Likelihood ratio test :*

Neyman Pearson has introduced a general method of test construction based on ratio of two density functions for testing a null hypothesis (simple or composite) against an alternative hypothesis (simple or composite). The ratio of the two density function is called likelihood ratio and this test is related to maximum likelihood estima

Let $x_1, x_2 \cdots x_n$ be a random sample of size n drawn from a population with p.d.f. $f(x, \theta_1, \theta_2 \cdots \theta_k)$ where $\theta_i \in \Omega$ $(i = 1, 2, \cdots k)$. Where θ_i's are paramet and Ω is the parametric space. Here the null hypothesis H_0 states that the parameters belongs to some subset ω of the parameter space Ω.

Then we want to test the null hypothesis

$$H_0 : (\theta_1, \theta_2, \cdots \theta_k) \in \omega$$

against the alternative hypothesis

$$H_1 : (\theta_1, \theta_2, \cdots \theta_k) \in \Omega - \omega$$

The likelihood function of the sample observations is given by $L = \prod_{j=1}^{n} f(x_j; \theta_1, \theta_2 \cdots \theta_k)$ —— ①

According to the principle of m.l.e. the likelihood equations for estimating the parameters θ_i's are

$$\frac{\partial \log L}{\partial \theta_i} = 0 \; ; \; (i = 1, 2, \cdots k)$$

Using these maximum likelihood equations we can obtain the m.l.e. for the parameters θ_i's as they are alowed to vary over the parameter space Ω and the subspace ω. Substituting these estimates in the likelihood function (1) we obtain the maximum values of the likelihood functions for variation of the parameters in Ω and ω respectively. Then the criterion for the likelihood ratio test is defined as the quo of these maxima and is given by —

$$\lambda = \frac{L(\omega)}{L(\Omega)} = \frac{\text{max } L \text{ under } H_0}{\text{max } L \text{ unconditionaly}}$$

Where $L(\omega)$ and $L(\Omega)$ are the maxima of the likelihood function (1) w.r.to parameters in the subspace ω and Ω respectively.

The quantity λ is a function of the sample observatio only and does not involve any parameter and as suc λ is a random variable being a function of randa variables. Obviously $\lambda > 0$ and $\omega \in \Omega \Rightarrow L(\omega) \leq L($

Hence $0 < \lambda < 1$

The likelihood ratio test principle states that the null hypothesis $H_0: (\theta_1, \theta_2 \cdots \theta_k) \in \omega$ is rejected if and only

$$\lambda(x_1, x_2 \cdots x_n) = \lambda < \lambda_0$$

Where λ_0 is so determined that the significance level of the test is given by

$$Pr(\lambda < \lambda_0 / H_0) = \alpha$$

A result that will simplify the test in the case of large samples is that $-2 \log_e \lambda$ is, under H_0, distributed approximately as a χ^2 with $df = \nu$. wher ν denotes the number of parameters specified by H_0.