p-chart for variable sample size

Method - I

Variable control limit :-

In this case we calculate 3σ limit for each sample seperately by using the formula -

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

where $\bar{p} = \frac{\Sigma d_i}{\Sigma n_i}$

	Sample size	no of defectives	p_i	$\frac{\bar{p}(1-\bar{p})}{n_i}$
1^{st}	10	3	$\frac{3}{10} = 0.3$	$\frac{0.18 \times 0.82}{10} = 0.01476$
2^{nd}	15	3	$\frac{3}{15} = 0.2$	$= 0.00984$
3^{rd}	20	4	$\frac{4}{20} = 0.2$	0.00738
4^{th}	25	5	$\frac{5}{25} = 0.2$	0.005904
5^{th}	30	3	$\frac{3}{30} = 0.1$	0.00492

$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i}$

$= \frac{18}{100}$

$= 0.18$

$(1-\bar{p}) = 0.82.$

LCL	UCL
$0.18 - 3\sqrt{0.01476}$	$3 + 3\sqrt{0.01476} = 0.544$
0	
0	0.430
0	0.437
0	0.411
0	0.390

Method – II (standardisation method).

We standardised the statistic p_i by the formula

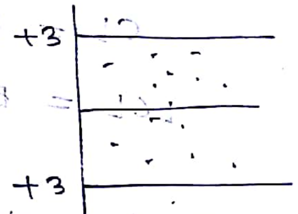
$$z_i = \frac{p_i - E(p_i)}{S.E(p_i)}$$

$$= \frac{p_i - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}}$$

where $\bar{p} = \frac{\sum d_i}{\sum n_i}$ and plot the z values against the corresponding sample no.

Since n is large $z_i \sim N(0,1)$ and hence

UCL = 3, CL = 0 and, LCL = −3.



c - chart

In many manufacturing or inspection situations, the sample size n, i.e, the area of opportunity is very large (since the opportunities for defect to occur numerous) and the probability p of the occurrence of a defect in any one spot is very small such that np is finite. In such situation we know that the statistical pattern of variations in data can be represented by poisson distribution and cosequently 3σ control limit based on poisson distribution are used. Since for a poisson distribution mean and variance are equal, if we assume that c is poisson variate with parameter λ we get

$E(c) = \lambda$ and $v(c) = \lambda$. Thus 3σ control limits for c-chart are given by

$$UCL = E(c) + 3\sqrt{v(c)} = \lambda + 3\sqrt{\lambda}$$

$$CL = E(c) = \lambda$$

$$LCL = E(c) - 3\sqrt{v(c)} = \lambda - 3\sqrt{\lambda}.$$

Case - I (standard specified)

If λ' is the specified value of λ

then $UCL = \lambda' + 3\sqrt{\lambda'}$

$CL = \lambda'$

$LCL = \lambda' - 3\sqrt{\lambda'}$

Case - II (standards not specified)

If the value of λ is not known, it is estimated by the mean no. of defects per unit. Thus, if c_i is the no. of defects observed on the ith $(i = 1, 2, \ldots, k)$ inspected unit then on estimate of λ is given by

$$\hat{\lambda} = \frac{\sum c_i}{K} = \bar{c}.$$

It can be easily seen that $\bar{c}$ is an unbiased estimator of λ.

The control limits in this case are given by

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$CL = \bar{c}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

Since c cannot be negative, if LCL given by above formula comes out to be negative it is regarded to be zero.

The central line is drawn at $\bar{c}$ and UCL and LCL are drawn at the values given by

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

The observed no. of defects on the inspected units are then plotted on the control chart. If all the points fall inside the control limits we can say that the process is in control, otherwise if at least one point is outside the control limit we can say that the process is out of control.