

Range Chart or R-chart

The chart constructed on using the range is termed as R-chart. R-chart is used to control the variability in the quality of products manufactured by a given process. The construction of R-chart involve the following steps -

(1) Measurement :- The first step in the construction of R-chart is to take measurements on the sample units selected at random from the product of the process. Attempt should be made to minimize the errors in measurements.

(2) Selection of Samples :- The sample obs. are taken at regular intervals of time and are classified into k -subgroups

having n_1, n_2, \dots, n_k obs. respectively.

The subgroups are called rational subgroups. Of possible the rational subgroups should be of equal size.

In practice 25 samples of size 4 each or 20 samples of size 5 each are taken.

(3) Calculation of range for each sample:

Let R_1, R_2, \dots, R_k be sample ranges of the k samples. We find $\bar{R} = \frac{R_1 + R_2 + \dots + R_k}{k}$.

(4) Computation of control limits :- In this case, the statistic under consideration is the sample range (\bar{R}) whose sampling distribution is given by

$$E(\bar{R}) = d_2 \sigma$$

$$S.E(\bar{R}) = D \sigma$$

where d_2 and D are functions of n .

Hence the control chart for R will be given by

$$LCL = d_2\sigma - 3D\sigma = D_1\sigma$$

$$CL = d_2\sigma$$

$$UCL = d_2\sigma + 3D\sigma = D_2\sigma$$

where $D_1 = d_2 - 3D$ and $D_2 = d_2 + 3D$

Case - I (standard given):

If σ be the specify value of σ then the R -chart will be based on

$$LCL = D_1\sigma$$

$$CL = d_2\sigma$$

$$UCL = D_2\sigma$$

Case - II (standard not given):

If σ is unknown then it is estimated by $\frac{\bar{R}}{d_2}$, since $E(\bar{R}) = d_2\sigma$. The R -chart will then become

$$LCL = d_2 \times \frac{\bar{R}}{d_2} - 3D \frac{\bar{R}}{d_2}$$

$$= \bar{R} - \frac{3D\bar{R}}{d_2}$$

$$= \bar{R} \left(1 - \frac{3D}{d_2}\right)$$

$$= \bar{R} D_3 \quad \left(\text{where } D_3 = 1 - \frac{3D}{d_2}\right)$$

$$CL = d_2 \times \frac{\bar{R}}{d_2} = \bar{R}$$

$$UCL = d_2 \times \frac{\bar{R}}{d_2} + 3D \times \frac{\bar{R}}{d_2}$$

$$= \bar{R} \left(1 + \frac{3D}{d_2} \right)$$

$$= \bar{R} D_4 \quad \left(\text{where } D_4 = 1 + \frac{3D}{d_2} \right)$$

(5) Construction of R-chart: To draw the R-chart on graph paper, we represent the sample nos. on a horizontal scale and the statistic (R) along the vertical scale. Sample ranges R_1, R_2, \dots, R_k are then plotted as dots against the sample nos. The central line should be drawn as a solid horizontal line. The UCL should be drawn as dotted horizontal line at the computed values. If the subgroup size is 7 or more, the LCL should be drawn on dotted horizontal line. However, it should be noted that if the subgroup size is 6 or less the LCL for R is 0.

(6) Interpretation of R-chart: If all the plotted points fall within the

control limits then we say that the process is in a state of statistical control but if one or more points fall outside the dotted lines, it indicates the process is out of control and the assignable cause to be traced out. Also any trend depicted by the plotted points indicates the presence of assignable causes. Sometimes the plotted points show a cyclic pattern. This is an indication of assignable causes.

Choice between \bar{x} -chart and R -chart:-

In order to examine whether a production process is in a state of control or not, both \bar{x} -chart and R -chart should be studied together. \bar{x} -chart indicates the variations between the samples while R -chart detects variations within the samples. So a production process is said to be under control if both the charts indicate that the process is in a state of control.

In practice it is better to consider R-chart first. If R-chart shows that dispersion (or variation) of the quality in the process is not in a state of control, it is better not to construct \bar{x} -chart until the quality dispersion is brought under control. If a point or a set of points fall outside the control limits in \bar{x} -chart, but R-chart indicates a state of control then we say that the process average, i.e., process level has shifted considerably.