**Two proportion:** Let, $P_1$ and $P_2$ be the proportions of two infinite population. Let, $x_1$ & $x_2$ denote the nos of members having the characteristics A in the random sample of size $n_1$ and $n_2$ drawn independent from the two populations. To test $H_0: P_1 = P_2$. Let, $X = x_1 + x_2$. Under $H_0: p_1 = p_2 = p$, say, ~~x=x~~ say, $X = x_1 + x_2 \sim Bin(n_1+n_2, p)$, where $x_1 \sim B(n_1, p)$, $x_2 \sim Bin(n_2, p)$, independently.

Under $H_0$, the conditional distribution of $X$. given that $x_1 + x_2 = x$ is given by the PMF:

$$P[X_1 = x_1 / x_1 + x_2 = x] = \frac{\binom{n_1}{x_1}\binom{n_2}{x - x_1}}{\binom{n_1+n_2}{x}}, \quad x_1 = 0, 1, \cdots n_1,$$

which is independent of $p'$.

If for given n.s', & the observed value of $x_1$ is $x_{10}$ and that of $X$ is $x_0$, then we have,

$$P_{H_0}[X_1 = x_1 / x_1 + x_2 = x_0],$$

$$= \frac{\binom{n_1}{x_1}\binom{n_2}{x_0 - x_1}}{\binom{n_1+n_2}{x_0}}, \quad x_0 = 0(1)n_1.$$

(a) $\boxed{H_1: P_1 > P_2}$ The p-value $= P_{H_0}[X_1 \geq x_{10} / x_1 + x_2] = x_0]$

$$= \sum_{x_1 \geq x_2} \frac{\binom{n_1}{x_1}\binom{n_2}{x_0 - x_1}}{\binom{n_1+n_2}{x_0}}$$

[N.T, if $P_1 > P_2$, we can except large value of $x_1$ given the total $x_1 + x_2' = x_0$]

If the p-value $\leq \alpha$, reject $H_0$ & if the p-value accept $H_0$ at '$\alpha$' value of singnificance.

(b) $\boxed{H_1 : p_1 < p_2}$ The p-value $= P_{H_0}[X_1 \leq x_{10} / X_1 + X_2 = x_0]$

$$= \sum_{x_1 \leq x_{10}} \frac{\binom{n_1}{x_1}\binom{n_2}{x_0 - x_1}}{\binom{n_1+n_2}{x_0}}$$

[N.T if $p_1 < p_2$, we can except large value of $x_1$ given the total $x_1 + x_2 = x_0$]

If the p-value $\leq \alpha$, reject $H_0$ & if the p-value $> \alpha$, accept $H_0$ at '$\alpha$' level of significance.

(c) $\boxed{H_1 : p_1 \neq p_2}$ the p-value

$$= 2 \min \{ P_{H_0}[X_1 \geq x_{10} / X_1 + X_2 = x_0], \; P_{H_0}[X_1 \leq x_0 / X_1 + X_2 = x_0] \}.$$

If p-value $\leq \alpha$, we reject $H_0$ & if the p-value $> \alpha$, accept $H_0$, at $\alpha$-level of singnificance.

Tests Related to poisson Distribution :-

(1) single population :- Let $X_1, X_2, \cdots X_n$ be a r.s from a $P(\lambda)$ popl$^n$. $\lambda$ unknown. To test $H_0 : \lambda = \lambda_0$.

N.T that, $Y = \sum_{i=1}^{n} X_i \sim P(n\lambda)$

For a given r.s $x_1, x_2, \cdots x_n$, let $y_0$ be the observed value of $Y$.

(a) $\boxed{H_1 : \lambda > \lambda_0}$

If $\lambda > \lambda_0$, we can expect $y > y_0$

the p-value $= P_{H_0}[y \geq y_0]$

$$= \sum_{y=y_0}^{\infty} e^{-n\lambda_0} \cdot \frac{(n\lambda_0)^y}{y!} = p, \text{ say}$$

If $p \leq \alpha$, we reject $H_0$ & if $p > \alpha$, accept $H_0$ at $\alpha$ level.

(b) $\boxed{H_1 : \lambda < \lambda_0}$

If $\lambda < \lambda_0$, we can expect $y < y_0$

the p-value $= P_{H_0}[y \leq y_0]$

$$= \sum_{y=0}^{y_0} e^{-n\lambda_0} \cdot \frac{(n\lambda_0)^y}{y!} = p, \text{ say}$$

If $p \leq \alpha$, reject $H_0$ & if $p > \alpha$, accept $H_0$ at '$\alpha$' level.

(c) $\boxed{H_1 : \lambda \neq \lambda_0}$

p-value $= 2 \min\{P_{H_0}[y \geq y_0], P_{H_0}[y \leq y_0]\}$

If $p \leq \alpha$, reject $H_0$ & if $p > \alpha$, accept $H_0$ at $\alpha$-level of significance.

(2) two populations: let $x_{11}, x_{12}, x_{13}, \ldots x_{1n_1}$ be a r.s from $P(\lambda_1)$ & $x_{21}, x_{22}, \ldots x_{2n_2}$ be a r.s from $P(\lambda_2)$ drawn independently.

Here, $Y_1 = \sum_{i=1}^{n_1} x_{1i} \sim P(n_1\lambda_1)$
$Y_2 = \sum_{i=1}^{n_2} x_{2i} \sim P(n_2\lambda_2)$
independently.

[and condition distn of $Y_1$ given $Y = y$ is

Then $Y = Y_1 + Y_2 \sim P(n_1\lambda_1 + n_2\lambda_2)$ under $H_0$.

$\text{Bin}\left(y, \frac{n_1}{n_1+n_2}\right)$.

<u>To test $H_0 : \lambda_1 = \lambda_2$ :</u>

Under $H_0$, $P[Y_1 = y_1 / Y_1 + Y_2 = y]$

$$= \binom{y}{y_1}\left(\frac{n_1}{n_1+n_2}\right)^{y_1}\left(\frac{n_2}{n_1+n_2}\right)^{y-y_1}, \text{ where } \lambda_1 = \lambda_2 = \lambda \text{ (say)}$$

let, for given r.s.'s, the observed value of $Y$ & $Y_1$ are $y_0$ and $y_{10}$ respectively.

Here, test will be based on $Y_1$ given $Y = y_0$, whose distⁿ is free from $\lambda$, under $H_0 : \lambda_1 = \lambda_2 = \lambda$.

(a) $\boxed{H_1 : \lambda_1 > \lambda_2}$

The p-value $= P_{H_0}[Y_1 \geq y_{10} / Y = y_0]$

$$= \sum_{y_1 \geq y_{10}} \binom{y_0}{y_1}\left(\frac{n_1}{n_1+n_2}\right)^{y_1}\left(\frac{n_2}{n_1+n_2}\right)^{y_0-y_1} = p, \text{ say.}$$

If $p \leq \alpha$, reject $H_0$ & if $p > \alpha$, accept $H_0$ at '$\alpha$' level.

(b) $\boxed{H_1 : \lambda_1 < \lambda_2}$

The p-value, $p = P_{H_0}[Y \leq y_{10} / Y = y_0]$

$$= \sum_{y_1 \leq y_{10}} \binom{y_0}{y_1}\left(\frac{n_1}{n_1+n_2}\right)^{y_1}\left(\frac{n_2}{n_1+n_2}\right)^{y_0-y_1}$$

(c) $\boxed{H_1: \lambda \neq \lambda_2}$

The p-value,

$$p = 2\min\{P_{H_0}[Y_1 \leq y_{10} / y_0],\ P_{H_0}[Y \geq y_{10} / Y = y_0]\}$$

If $p \leq \alpha$, reject $H_0$ & if $p > \alpha$, accept $H_0$ at $\alpha$ level of significence.

Dt: 8/03/21

To test the null hypothesis $H_0: \beta = \beta_0$, against $H_1: \beta \neq \beta_0$

Let, $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ be n pairs of observation's drawn from ~~a~~ (Bivariate Normal Dis$^n$) $BNN(\mu_1, \mu_2, \sigma_1, \sigma_2 \& \rho)$

The sample regression coefficient of $y$ on $x$ is difined by,

$$b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

Now, $E(b) = \beta$ & $V(b) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2}$

Case-(i): $\sigma^2$ is known:

If $\sigma^2$ is known then the test statistic

$$Z = \frac{b - E(b)}{S.E(b)} = \frac{b - \beta}{\sqrt{\frac{\sigma^2}{\sum(x_i - \bar{x})^2}}}$$

Under $H_0$ $Z$ becomes,

$$Z = \frac{b - \beta_0}{\sqrt{\frac{\sigma^2}{\sum(x_i - \bar{x})^2}}} \sim N(0, 1)$$

Decision :

If $|Z| > Z_{\alpha/2}$ we reject $H_0: \beta = \beta_0$ against $H_1: \beta \neq \beta_0$, ~~we a~~ otherwise we accept the null hypothesis :

Case-II ($\sigma^2$ is unknown) :

If $\sigma^2$ is unknown then it is estimated by $\hat{\sigma}^2 = S_{y.x}^2$ ~~variab~~ $= \frac{\sum\{(y_i - \bar{y}) - b(x_i - \bar{x})\}^2}{n-2}$

To test the hypothesis $H_0: \beta = \beta_0$ we use the test statistic under $H_0$ :

$$t = \frac{b - \beta_0}{\sqrt{S_{y.x}^2 / \sum(x_i - \bar{x})^2}} \sim t_{n-2}$$

Decision :

If $|t| > t_{\alpha/2, n-2}$ we reject $H_0: \beta = \beta_0$ against, $H_1: \beta \neq \beta_0$, otherwise we accept the null hypothesis.

Q*( let us consider the following simple regression equation involving to variable X & Y as $Y = \alpha + \beta x + \epsilon$, where $\epsilon$ is the unobservable random error variable & we assumed that $\epsilon \sim N(0, \sigma_\epsilon^2)$.

we assum that,(i) Y must me a random variable, but X may be either non-random or random.

(ii) Distn^n Y given $X = x$ is normal with mean $E(Y/x) = \alpha + \beta x$ & variance $\sigma^2$.) .

Z Theorem:

Here, $\hat{\alpha} = a = (\bar{y} - b\bar{x})$ & $\hat{\beta} = b = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

now, $E(a) = \alpha$ & ~~V(a)=~~

$$V(a) = \sigma^2 \left\{ \frac{1}{n} + \frac{1}{\Sigma(x_i - \bar{x})^2} \right\}$$

Case-1 : ($\sigma^2$ is Known)

To test the hypothesis we use the test statistic. under $H_0 : \alpha = \alpha_0$

$$Z = \frac{a - \alpha_0}{\sqrt{\sigma^2 \left\{ \frac{1}{n} + \frac{1}{\Sigma(x_i - \bar{x})^2} \right\}}} \sim N(0,1)$$

Decion :

If $|z| > z_{\alpha/2}$ we reject $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha \neq \alpha_0$, otherwise we accept the $H_0$.

Case-II : ($\sigma^2$ is unknown)

If $\sigma^2$ is unknown then it is estimated by $\hat{\sigma}^2 = S_{yx}^2 = \dfrac{\sum \{(y_i - \bar{y}) - b(x_i - \bar{x})\}^2}{n-2}$.

To test the hypothesis we use the test statistic under $H_0 : \alpha = \alpha_0$

$$t = \frac{a - \alpha_0}{\sqrt{S_{yx}^2 \left\{ \frac{1}{n} + \frac{1}{\Sigma(x_i - \bar{x})^2} \right\}}} \sim t_{n-2} .$$

Decion :

If $|t| > t_{\alpha/2, n-2}$ we reject $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha \neq \alpha_0$; otherwise we accept the $H_0$.

3: Theorem

To test the hypothesis $H_0 : \beta_1 = \beta_2$ against $H_1 : \beta_1 \neq \beta_2$.

*H.w - - -

Case - II: $\sigma_1^2$ & $\sigma_2^2$ are unknown but known to be equal. i.e $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

(If $\sigma_1^2 = \sigma_2^2 = \sigma^2$, are equal).

then $Z = \dfrac{(b_1 - b_2)}{\cancel{\sigma^2}\sqrt{\cancel{\sigma^2}\left( \frac{1}{\Sigma(x_{1i} - \bar{x}_1)^2} - \frac{1}{\Sigma(x_{2j} - \bar{x}_2)^2}\right.}}$

the pooled estimator of $\sigma^2$ is $S_y$

$$S_{yx}^2 = \frac{\sum_i \{(y_{1i} - \bar{y}_1) - b_1(x_{1i} - \bar{x}_1)\}^2 + \sum_j \{(y_{2j} - \bar{y}_2) - b_2(x_{2j} - \bar{x}_2)\}^2}{n_1 + n_2 - 4}$$

Here, the test statistic under $H_0$ will be

$$t = \frac{b_1 - b_2}{\sqrt{S_{yx}^2 \left\{ \frac{1}{\Sigma(x_{1i} - \bar{x}_1)^2} + \frac{1}{\Sigma(x_{2j} - \bar{x}_2)^2} \right\}}} \sim t_{n_1 + n_2 - 4}$$

Decision:

If $|t| > t_{\alpha/2,\, n_1+n_2-4}$ we reject the $H_0$:

$H_0 : \alpha_1 = \alpha_2$

$\hat{\alpha}_1 = a_1 = \bar{y}_1 - b_1 \bar{x}_1$

$\hat{\alpha}_2 = b_1 = \bar{y}_2 - b_2 \bar{x}_2$

To test the hypothesis $H_0 : \alpha_1 = \alpha_2$, against the alternative $H_1 : \alpha_1 \neq \alpha_2$.

$E(a_1) = \alpha_1$, $E(a_2) = \alpha_2$, ~~$V(a_1$~~

$$V(a_1) = \sigma_1^2 \left\{ \frac{1}{n_1} + \frac{1}{\Sigma(x_{1i} - \bar{x})^2} \right\}$$

$$V(a_2) = \sigma_2^2 \left\{ \frac{1}{n_2} + \frac{1}{\Sigma(x_{2j} - \bar{x})^2} \right\}$$

$H_0 : \alpha_1 = \alpha_2$, statistic: $(a_1 - a_2)$.

$E(a_1 - a_2) = \alpha_1 - \alpha_2$

$$V(a_1 - a_2) = \sigma_1^2 \left\{ \frac{1}{n_1} + \frac{1}{\Sigma(x_{1i} - \bar{x})^2} \right\} + \sigma_2^2 \left\{ \frac{1}{n_2} + \frac{1}{\Sigma(x_{2j} - \bar{x})^2} \right\}$$

test statistic

$$Z = \frac{a_1 + a_2}{\sqrt{\sigma_1^2\left\{\frac{1}{n_1} + \frac{1}{\Sigma(x_{1i} - \bar{x}_1)^2}\right\} + \sigma_2^2\left\{\frac{1}{n_2} + \frac{1}{\Sigma(x_{2j} - \bar{x}_2)^2}\right\}}} \sim N(0,1)$$

Consider unbiasedness

Decion:

If $|Z| \geq Z_{\alpha/2}$, we reject $H_0: \alpha_1 = \alpha_2$ against, $H_1: \alpha_1 \neq \alpha_1$ otherwise we accept $H_0$

Case-II

If $\sigma_1^2$ & $\sigma_2^2$ are unknown but know to be equal.

The pooled estimation of $\sigma^2$ is ~~define~~ given by,

$$S_{yx}^2 = \frac{\Sigma(y_i - \hat{y})^2}{n_1 + n_2 - 4}$$

$$t = \frac{a_1 - a_2}{\sqrt{S_{yx}^2\left[\left\{\frac{1}{n_1} + \frac{1}{\Sigma(x_{1i} - \bar{x}_1)^2}\right\} + \left\{\frac{1}{n_2} + \frac{1}{\Sigma(x_{2j} - \bar{x}_2)^2}\right\}\right.}} \sim t_{n_1 + n_2 - 4}$$

Decion:

$|t| > t_{\alpha/2, n_1 + n_2 + 4}$ we reject the $H_0$ otherwise we accept the $H_0$.

$\chi^2, \tau, F,$ midium (cs)

$\chi^2$, m.l.p, -near, -wkn-maginl, $\chi^2$ division

# Test for two proportions (Exact test)

Dt 10/03/21

## Binomial Distribution:

G.K Hypergeometric di

$H_0 : P_1 = P_2$

$H_1 : P_1 > P_2$

$H_1 : P_1 < P_2$

$H_1 : P_1 \neq P_2$

* supposed $X_1$ & $X_2$ are two independently poisson random variable with $E(X_k) = \mu_k$, $k = 1, 2$. Find the regression coefficient ($\beta$) on $X_1$ on $X_1 + X_2$. Carry out a suitable exact test for $H_0 : \beta = \frac{1}{2}$, against, $H_1 : \beta \neq \frac{1}{2}$.

→ $X_1 \sim P(\mu_1)$
$X_2 \sim P(\mu_2)$ } independent

Regration con
$\frac{E(Y/X)}{E(X/Y)}$

$X_1 + X_2 \sim P(\mu_1 + \mu_2)$

$P(X_1 / X_1 + X_2 = x) \sim B\left(x, \frac{\mu_1}{\mu_1 + \mu_2}\right)$

$E(X_1 / X_1 + X_2 = x) = x \cdot \frac{\mu_1}{\mu_1 + \mu_2} = \beta x$ , $\beta = \frac{1}{2}$

$\therefore \beta = \frac{1}{2}$

$\Rightarrow \frac{\mu_1}{\mu_1 + \mu_2} = \frac{1}{2}$

or, $\mu_1 = \mu_2$

M-12

→ Here, $x_1 \sim P(\mu_1)$ & $x_2 \sim P(\mu_2)$ independently

$\therefore x_1+x_2 \sim P(\mu_1+\mu_2)$,

we know that,

if $x_1 \sim P(\lambda_1)$ & $x_2 \sim P(\lambda_2)$ ~~then~~, indepedtly

then $\left(x_1 / x_1+x_2\right) \sim B\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$

$\therefore \left(x_1 / x_1+x_2 = x\right) \sim B\left(x, \frac{\mu_1}{\mu_1+\mu_2}\right)$

$\therefore$ The regration of $(x_1 / x_1+x_2)$ is

$E\left(x_1 / x_1+x_2 = x\right) = x \frac{\mu_1}{\mu_1+\mu_2}$, which is

linear in $x$.

Clearly $p = \frac{\mu_1}{\mu_1+\mu_2}$.

Hence, $H_0: p = \frac{1}{2}$

$\Rightarrow H_0: \frac{\mu_1}{\mu_1+\mu_2} = \frac{1}{2}$

$\Rightarrow H_0: \mu_1 = \mu_2$

Hence, Here we want to test the hypothesis $H_0: \mu_1 = \mu_2$. against $H_1: \mu_1 \neq \mu_2$.

To sample mean test

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1st unit – Math, Unbaistnes, Cosestency, so,

$\Sigma |x_i| \to N$

$x_{(n)}+1$ is the unbaisted estimator $\theta$

invariant property – $\frac{\Sigma x_i}{n} \left(\frac{\Sigma Y_i}{n}\right.$

maximul BV(N)

method of moment

$\to (r,K)$ $a+b-c$

$a+b+c$.

$5\times 17.41$

## Two sample Mean test:

Let, $x_{11}, x_{12}, \dots x_{1n_1}$ be a random sample of size $n_1$ drawn from $N(\mu_1, \sigma_1^2)$ population.

Let, $x_{21}, x_{22}, \dots x_{2n_2}$ be another random sample of size $n_2$ drawn from $N(\mu_2, \sigma_2^2)$ population.

Both the sample are independend.

Here, we want to test the ~~statistic H~~ null hypothesis $H_0: \mu_1 = \mu_2$, against the alternative $H_1: \mu_1 \neq \mu_2$.

At the pre-specified value of $\alpha$ the null hypothesis $H_0: \mu_1 = \mu_2$ alternatively expressed as $H_0: \mu_1 - \mu_2 = 0$.

Here, the sample mean $\bar{x}_1 \sim N(\mu_1, \sigma_1^2/n_1)$ & $\bar{x}_2 \sim N(\mu_2, \sigma_2^2/n_2)$ to test the hypothesis, we can take $(\bar{x}_1 - \bar{x}_2)$ as an ~~approppreate~~ apprppriate estimator of $(\mu_1 - \mu_2)$ & $\bar{x}_1 - \bar{x}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$

Case-1: ($\sigma_1^2$ & $\sigma_2^2$ are known).

In this case we use the test statistics

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

Under $H_0: \mu_1 = \mu_2$, the statistic becomes.

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

## Decision:

If $|z| \geq z_{\alpha/2}$ we reject $H_0: \mu_1 = \mu_2$ againsts the alternative $H_1: \mu_1 \neq \mu_2$, otherwise we accept $H_0: \mu_1 = \mu_2$.

**Case-2**: ($\sigma_1^2$ & $\sigma_2^2$ are unknown) ~~but known~~

$\sigma_1^2$ & $\sigma_2^2$ are unknown but known to be equal.

i.e. $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Here, we use a pooled estimator of $\sigma^2$.

i.e. $$\hat{\sigma}^2 = s^2 = \frac{\sum(x_{1i} - \bar{x}_1)^2 + \sum(x_{2j} - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

To test the hypothesis we used the test statistic,

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$$

Under $H_0$, $t$ becomes,

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

## Decision:

If $|t| \geq t_{\alpha/2, n_1 + n_2 - 2}$ we reject $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$, otherwise we may accept the null hypothesis.