3}{\sqrt{n}}$

Case - 1

Standards given :- If the values for μ and σ are specified as μ'

and σ', the control chart for $\bar{x}$ will be given by

$$LCL = \mu' - A\sigma'$$

$$CL = \mu'$$

$$UCL = \mu' + A\sigma'$$

where $A = \frac{3}{\sqrt{n}}$.

Case - 2 :- Standard not given :- If μ and σ are not known then we use ~~there~~ estimates provided by the k - saples drawn.

The relations $E(\bar{\bar{x}}) = \mu$

$$E(\bar{R}) = d_2\sigma$$

$$E(\bar{S}) = C_2\sigma$$

(C_2 and d_2 are cont depending on the saple size n)

Provide us an ~~estimal~~ with an estimate of μ and 2 - alternative estimates of σ namely $\hat{\mu} = \bar{\bar{x}}$ ——— ①

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \quad \text{——— ②}$$

and $\hat{\sigma} = \frac{\bar{S}}{C_2}$ ——— ③

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In case one uses the estimates ① and ②, the chart for $\bar{x}$ will be based on.

$$LCL = \bar{\bar{x}} - \frac{3}{\sqrt{n}} \frac{\bar{R}}{d_2} = \bar{\bar{x}} - A_2 \bar{R}$$

$$CL = \bar{\bar{x}}$$

$$UCL = \bar{\bar{x}} + \frac{3}{\sqrt{n}} \frac{\bar{R}}{d_2} = \bar{\bar{x}} + A_2 \bar{R}$$

$$\left(\text{Where } A_2 = \frac{3}{\sqrt{n}\, d_2}\right)$$

On the other hand if ① uses the estimates ① and ③ the chart for $\bar{x}$ will be given by.

$$LCL = \bar{\bar{x}} - \frac{3}{\sqrt{n}} \cdot \frac{\bar{s}}{c_2} = \bar{\bar{x}} - A_1 \bar{s}$$

$$CL = \bar{\bar{x}}$$

$$UCL = \bar{\bar{x}} + \frac{3}{\sqrt{n}} \cdot \frac{\bar{s}}{c_2} = \bar{\bar{x}} + A_1 \bar{s}$$

$$\left(\text{Where } A_1 = \frac{3}{c_2 \sqrt{n}}\right)$$

(V) <u>Construction of $\bar{x}$-chart</u> :- To draw the $\bar{x}$-chart on a graph paper we represent the sample nos on a horizontal scale at the bottom of the control chart. and the statistic ($\bar{x}$) along the

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vertical scale. Sample means $\bar{x}_1, \bar{x}_2$ --- $\bar{x}_K$ are then plotted as dots against the K-sample nos. The central line is drawn as dark horizontal line. The UCL and LCL are drawn as dotted horizontal line at the computed values.

(vi) Interpretation of $\bar{x}$-chart :- If all the ~~points~~ plotted points fall within the control Limits, then we say that the process is in a state of statistical control. But if one or more points fall outside the two dotted lines, it indicate that the process is out of control. and the assignable causes are to be traced out. Also any trend depicted by the plotted point indicates presence of assignable causes. Sometimes the plotted point show a cyclic pattern. This is an indication of assignable causes. Once the SQC units reports the presence of assignable causes, the job of finding out the causes is left to the production engineer.