Q. 47. Write a breif note on Random Variable.

Ans. A real-valued function defined over the sample space of a Random experiment is called a random variable (*r.v.*)

An *r.v.* is usually denoted by X, Y, Z, U, V etc.

There are mainly two types if *r.v.s.* (*i*) Discrete (*ii*) Continous

Discrete *r.v.* : An *r.v.* X is said to be discrete if the range of X is either finite or countably infinite.

e.g. If a coin is tossed, the sample space is S = {H, T}, then

$$X(s) = 1 \text{ if } s = H \text{ i.e. if head}$$
$$= 0 \text{ if } s = T \text{ i.e. if Tail}$$

is an *r.v.* Which takes the value 0 and 1.

Continuous *r.v.* : An *r.v.* X is said to be continuous if it can take any value in an interval or in union of intervals on the real line.

e.g. Suppose we are observing the height (X) of a person. Then X is a continous *r.v.* as it can take any value with in a certain range.

[**Note** : Some times we may come across *r.v.s* which are partly discrete and partly continuous such *r.v.s.* are known as Mixed *r.v.*

Consider the game. A player tosses an unbaised coin, if he gets Tail, he gets point '2'. If the outcome is Head, the player spins a balanced spinner that has a scale from 0 to 1 and gets that fraction as a point, where the pointer of the spinner come to rest. Now if X denote the variable of point received by a player, then clearly X is an r.v. which can take values from 0 to 1 and 2. Thus the r.v. X is partly continuous, as it can take all values in the interval [0, 1] and partly discrete as it can take the value 2. Such an r.v. is referred to as **Mixed** r.v.

(1) If X is an *r.v.* then $\frac{1}{X}, X^2, |X|$ are also *r.v.*

(2) If X_1 and X_2 are two *r.v.s.* and a and b are two constants at least one non zero then (*i*) $aX_1 + bX_2$ is also an *r.v.* In paraticular $X_1 + X_2, X_1 - X_2, aX_1, bX_2, aX_1 + b$ etc. are also r.v.s. (*ii*) $X_1 X_2$ is also a *r.v.*

(3) If $X_1, X_2, \ldots, X_n$ are n *r.v.s.* then max $[X_1, X_2, \ldots, X_n]$, min $[X_1, X_2, \ldots, X_n]$ are also *r.v.s.*

(4) If X is an *r.v.* and f a continuous function then $f(X)$ is also an *r.v.*

Q. 48. What is p.m.f. ?

Ans. Probability Mass Function (p.m.f.) : Let X be an discrete *r.v.* taking values $x_1, x_2, \ldots$ Let $p_i = P[X = x_i] = p(x_i), i = 1, 2, \ldots$ be the probability that X takes the value x_i. Then the function $p(.)$ is called the *p.m.f.* of X if it satisfies the following conditions.

(*ii*) $p(x_i) \geq 0$ for all i

(*ii*) $\sum_{i=1}^{\infty} p(x_i) = 1$

The set of values (x_i, p_i), $i = 1, 2, \ldots$ is called the probability distribution of X.

e.g. In tossing a fair coin. S = {H, T}

Let X = 1 if Head occurs

= 0 if Tail occurs

Since the coin is fair the *p.m.f.* is given by

$$P[X=1] = P[H] = \frac{1}{2}$$

$$P[X=0] = P[T] = \frac{1}{2}$$

Note : If X is an *r.v.* and Y = g(X) is also an *r.v.*, then the p.m.f. of Y is given by

$$f_y(y) = \sum_{x : g(x) = y} f_X(x), \; f_X(x) \text{ is the p.m.f. of X}$$

In particular if X has p.m.f. $f_X(x)$ then the p.m.f. of some important functions of X are as follows.]

(*i*) Y = g(X) = — X

$$\therefore f_y(g) = P[Y = y] = P[-X = y] = P[X = -y] = f_X(-y).$$

(*ii*) Y = g(X) = max {0, X}

$$\therefore \quad f_y(y) = P[Y = y] = P[\max(0, X) = y]$$

$$= \begin{cases} \sum_{x \leq 0} f_X(x) & , \text{if } y = 0 \\ f_X(y) & , \text{if } y > 0 \end{cases}$$

(*iii*) Y = g(X) = min {0, X}

$$\therefore \quad f_Y(y) = P[Y = y] = P[\min(O, X) = y]$$

$$= \begin{cases} \sum_{x \geq 0} f_X(x) & , \text{if } y = 0 \\ f_X(y) & , \text{if } y > 0 \end{cases}$$

(*iv*) Y = g(X) = |X|

$$\therefore \quad f_Y(y) = P[Y = y] = P[1 \times 1 = y$$

$$= P[X = y \cup X = -y]$$

$$= \begin{cases} f_X(y) + f_X(-y) & , \text{if } y \neq 0 \\ f_X(0) & , \text{if } y = 0 \end{cases}$$

Q. 49. What is p.d.f. ?

Ans. (**Probability density function (p.d.f.)** : Let X be a continuous *r.v.* taking values in the interval $a \le X \le b$. Then the function $f(x)$ is said to be the *p.d.f.* of X if it satisfies the following conditions.

(*i*) $f(x) \ge 0$ for all $x \in [a, b]$

(*ii*) $\int_a^b f(x)\, dx = 1.$

and for any two numbers $\alpha, \beta (\alpha < \beta)$ in the interval $[a, b]$

$$\int_\alpha^\beta f(x)\, dx = P[\alpha < X < \beta].$$)

e.g. If X is an *r.v.* and $a < X < b$. Then $f(x) = \frac{1}{b-a}$ defines the *p.d.f.*

Since $f(x) = \frac{1}{b-a} \ge 0$ for all $x \in [a, b]$

and $$\int_a^b f(x)dx = \frac{1}{b-a}(b-a) = 1.$$

It should be noted that for an continuous *r.v.* X, $P[X = x] = 0$ and $f(x)dx$ gives the probability that X lies in a very small interval of length dx and is known as the **probability differential** of X.

Q. 50. Define Distribution function. Discuss its properties.

Ans. Distribution function (or Cumulative distribution function) :

The distribution function $F_X(x)$ of an *r.v.* X is defined by

$$F_X(x) = P[X \le x]$$

Properties of distribution function

(*i*) $a < X \le b$ then $P[a < X \le b] = F(b) - F(a)$

(*ii*) If $x \le y$ then $F(x) \le F(y)$ *i.e.* F is monotonically non-decreasing.

(iii) $\lim_{x \to \infty} F(x) = 0$

(iv) $\lim_{x \to \infty} F(x) = 1$

(v) F is continuous to the right *i.e.*

$$F(x+0) = F(x)$$

If X is discrete *r.v.* taking values $x_1, x_2, ...$ with $P[X = x_i] = p_i$ then

$$F(x_i) = P[X \le x_i] = \sum_{t=1}^{i} p_t$$

If X is a continuous *r.v.* and $a \le X \le b$ with *p.d.f.* $f(x)$, Then

$$F(x) = \int_a^x f(x)dx.$$

If X is continuous then

$$\frac{d}{dx}F(x) = f(x), \text{ provided the derivatives exists.}$$

And if X is discrete $P(X=x_j) = F(x_j)—F(x_{j-1})$.

Q. 51. 'The number of heads obtained by tossing two coins is a discrete random variable'. Discuss.

Ans. Tossing two coins is a random experient with sample space S = {HH, HT, TH, TT}. Let X be the number of heads obtained in tossing two coins once. Then clearly X is a *r. v.* and the range of X contains only 3 values namely 0, 1 and 2. Thus the *r.v.* X takes finite number values. Hence is a discrete *r.v.*

Q. 52. Represent the following distribution by a historgam.

Variable (X) :	0	1	2	3
Probability p(X) :	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

Ans. [A probability histogram of the p.m.f is a graphical represntation in which a rectangle of height p(X) and width 1, centred at x is drawn for each value x of X. Such representation often gives better insight of the probability distribution.]