process ...

Control Chart for S.D or σ chart :-

Since S.D is an ideal measure of dispersion, a combination of control cha[rts] for mean and S.D (known as $\bar{x}$ and σ-c[harts]) is theoretically more appropriate than a combination of $\bar{x}$ and R-charts for controlling process average and proces[s] variability.

The construction of σ-ch[art] ~~in~~ involves the following steps-

(i) Measurements: The first step in the construction of $\bar{x}$-chart is to take measurement on the sample units selected at random from the product of the process. Attempt should be made to minimize the errors in measurements.

ii) Selection of samples or subgroup: The sample obs. are taken at regular intervals of time and are classified into K-subgroups having $n_1, n_2, --- n_K$ obs. respectively. The subgroups are called rational subgroups. If possible the rational subgroups should be of equal size. In practice 25 samples of size 4 each for 20 samples of size 5 each are taken.

iii) Calculation of S.D for each sample:-

Let $S_1, S_2, --- S_K$ be the sample standard deviation, we define

$$\bar{s} = \frac{S_1 + S_2 + S_3 + \cdots + S_K}{K}.$$

(iv) Computation of control limits: In this case the statistic under consideration is the sample standard deviation (s) whose sampling distribution is given by

$$E(\bar{s}) = c_2\sigma \quad \text{and}$$

$$S.E(\bar{s}) = \sigma\sqrt{\frac{n-1}{n} - c_2^2}$$

Hence the control chart for σ will be given by -

$$LCL = c_2\sigma - 3\sigma\sqrt{\frac{n-1}{n} - c_2^2} = B_1\sigma$$

where $B_1 = c_2 - 3\sqrt{\frac{n-1}{n} - c_2^2}$

$$CL = c_2\sigma$$

$$UCL = c_2\sigma + 3\sigma\sqrt{\frac{n-1}{n} - c_2^2} = B_2\sigma.$$

where $B_2 = c_2 + 3\sqrt{\frac{n-1}{n} - c_2^2}$

Case - I (Standard given)

If σ' be the specified value of σ then the σ chart will be based on

$$LCL = B_1\sigma'$$

$$CL = c_2\sigma'$$

$$UCL = B_2\sigma'$$

Case - II (standard ~~rest~~ not given)

In this case one uses the estimate $\frac{\bar{s}}{c_2}$ for σ since $E(\bar{s}) = c_2\sigma$. The σ chart will then become -

$$LCL = B_1 \frac{\bar{s}}{c_2} = B_3 \bar{s} \quad \text{where } B_3 = \frac{B_1}{c_2}$$

$$CL = c_2 \frac{\bar{s}}{c_2} = \bar{s}$$

$$UCL = B_2 \frac{\bar{s}}{c_2} = B_4 \bar{s}, \quad \text{where } B_4 = \frac{B_2}{c_2}.$$

(v) Construction of σ - chart : To draw the σ chart on a graph paper we represent the sample nos. on a horizontal scale and the statistic s along the vertical scale. Sample standard deviation $s_1, s_2, --- s_k$ are then plotted as dots against the sample nos. The central line should be drawn as a solid horizontal line. The upper and lower control limits should be drawn as dotted horizontal lines at the computed values.

(vi) Interpretation of σ - chart : If all the plotted point falls within the control limits then we say that the process is in a state of statistical control. But if one or more point falls outside the dotted lines it indicate the process ~~is out~~ of control and the assignable causes are to be traced out. Also any trend depicted by the plotted points indicates presence of assigna causes.

Control charts for attributes :-

(i) control charts for number of defectives or np - charts or D - charts :-

When the quality characteristic is an attribute on each item is recorded as either defective or non-defective, to judge whether the process is in control, one has to as certain whether the population fraction defective p is the same for all subgroups. The judgement may be based on the no. of defectives (d) in the sample (subgroup).

If the sample size be n and the no. of defective items in each the sample be d then $\frac{d}{n}$ will be the sample fraction defective $\hat{p}$. Hence $\hat{p} = \frac{d}{n}$ or $n\hat{p} = d$. The sampling distribution of the statistic is given by -

$$E(d) = np$$

$$\text{and } S.E(d) = \sqrt{np(1-p)}$$

p being the process (population) fraction defective. Hence the control chart will be given by -

$$LCL = np - 3\sqrt{np(1-p)}$$

$$CL = np$$

$$UCL = np + 3\sqrt{np(1-p)}$$

Case - I (standard given)

If p' be the specified value of p then the control limits for np - chart are -

$$LCL = np' - 3\sqrt{np'(1-p')}$$

$$CL = np'$$

$$UCL = np' + 3\sqrt{np'(1-p')}$$

Case - II (standard not given)

Suppose we take K independent random sample each of fixed size n from the given process.

Let di be the no. of defective units in the ith sample. Then the fraction defective p_i for the ith sample is given by

$$p_i = \frac{d_i}{n}$$

$$\therefore d_i = np_i$$

If p is not known then it's unbiased estimate $\bar{p}$ based on K samples is used and is given by

$$\bar{p} = \frac{\Sigma d_i}{\Sigma n} = \frac{\Sigma d_i}{nk} = \frac{\Sigma n p_i}{nk} = \frac{\Sigma p_i}{k}.$$

The lines of the control chart will then be -

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$CL = n\bar{p}$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

If LCL comes out -ve then it is to be taken as zero.

Construction of np-chart :-

The np-chart is drawn on a graph paper by taking the sample no. along the horizontal scale. sample d values are plotted on the graph against the sample nos. The central line is drawn as a dark horizontal line and UCL and LCL are plotted as dotted horizontal lines at the computed values.

Interpretation of np- chart : If all the sample points fall within the control limits, the process is deemed to be in statistical control. If any point lies outside the control limit, it is concluded that the process is out of control. The points which lie outside the upper control limit are called high spots. This points indicated deterioration in the production process. Such a situation should immediately be reported to the production incharge. Again the points which fall below the control line, they are called low spots. This points show an improvement in the production process or give higher assurance of the good quality of the products.