Relativistic transformation of $\vec{E}$ and $\vec{B}$

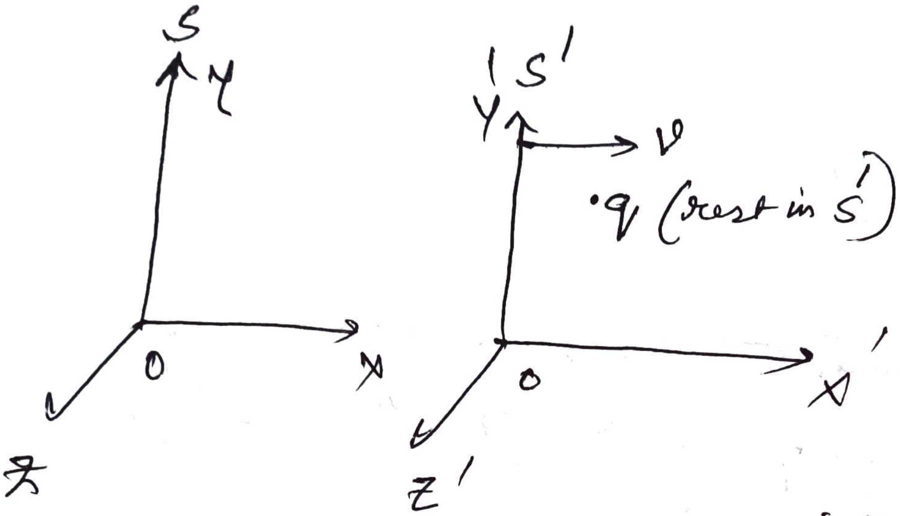
We have the electromagnetic force on a particle ~~of~~ moving with velocity u and charge q is

$$F = q(E + u \times B) \longrightarrow (1)$$

i. Electric force on a a charge q having field intensity E is

$$F_e = qE$$

ii. Magnetic force on a charge q moving with velocity v in a magnetic field ~~B~~ B is $F_B = q(v \times B)$



Description of the situation

1. S and S' are two inertial frame where S' is moving with const translational velocity v w.r. to S along +ve x direction.

2. q a charge is at rest w.r. to S'.

∴ The electric field in S' is E' and magnetic field B'.

$F_e' = qE'$

$F_B' = 0$ as charge is at rest

3. For the same charge w.r.t S

The charge is now in motion with constant translational velocity v along $x x'$ axes.

∵ The three component of velocity of the charge particle is

$$\left.\begin{array}{l} v_x = v \\ v_y = 0 \\ v_z = 0 \end{array}\right\} \longrightarrow (1)$$

According to relativity we have the transformation of force as (Betⁿ S and S')

$$\left.\begin{array}{l} F_x' = F_x \\ F_y' = \dfrac{F_y}{\sqrt{1 - v^2/c^2}} = \gamma F_y \\ F_z' = \gamma F_z \end{array}\right\} (2)$$

Now for the present case

$$F_x = q\left[E_x + (v \times B)_x\right]$$

$$= q\left[E_x + (\underbrace{v_y B_z - v_z B_y}_{0})\right]$$

$$= qE_x$$

$$\therefore F_x' = F_x = qE_x'$$

$$\Rightarrow E_x = E_x' \longrightarrow (3)$$

for

$$F_y = qE_y$$

Using the transformation relation

$$F_y = \frac{F_y'}{\gamma} = \frac{qE_y'}{\gamma}.$$

$$\Rightarrow \frac{qE_y'}{\gamma} = q\left(E_y + (v\times B)_y\right)$$

$$\Rightarrow E_y' = \gamma\left(E_y + \left(\underbrace{v_z}_{0} B_x - v_x B_z\right)\right)$$

$$\Rightarrow E_y' = \gamma\left(E_y - vB_z\right) \longrightarrow (4)$$

$$\text{lly} \quad E_z' = \gamma\left(E_z + vB_y\right) \longrightarrow (5)$$

Combining (3), (4) and (5)

$E_x' = E_x$	$E_x = E_x'$
$E_y' = \gamma\left(E_y - vB_z\right)$	$E_y = \gamma\left(E_y' + vB_z'\right)$
$E_z' = \gamma\left(E_z + vB_y\right)$	$E_z = \gamma\left(E_z' - vB_y'\right)$

$$\left.\begin{aligned} E_{\parallel}' &= E_{\parallel} \\ E_{\perp}' &= \gamma\left(E_{\perp} + (v\times B)_{\perp}\right) \end{aligned}\right\} \longrightarrow (6)$$

The magnetic component of force follow the following set of transformation.

$$B_x' = B_x$$
$$B_y' = \gamma\left(B_y + \frac{v}{c^2} E_z\right)$$
$$B_z' = \gamma\left(B_z - \frac{v}{c^2} E_y\right)$$

$$B_x = B_x'$$
$$B_y = \gamma\left(B_y' - \frac{v}{c^2} E_z'\right)$$
$$B_z = \gamma\left(B_z' - \frac{v}{c^2} E_y'\right)$$

Combining the above three equations

$$\left.\begin{aligned} B_{\parallel}' &= B_{\parallel} \\ B_{\perp}' &= \gamma\left[B_{\perp} - \frac{1}{c^2}(v \times E)_{\perp}\right] \end{aligned}\right\} \quad 7$$