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Manual X

INDIRECT TECHNIQUES FOR DEMOGRAPHIC ESTIMATION

A collaboration of the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat with the Committee on Population and Demography of the National Research Council, United States National Academy of Sciences

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The term "country" as used in the text of this publication also refers, as appropriate, to territories or areas.

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FOREWORD

The Population Division of the Department of International Economic and Social Affairs of the United Nations has been concerned with the measurement of demographic phenomena for over 30 years. The research programme of the Division has been designed to improve the collection, analysis and understanding of demographic trends and policies; and its analytical studies on mortality, fertility, urbanization and population structure have been widely used by both Governments and scholars.

A series of technical manuals has also been published by the Population Division, describing methods of demographic analysis and demographic estimates and projections needed for the purposes of economic and social policies. These manuals deal with methods of estimating total population for current dates, appraising the quality of basic data for population estimates, projecting population by sex and age, estimating basic demographic measures from incomplete data, projecting the economically active population, measuring internal migration, and projecting households and families. The most recent manuals are *Manual VIII: Methods for Projections of Urban and Rural Population*[1] *and Manual IX: The Methodology of Measuring the Impact of Family Planning Programmes on Fertility*.[2]

In the 1960s, many Governments became increasingly aware of the implications of the unprecedented population trends for economic and social development and began to consider policies designed to moderate population growth. However, adequate demographic data, especially in developing countries, were lacking. Recognizing that there was a serious gap between the quantitative information about populations and the amount and quality of data actually available, the Population Commission, at its twelfth session in 1963, recommended that a manual be prepared on methods of estimating fundamental demographic measures from incomplete data, since it was apparent that there was a need for a manual that would make it possible for a demographer/statistician to derive the maximum of reliable information from data in a census or demographic survey. At the request of the United Nations, Ansley J. Coale and Paul Demeny, with the assistance of R. D. Eden, Erna Härm and S. B. Mukherjee, prepared *Manual IV: Methods of Estimating Basic Demographic Measures from Incomplete Data*,[3] which was published in 1967. Its importance and usefulness were quickly recognized by demographers and statisticians throughout the world. *Manual IV* was reprinted in 1969 and 1978 to accommodate the many requests.

Since the publication of *Manual IV*, a number of new techniques for estimating fertility and mortality levels and trends have been developed and many of the earlier methods have been refined. At its nineteenth session in 1977, the Population Commission recommended that *Manual IV* be revised to take into account the advances in methodology and the changes in the demographic situation in recent years.[4] Since such a revision was already under way through a project initiated by the Committee on Population and Demography of the Assembly of Behavioral and Social Sciences of the National Research Council, United States National Academy of Sciences, it was decided that the recommendations of the Population Commission would be best served by joint efforts to develop the present manual.

The Committee on Population and Demography was established in April 1977 under the Assembly of Behavioral and Social Sciences of the National Research Council. Funded by the United States Agency for International Development, the Committee has undertaken three major tasks: (*a*) to evaluate available evidence and prepare estimates of levels and trends of fertility and mortality in selected developing countries; (*b*) to improve the technologies for estimating fertility and mortality when only incomplete or inadequate data exist (including techniques of data collection); and (*c*) to evaluate the factors determining the changes in birth rates in developing countries.

The Committee approached the first task through careful assessment, by internal and external comparison, and through analysis, by application of the most reliable methods known, of all the data sources available. Each of the country studies undertaken therefore consisted of the application of a range of methods to a number of data sets. Best estimates of levels and recent trends were then developed on the grounds of their consistency and plausibility, and the robustness of the individual methods from which they were derived. Several country reports on levels and trends of fertility and mortality have been published.[5]

The Committee's second task, refinement of methodology, was seen as a by-product of achieving the first. The application of particular methods to many different data sets from different countries and referring to different time periods would inevitably provide valuable information about the practical functioning of the

[1] United Nations publication, Sales No. E.74.XIII.3.

[2] United Nations publication, Sales No. E.78.XIII.8.

[3] United Nations publication, Sales No. E.67.XIII.2.

[4] *Official Records of the Economic and Social Council, Sixty-second Session, Supplement No. 4*, para. 186.

[5] Available from the National Academy Press, 2101 Constitution Avenue, N.W., Washington, D.C. 20418, United States of America.

methods themselves. Particular data sets might also require the development of new methodology or the refinement of existing techniques.

Manual X has its origins in the first task of the Committee on Population and Demography, but it has grown to include the presentation of the methodological developments resulting from the Committee's work on its second task. When the Committee was first considering the establishment of panels to prepare country studies, it recognized that much of the valuable methodological work carried out in the preceding decade was widely scattered through technical publications which were difficult to obtain. To facilitate the panel's estimation work, the Committee decided that it would be useful to have a manual of the most up-to-date analytical methods. As the work of the Committee progressed, several new methods were developed and a number of existing methods were refined. In view of the United Nations work in this area, it was decided that the manual, rather than being a mere tool in the preparation of country studies, should become an up-to-date comprehensive demographic estimation manual to be developed with the collaboration of the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat.

The project that is the subject of this report was approved by the Governing Board of the National Research Council, whose members are drawn from the councils of the National Academy of Sciences, the National Academy of Engineering and the Institute of Medicine. The members of the Committee responsible for the report were chosen for their special competences and with regard for appropriate balance. The report has been reviewed by a group other than the authors, according to procedures approved by a committee consisting of members of the Academies and the Institute.

The United Nations wishes to thank Kenneth Hill and Hania Zlotnik of the Committee staff and James Trussell of the Office of Population Research, Princeton University, who wrote *Manual X*. Hania Zlotnik has also prepared computer programs to apply some of the techniques described in this work. These are published in a separate volume entitled *Computer Programs for Demographic Estimation: A User's Guide*.[6]

The United Nations also wishes to thank the members of the Committee on Population and Demography for their contribution to this important study: Ansley J. Coale (Chairman), Office of Population Research, Princeton University; William Brass, Centre for Population Studies, London School of Hygiene and Tropical Medicine; Lee-Jay Cho, East-West Population Institute, East-West Center, Honolulu; Ronald Freedman, Population Studies Center, University of Michigan; Nathan Keyfitz, Department of Sociology, Harvard University; Leslie Kish, Institute for Social Research, University of Michigan; W. Parker Mauldin, Population Division, The Rockefeller Foundation; Jane Menken, Office of Population Research, Princeton University; Samuel Preston, Population Studies Center, University of Pennsylvania; William Seltzer, Statistical Office, United Nations; Conrad Taeuber, Kennedy Institute, Center for Population Research, Georgetown University; Etienne van de Walle, Population Studies Center, University of Pennsylvania; Robert J. Lapham, Study Director.

Acknowledgement is also due to the United Nations Fund for Population Activities, whose financial grant made possible the printing of this *Manual*.

[6] Report No. 11 of the Committee on Population and Demography; available from the National Academy Press.

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Explanatory notes

The following symbols have been used in the tables throughout the report:

Three dots (. . .) indicate that data are not available or are not separately reported.

A hyphen (-) indicates that the item is not applicable.

A dash (—) indicates that the amount is nil or negligible.

A minus sign (—) indicates a deficit or decrease, except as indicated.

A full stop (.) is used to indicate decimals.

A slash (/) indicates a crop year or financial year, e.g., 1970/71.

Use of a hyphen (-) between dates representing years, e.g., 1971-1973, signifies the full period involved, including the beginning and end years.

Details and percentages in tables do not necessarily add to totals, because of rounding.

Reference to "dollars" ($) indicates United States dollars, unless otherwise stated.

Reference to "billion" indicates a thousand million.

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INTRODUCTION

A. ORIGINS AND SCOPE OF *Manual X*

Manual X was originally conceived as an aid to population experts, mainly from developing countries, to carry out the best possible evaluation and exploitation of local data sources. It was also meant to serve as the framework on which the detailed country reports prepared by the Committee on Population and Demography and its panels should be based. Use of preliminary sections of the *Manual* to carry out the Committee's work programme demonstrated that it is a useful research tool, disseminating the most recent techniques of demographic estimation. The collaboration of the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat with the Committee on Population and Demography assures an even wider audience for the *Manual*, making it yet more important that users understand something of its origins in order to be forewarned of the limitations of the presentation.

Manual X is particularly valuable because it is the most complete compilation to date of techniques suited for the analysis of incomplete or defective demographic data. Even in this respect, however, the *Manual* is not perfect. During the past four years, the field of demographic estimation has expanded considerably, and the task of maintaining the *Manual* completely up-to-date was beyond the capabilities of its authors. Furthermore, as new methods were proposed and tested, it frequently became apparent that their robustness to error was not as high as expected. Therefore, in several instances, methods included in preliminary versions of the *Manual* had to be altered. It is possible that some of the methods included in the present version will require future modifications to improve their performance. Thus, in terms both of scope and of detail, this volume cannot be considered the last word on demographic estimation.

The second quality of *Manual X* is that it is "user-oriented". Emphasis has been given to the way in which estimation techniques are applied. Detailed examples are provided, and the overall tendency has been to include too much detail rather than too little. This strategy contrasts sharply with the practice normally followed in papers published by professional journals, often the only published sources of information about recently developed methods. Obviously, those already familiar with the methods described may consider such a level of detail unnecessary or overwhelming. However, experience has shown that the detail is useful for the less sophisticated user, for whom it is essential that no errors resulting from the faulty application of a method be confounded with those already present in the data used.

As a consequence of this user-oriented approach, a conscious effort has been made to simplify as much as possible the description of each method and of the rationale underlying it. Such a strategy is certainly in agreement with the "how to" character of the *Manual*, but it reduces its theoretical significance. Therefore, the user of *Manual X* will find information concerning the available techniques, the basic hypotheses underlying them, certain guiding principles about cases in which it is not appropriate to use given methods, detailed examples of how to apply the techniques proposed and some guidance as to the interpretation of the results they yield; but even the mastery of the entire *Manual* will provide the user with little insight into how new techniques or alternative approaches to available techniques can be developed. In this sense, the *Manual* is more a tool than a general theory of indirect estimation. The user who is interested in a more theoretical treatment should consult the papers available in the demographic literature, references to which are provided in the text.

A further limitation of the *Manual* is that it deals mainly with the estimation of fertility and mortality in developing countries. There are other demographic processes affecting the populations of these countries (migration, for example) which are not treated here. There is no theoretical reason that the techniques described should not be applied to accurate data from developed countries (and indeed many of them have been, with generally very satisfactory results) except that such applications are not necessary from the point of view of evaluating the data, though they are often of interest in evaluating the methodology itself. One area of demographic analysis for developed countries in which the techniques have proved useful is historical demography, where the data cannot always be assumed to be accurate. As long as the underlying assumptions are more or less met, the methods presented can be applied under any conditions, though their use may not be necessary.

Perhaps the most serious limitation of *Manual X* is that it does not provide sufficient guidance for the assessment of results, an aspect of analysis that is also somewhat perfunctorily considered in the literature. The reason for this omission is that the task of assessment is extremely complex, varying widely from application to application. Hence, it is almost impossible to lay down general rules to perform it. This limitation is exacerbated to some degree by the modular structure of the *Manual*. Indeed, since every section is designed to be as self-contained as possible, most examples are treated independently, leaving little room for the inter-method, intertemporal or interdata comparisons that are so illuminating in actual analysis. Because the country

studies prepared for the Committee on Population and Demography of the National Research Council attempt to establish coherent pictures of the demographic evolution of a population, taking into account all the relevant data available, they provide more insights into the assessment of results from different methods, data sets and time periods, and into the way that sense can be made of a variety of imperfectly consistent estimates. Therefore, it is recommended that users of the *Manual* refer to the country reports listed at the end of this introduction as examples of how the results yielded by the methods described herein can be used effectively in combination rather than in isolation.

B. Definition of indirect techniques in demographic estimation

Demographic estimation consists of the attempt to measure values of basic demographic parameters, such as the birth rate, the death rate or the level of total fertility, under less than perfect conditions. These basic parameters indicate the way in which a population will evolve, in terms of size and age structure, over time.

The term "indirect" used to qualify some of the techniques used in demographic estimation has its origin in the fact that such techniques produce estimates of a certain parameter on the basis of information that is only indirectly related to its value. The classic example is the use of the proportion of children dead among those ever borne by women aged 20-24 years to estimate the probability of dying before age 2. The observed proportion of children dead is clearly related to mortality conditions, but it is not a pure mortality measure because it is affected by other, non-mortality parameters. In order to transform this proportion into the desired life-table function, the other parameters must be allowed for, generally by using procedures founded on demographic models. Therefore, not only is the information used "indirect", but the procedure followed, although considerably simplified in practice, is by no means straightforward theoretically. The extent of indirectness varies greatly, however, among procedures, in terms both of the reliance on models and of the number of unwanted factors that have to be allowed for. The term "indirect" is therefore used to describe any estimation method that depends upon models or uses consistency checks, or indeed uses conventional data in an unconventional way.

C. Need for indirect estimation

Traditionally, demographic estimation has been based on data collected by censuses and by a vital registration system. A continuous registration system usually has the task of recording vital events (births, deaths, marriages, divorces etc.) as they occur. When this system is coupled with periodic counts of the population (censuses), the calculation of demographic parameters becomes possible. Assuming that both the registration of vital events and the census counts were perfect, demographic parameters could be calculated directly from the data reported and there would be no need for indirect estimation. Unfortunately, however, in many countries today, either the data-collection systems described above do not exist or their performance is so poor that the estimates obtained directly from the data they produce are severely flawed.

Some of the possible deficiencies of a vital registration system, where it exists at all, may be outlined. The main deficiency is its failure to record all vital events as they occur. For example, it is well known that births may go unregistered for several years. Only when the child is ready to join the public education system, or some other type of organization for which a birth certificate is required, is the birth reported. In countries where a sizeable proportion of the population has never attended school or been in any way connected with official organizations demanding identity certificates, many births may never be registered. The same may be true of the births of children who die very young; in such cases, the parents may consider the registration of either the birth or the death to be futile. Adult deaths are likely either to be reported near the time of their occurrence or not at all. In most countries, a death must be registered before a burial permit is issued. Hence, more people may find it necessary to report deaths soon after they occur, especially in urban areas where burial grounds are often restricted to certain areas whose administrators are closely supervised by government officials. However, the necessity for a burial permit may be practically non-existent in rural areas, where ties with the government administration are weaker. Therefore, it is not surprising to find that, in spite of the legal necessity of registering deaths, many of them are never recorded. The vital registration system may also be deficient in recording characteristics of events, such as age at death, age of mother at a birth, or mother's parity after a birth.

Censuses, the second component of the traditional demographic estimation input, are also far from yielding perfect data. They suffer mainly from two types of error: the failure to enumerate all the members of the relevant population (though, occasionally, some censuses have also produced population overcounts) and poor age-reporting on the part of the population canvassed. Differential coverage of the population by age and sex is also very often present (young children, especially, tend to be undercounted to a greater extent than the adult population), and its effects cannot always be separated from those of age misstatement.

D. Evolution of indirect estimation and the contents of *Manual X*

Faced with the impossibility of obtaining reasonable measures of demographic parameters directly from the traditional data sources, demographers have developed a set of techniques that allow their indirect estimation. The development of these techniques has taken two main courses: either the search for robust methods to analyse data that have been collected by the traditional systems (such as a method designed to estimate a death

rate from vital-registration data of uncertain accuracy), or the search for questions that can be answered with reasonable accuracy and that provide enough information about a certain demographic phenomenon to permit the indirect estimation of its level (for example, the use of information on the incidence of orphanhood to estimate adult mortality).

Because of their reliance upon special types of questions, indirect methods of estimation of the second type have come to be closely associated with sample surveys. Indeed, the sample survey provides perhaps the best means for collecting the data required for these techniques.[1]

During the first attempts to obtain estimates of demographic parameters from data gathered by sample surveys, the questions tried were the straightforward queries suggested by the traditional data sources. If a birth rate was to be estimated, the number of births occurring during a given year had to be known; hence, the question "How many births have occurred in this household during the past year?" was to be asked. It was soon realized that one could do better by asking each woman of childbearing age in the household: "How many children have you had during the past year?"; and recording her age. With that information, age-specific fertility rates could also be estimated. If an estimate of the death rate was needed, a question on deaths in the household during the year preceding the survey would be used. If a death had occurred, recording the age and sex of the deceased would permit the estimation of age-specific and sex-specific central mortality rates from which a life table could be derived. Thus, a complete demographic profile was at hand, just by asking two or three simple questions.

Unfortunately, the results obtained from these surveys did not match expectations. The death rates obtained were almost invariably too low and the birth rates usually seemed implausible. The failure of the question on deaths was explained by arguing that people do not like to talk of such events; they wish to forget them. Also, it was argued that the occurrence of a death within a household might bring about the household's disintegration, so that the households existing at the time of the survey would be more likely not to have experienced deaths of any of their adult members during the preceding year than would those which were in existence at some time during the course of the year.

A further reason postulated for the poor performance of these questions was the existence of a misconception on the part of the respondent of the length of the time period being alluded to. The vague expression "during the preceding year" might not be interpreted by those being interviewed as exactly the 12 months preceding the interview, but instead as a longer or shorter period, according to cultural factors.

Further trials with other ways of phrasing these questions have shown that their performance may be improved to some extent by making them more specific. For example, the period being considered should be specified as exactly as possible, preferably by quoting dates that have special relevance for the population being interviewed. Thus, when a survey is taking place during the second half of November 1978, for example, it may be better to ask "How many children have you had since 1 January of this year?" than "How many children have you had since 22 November 1977?" The first version of the question does not cover a complete year, but it would perform better than the second in a calendar-conscious society because 22 November is not likely to be a significant date for the respondents. The systematic errors introduced by restricting the information collected to periods that are shorter than a year in length are easier to correct than are those arising from incorrect responses.

Yet, although the data produced by these questions (or by the most recent and best version available to explore recent births—namely, "What was the date of your most recent birth?"—posed to all women of reproductive ages) are better, they are still far from perfect. Omission and recall errors are always present to varying degrees so that special analytical tools are necessary for the assessment and correction of the basic data. Some 20 years ago, Brass[2] proposed a method that made possible the adjustment of the reported data on births in the past year. This method is based on the simple idea of comparing the declared lifetime, or cumulated, fertility of young women (children ever born) with cumulated period fertility obtained from a question on births in the past year. Brass argued that although the number of births reported as occurring in the year before the survey might not be correct, the proportionate error might be more or less constant with the mother's age, so that the age-pattern of recent fertility could be accepted even if the level could not. Similarly, one would expect younger women to report their lifetime fertility more accurately, since their births would have occurred fairly recently, so that their level of lifetime fertility could be considered reliable. The correction proposed by Brass was to adopt the two most reliable parts of the information available, taking the age pattern of fertility from the observed number of births in the past year and scaling it to match the level of fertility indicated by the lifetime fertility reported by younger women.

That idea gave rise to a series of methods exploiting the data on children ever born, obtained by asking women of reproductive ages the question: "How many children, who were born alive, have you ever had?" These methods are presented in chapter II. As far as possible, all of them take into account the usual deficiencies of data on children ever born, such as the tendency of older women to declare fewer children than the number they have really borne. This tendency has

[1] For an in-depth analysis of the various data-collection techniques now available, see Panel on Data Collection, *Collecting Data for the Estimation of Fertility and Mortality*, National Research Council Committee on Population and Demography Report No. 4 (Washington, D.C., National Academy Press, 1981).

[2] W. Brass, "Uses of census or survey data for the estimation of vital rates", (E/CN.14/CAS.4/V.5), paper prepared for the African Seminar on Vital Statistics, Addis Ababa, 14-19 December 1964.

been explained by arguing that older women may either omit those children who have died or report only those who still live with them. Omission may be minimized by asking more specific questions, such as: "How many children do you have who live with you?", "How many children do you have who live elsewhere?", and "How many children have you had who have died?" The sum of the responses to these questions gives the desired number of children ever born.

A by-product of the data on children ever born gathered by the set of three questions just presented is the number of children surviving among those ever born. The ratio of these two numbers, that is, the proportion of the children who are still alive, has long been recognized as an indicator of child mortality, and Brass[3] was the first to suggest a method to transform these ratios (computed for women in each five-year age group) into life-table measures of mortality. Some of the variants of his original method are presented in chapter III. They constitute probably the most powerful techniques of indirect demographic estimation, especially in view of the difficulty in obtaining valid estimates of child mortality from registration data where, as pointed out earlier, deaths of young children are particularly susceptible to omission.

In the area of adult mortality estimation, two approaches were developed almost simultaneously. The first was based on the possibility of gathering data that would indirectly provide information on adult mortality. The demographic sample survey provides an ideal means of collecting these data, because it allows the types of questions that are expected to provide reasonably reliable results to be tested without great expense. So far, two questions have proved useful for the estimation of adult mortality. One proposed by Brass and Hill[4] to estimate female mortality investigates the maternal orphanhood status of each member of the population ("Is your mother alive?"); the other is used to estimate male or female mortality by investigating the survival status of the first spouse of women or men, respectively, who have been married at least once ("Is your first husband still alive?"; "Is your first wife still alive?"). Responses to these questions have been found to exhibit some characteristic biases. Young children whose true mothers have died are very often living with adoptive mothers; and the respondent to a question on survival of mother is not likely to make the distinction when answering, either because he or she simply does not know about the adoption or because he or she feels that the matter should not be reported. (Indeed, with young children, the "respondent" is not really the respondent at all, since most information is provided by an adult household member.) Therefore, the true incidence of orphanhood at younger ages is often underreported and the adult mortality of young adult females cannot be estimated with confidence. Data on widowhood, on the other hand, is affected by remarriage. To avoid this problem, only the survival of the first spouse should be investigated, and the question should make this distinction as clear as possible. Furthermore, in countries where a substantial proportion of the unions are of a consensual type (that is, not legalized) one must make it clear to the respondent that "spouse" and "consensual union partner" are equivalent. When phrased properly, this question can yield data of good quality. All existing methods of adult mortality estimation based on orphanhood and widowhood data are discussed in chapter IV.

In the second approach to adult mortality estimation, Brass[5] tried to make use of already existing data, obtained either by a registration system or by a question on deaths in the past year asked in a survey, on the number of deaths classified by age and sex occurring during a given period (usually a year) to a certain population of known age distribution. A simple model associating the reported age distribution of deaths with the age distribution of the population was developed, based on the assumption that deaths are equally underreported at each age and that the population is stable, and was used as a method allowing the estimation of the extent of relative underregistration. This method and a similar technique proposed by Preston and Coale[6] are presented in chapter V.

Most of the techniques described above are applied to data that are generally gathered by sample surveys, though it is primarily cost factors which prevent them from being gathered by censuses, which are, after all, only a special type of survey. Yet, typically, censuses have not included many of the questions necessary for indirect estimation, in part because of the necessity of maintaining the census questionnaires short in order not to prejudice the overall quality of the data. Hence, it is fairly common for censuses to establish only the age and sex distribution of the population being counted and, perhaps, to gather some information on household structure, as far as demographic information is concerned. Only three questions are required to fulfil this goal: one on age; one on sex; and one exploring the relationship of each reported household member to the head of the household. Almost every census carried out during the twentieth century has included questions on age and sex, so that age distributions of the population by sex are available for many countries for different points in time. The development of mathematical models relating different aspects of population growth to the observed age distribution[7] has given rise to several

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[3] *Ibid*

[4] W. Brass and K. H. Hill, "Estimating adult mortality from orphanhood", *International Population Conference, Liège 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 3, pp. 111-123.

[5] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, North Carolina, Carolina Population Center, Laboratories for Population Statistics, 1975).

[6] Samuel H. Preston, Ansley J. Coale, James Trussell and Maxine Weinstein, "Estimating the completeness of reporting of adult deaths in populations that are approximately stable", *Population Index*, vol. 46, No. 2 (Summer 1980), pp. 179-202.

[7] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966); and Ansley J. Coale, *The Growth and Structure of Human Populations: A Mathematical Investigation* (Princeton, New Jersey, Princeton University Press, 1972).

techniques of estimation based on the age structure of the population at a given time.[8] These techniques often make use of sets of mortality models or of models of age distributions of theoretically stable populations. Such sets are known as "model life tables" and "model stable populations", and some of the sets available are described in chapter I. The estimation techniques that use them are presented in chapter VII.

Traditionally, the information yielded by a census question on relationship to the head of the household was not used for demographic estimation purposes. That situation changed when Cho[9] suggested a method that uses the stated relationship to head of the household to identify, where possible, the mother of each enumerated child. Once the child's mother is identified, the age of the child and that of the mother are available; and it is possible to compute age-specific fertility rates for each of the 10 or 15 years preceding the census whenever estimates of child and female adult mortality are available. In countries where young children are not differentially undercounted, where most young children live with their mothers and where age-reporting is reasonably accurate, this method yields good results. It is especially suited for the study of the age structure of fertility—provided, of course, that age-misreporting is not severe. Since this method requires for its application estimates of demographic parameters that are usually obtained by methods described in earlier chapters, it is presented in chapter VIII.

Lastly, if a country possesses a relatively good death registration system, it may be possible to compare its performance with indicators of mortality provided by successive censuses, since the difference between counts of the same cohort at two different points in time is, in the absence of net migration, only the result of mortality. A method that, by comparing death registration and census counts, permits the estimation of death underregistration and relative census completeness of enumeration[10] is presented in chapter IX.

E. USE OF *Manual X*

Manual X is intended to describe some of the more promising techniques currently available to make indirect estimates of demographic parameters. Each of the techniques presented is based on a more or less simplified model of reality, where by "model" is meant a set of mathematical relations between relevant demographic variables. Some understanding of the model underlying each technique is essential for its adequate use, and to understand a model is to have a clear idea of the hypotheses or assumptions on which it is based. In describing each technique in this *Manual*, the assumptions on which its underlying model is based are always emphasized.

The models underlying the techniques presented often take into account some of the flaws that are likely to be present in actual data. For example, the technique that allows the estimation of total fertility by comparing cumulated current fertility with reported average parity specifically assumes that recent fertility has the right shape but not necessarily the right level. In other words, it takes into account the deficiencies noticed in data referring to recent fertility and, in consequence, it is designed to be robust to the typical errors found in actual data. As far as possible, other techniques are also based on models that make some allowance for the existence of typical data errors and are therefore well suited to permit the estimation of demographic parameters from data sets that are less than ideal.

However, even though these techniques are quite powerful, they do not solve all the problems that arise in demographic estimation. For example, their performance is not totally immune to the effects of age-misreporting, nor is it entirely adequate when differential errors by age are present. Therefore, the analyst using these techniques is well-advised to assess fully the quality of the data being analysed before accepting or rejecting the parameter values yielded by particular indirect methods. If a range of indirect techniques can be applied in a particular case, it is unlikely that their results will be entirely consistent. The choice of a final estimate among the possible candidates will be based on knowledge of the ways the methods generally perform, on knowledge of the particular data sets being used, and on knowledge of the country itself and its customs. Such judgements are based largely on experience, and no attempt has been made in this *Manual* to provide general rules for them beyond indicating how particular errors affect the results of particular methods.

The strategy followed in producing *Manual X* has been to describe in the simplest possible way the assumptions and, if necessary, the actual models on which each technique is based; and to provide the reader with a detailed description of the actual application of each method. The organization of each section describing a technique consists usually of four subsections. The introductory portion describes the basis of the method and its rationale; the second lists the data required for its application; the third describes, in general terms, how the method is applied; and the fourth is an example of its application to an actual data set. Both the subsection describing the computational procedure and that corresponding to the detailed example are divided into different and clearly labelled steps.

As far as possible, the section describing a particular technique is intended to be independent from other sections of the *Manual*, so that the user may refer to it without having to work through the entire volume. Occasionally, when a section is not entirely self-contained or self-explanatory, the reader is referred to

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[8] *Manual IV: Methods of Estimating Basic Demographic Measures from Incomplete Data* (United Nations publication, Sales No. E.67.XIII.2).

[9] Lee-Jay Cho, "The own-children approach to fertility estimation: an elaboration", *International Population Conference, Liège, 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 2, pp. 263-280.

[10] S. Preston and K. Hill, "Estimating the completeness of death registration", *Population Studies*, vol. 34, No. 2 (July 1980), pp. 349-366.

other parts of the *Manual* where the necessary concepts or procedures are explained. However, the user should always read the introductory comments of a chapter before using a method from that chapter, since the general characteristics of the data are described in that section.

In general, all the techniques presented are fairly easy to apply. The procedures described are especially suited for the analyst who possesses a hand calculator and some patience, and who is willing to carry out sometimes lengthy computations carefully. The background required to understand the assumptions and the procedures described in chapters II-IX consists of a familiarity with basic demographic measures and a command of algebra. However, it is realized that to grasp fully the mathematical derivation of the demographic models presented in chapter I, a good command of statistics and of calculus is necessary. This chapter, although presenting some background material that is very useful in understanding the other chapters of the *Manual*, does not conform to their level of simplicity; and although it is recommended that every user read it, full understanding of its contents is not necessary for the successful application of the methods described.

It is hoped that this *Manual* will fulfil two functions: as a didactic tool especially suited for those wishing to learn how to perform indirect demographic estimation; and as a handbook for the experienced demographer who needs a quick guide to the application of a variety of demographic estimation techniques. It is strongly recommended that both types of users read this introduction entirely before approaching the techniques themselves, as it contains the information necessary to understand the conventions and notations used throughout the *Manual*. Furthermore, when only one method in a given chapter is of interest to the user, it is recommended that the introductory section of the chapter be read before focusing on the method itself. This section usually states the assumptions on which the method is based and the errors typically found in the type of data being used. It should not, therefore, be overlooked.

The techniques presented have been grouped into chapters according to the type of data they require and not according to the type of estimates they yield. Therefore, several chapters need to be considered when one is interested in parameter types rather than in data types. For example, age-specific fertility rates may be estimated from data on children ever born or from census or survey data where the identification of own-children is possible; and while all the techniques using data on children ever born to estimate fertility are presented in chapter II, the own-children method of fertility estimation is presented in chapter VIII, together with other reverse-survival techniques. The same observation could be made about adult mortality estimation, which can be performed either when a distribution of deaths by age and sex exists or when data on orphanhood and widowhood or even when successive age distributions are available. It is hoped that table 1 and the annotated index tables of each chapter may make the selection of the techniques applicable to different data sets straightforward.

To conclude, it should be pointed out that although all the examples presented are based on real data, the results obtained from the application of a certain method to an isolated set of information should not be considered definitive. Frequently, the application of a method requires several pieces of information obtained from different data sets or sources; and it is impossible within the scope of this *Manual* to assess in each case whether these external pieces of information are satisfactory. In other instances, the application of a certain method of estimation reveals inconsistencies in the data that make the results suspect. In such cases, further exploratory and explanatory work would be required to resolve these inconsistencies. Yet, since the purpose of *Manual X* is to illustrate the way in which the methods

TABLE 1. SCHEMATIC GUIDE TO *Manual X*

Chapter		Type of information	Estimated parameters
II	Estimation of fertility based on information about children ever born	Children ever born Births in the past year Number of women Total population	Total fertility Age-specific rates Birth rate
III.	Estimation of child mortality from information on children ever born and children surviving	Children surviving Children dead Children ever born Number of women	Probabilities of surviving from birth $_1(2)$; $_1(3)$; $_1(5)$; $_1(10)$, $_1(15)$; $_1(20)$
IV.	Estimation of adult survivorship probabilities from information on orphanhood and widowhood	Respondents with mother alive Respondents with mother dead	Probabilities of surviving from age 25 to age $25 + x$, for $x = 10, 15, \ldots 60$ among females
		Respondents with father alive Respondents with father dead	Probabilities of surviving from age 22.5 (or 37.5) to age $35 + x$ (or $40 + x$) for $x = 10, 15, \ldots 55$ among males

TABLE I (continued)

Chapter	Type of input data	Estimated parameters
	Ever-married female population with first husband alive Ever-married female population with first husband dead	Probabilities of surviving from age 20 to age x, for $x = 25, 30, \ldots, 60$ among males
	Ever-married male population with first wife alive Ever-married male population with first wife dead	Probabilities of surviving from age 20 to age x, for $x = 25, 30, \ldots, 55$ among females
V. Estimation of adult mortality from information on the distribution of deaths by age	Deaths classified by age Population classified by age Growth rate	Adjustment factor for deaths Adjusted age-specific central mortality rates for ages over 10
VI. Derivation of a smooth life table from a set of survivorship probabilities	Estimates of child mortality Estimates of adult mortality from orphanhood, or widowhood, or death registration	Complete life table
VII. Fertility and mortality estimation using model stable age distributions	Growth rate Child mortality ($_1q_0$) Age distribution	Birth rate Death rate Gross reproduction rate Life table
VIII. Estimation of fertility by reverse-survival methods	Population classified by age Estimates of child mortality Growth rate	Birth rate
	Enumerated children by single year of age and single year of mother's age Female population by single years Estimates of child mortality Estimates of female adult mortality	Age-specific fertility rates for the 10 or 15 years preceding enumeration Total fertility for the same time period
IX. Estimation of adult mortality using successive census age distributions	Population enumerated at two points in time five or 10 years apart, classified by five-year age group	Life table
	Population enumerated at two points in time classified by age Recorded intercensal deaths classified by age	Completeness of death registration Completeness of census enumeration Life table

are applied and not to obtain complete, coherent demographic profiles of the cases studied, a thorough validation of the results obtained has not generally been attempted. Hence, although this *Manual* is well suited to instruct the user on how to obtain the pieces of the puzzle, it does not go far enough in showing how to put them together to form a consistent picture of the demographic situation of the population concerned.

F. Definitions and Conventions

This section discusses briefly the conventions used throughout this *Manual* concerning both notation and some general definitions. Of course, the specific definitions needed in particular chapters are given there; but in order to avoid repetition, frequently used symbols are defined in this section.

First to be discussed is notation. Because of the wide variety of concepts used, it has been impossible not to assign different meanings to the same symbol in different chapters. Even though this practice clearly violates strict mathematical conventions, its flexibility far outweighs the slight confusion, if any, that it may cause. In order to avoid such confusion completely, the reader must be aware of the cases in which different meanings are associated with the same symbol. Within any chapter, a given symbol always refers to the same type of data; however, its exact algebraic definition may vary according to indices associated with it. For example, the symbol N usually represents the number of persons in some age group, but $N(x)$ is the symbol used to represent the number of persons in the age group from x to $x+1$; ${}_5N_x$ is the number of persons whose exact ages range from x to $x+5$, $N(x+)$ represents the number of persons aged x and over, and $N(x-)$ is the number of persons below age x. Of course, the exact definition of

each of these variations of the symbol N is stated within the chapter where it is used, but it would be helpful for the user to remember these frequently used variations of meaning.

Another example is the use of P to represent average parity. Usually, $P(i)$ represents the average parity of women whose ages range from $x_1 = 10+5(i)$ to $x_2 = 10+5(i+1)$, although occasionally it represents the average parity of women who were first married between $y_1 = 5(i-1)$ and $y_2 = 5(i)$ years before the survey. Furthermore, in sections where more than one survey is considered, $P(i, t)$ is used to represent the average parity of women in group i at time t.

Some symbols have different meanings in different chapters. Since this *Manual* is organized in a modular fashion, with each chapter or module being largely independent of other chapters, this misuse of symbols is not expected to cause problems. For example, it will be quite clear to the reader that in chapter VII, $C(x)$ is used to represent the proportion of the population under age x, while in chapter V, the letter C is used to represent the estimated completeness factor for death registration. Since each of these definitions of C is used consistently within each chapter, it is unlikely that confusion will arise, as long as it is remembered that chapters should be treated as modules.

On the other hand, some symbols are used consistently throughout the *Manual*. These symbols are defined below:

b: birth rate. More accurately called the "crude birth rate", b is the ratio of the number of births occurring in a population during a given period and the number of person-years lived by the population during that time period. The latter figure is usually approximated by the size of the population at the mid-point of the period, multiplied by the length of the period in years. This rate is usually expressed in births per 1,000 population; that is, its true value is multiplied by 1,000. Yet, in this *Manual*, this convention is not followed. Therefore, all birth rates quoted are numbers smaller than unity, which require multiplication by 1,000 to be comparable to figures cited elsewhere;

d: death rate. The complete name of d is the "crude death rate". It is the ratio of the number of deaths occurring in a population during a certain time period and the person-years lived by the population during that time period. An approximation of this number of person-years lived is often obtained by multiplying the mid-period population by the period length in years. Usually, death rates are also expressed per 1,000 population (that is, their true value is multiplied by 1,000), but this practice is not followed here. Hence, all death rates quoted in the *Manual* are numbers smaller than unity;

r: growth rate. More properly known as the "crude rate of natural increase," r is the difference between the birth and death rates defined above. Crude rates of natural increase are often expressed as percentages. This practice is entirely avoided in this *Manual*, but especially in this case, the reader must be aware of it so as to be able to make the correct comparisons between the figures quoted here and those presented in other sources. As an example, note that a value of $r = 0.0280$, the form in which growth rates are expressed herein, is equivalent to a growth rate of 2.80 per cent in conventional form;

$q(x)$: probability of dying. Throughout the *Manual*, the life-table probability of dying between birth and exact age x is denoted by $q(x)$. Being a probability, the value of $q(x)$ cannot be greater than one. Although its value can also be quoted as per 100 (per cent) or per 1,000, it is not the usual practice to do so. Hence, the reader will find that the $q(x)$ values given here have the same form as those generally used elsewhere;

$l(x)$: probability of surviving. The value of $l(x)$ is, in probability terms, the complement of $q(x)$, that is, $l(x) = 1.0 - q(x)$. By definition, $l(x)$ is the probability of surviving from birth to exact age x; and it is conventional practice to express the $l(x)$ values in per 1,000 or per 10,000 form. This practice is especially prevalent in the construction of life tables where $l(x)$ is also interpreted as the number of persons of exact age x in a stationary population with radix $l(0)$. The arbitrary selection of the value of $l(0)$ amounts to multiplying all the true probability values of $l(x)$ by a constant, in order to avoid using decimal points. In most of this *Manual*, $l(0)$ has the value of one; and therefore, $l(x)$ is a probability value never greater than one. However, an exception to this convention occurs in section D of chapter IX, where the radix of the estimated $l(x)$ values cannot be known, therefore making impossible their translation into true probabilities of survival from birth. The user should make a note of the general convention and of this main exception in order to make allowance for the final position of the decimal point when making calculations or comparisons with respect to other estimates;

e_x: expectation of life. The symbol e_x is used to denote the average expectation of life of a group of survivors on reaching age x. The expectation of life at birth is denoted by e_0. The value of e_x represents the average number of years that the members of the stationary (life-table) population alive at exact age x may expect to live. The reciprocal of e_x ($1.0/e_x$) is the death rate over age x for the stationary population. Another name for e_x is "life expectancy at age x";

${}_5m_x$: central death rates, specific by age. The symbol ${}_5m_x$ is used to represent central death rates calculated directly from the number of deaths occurring to the population in the age group from x to $x+5$ and the person-years lived. When the equivalent measure is calculated for a stationary population (life table), an upper-case M is used to indicate this fact (${}_5M_x$);

P: average parity. Demographers define parity as the number of live births a woman has had. Average parity is, therefore, the average number of live births per woman, or the average number of children ever born per woman. The use of the index i to denote that average parities refer to different age groups of women has become common practice. Therefore, instead of denoting the average parity of women in age group 15–19 by ${}_5P_{15}$, as the usual actuarial notation would dictate, the

symbol $P(1)$ was selected. Ease in typing $P(i)$ rather than ${}_iP_x$ has led to the widespread acceptance of the former notation, and it was adopted for this *Manual*. In fact, it has even been abused, in a sense, because i represents both age and marriage-duration groups, according to the case at hand, and it was extended by using it to index some variables other than parity. Because of the simplicity and frequent use of the indexing variable i in this *Manual* and elsewhere, the user is advised to become familiar with it;

$f(i)$: age-specific fertility rate. In this *Manual*, $f(i)$ represents the age-specific fertility rate of women whose age ranges from $x_1 = 10+5i$ to $x_2 = 10+5(i+1)$. In general, the index i used for parities is used in an identical fashion for age-specific fertility rates. Thus, $f(1)$ represents the age-specific fertility rate for women aged 15-19;

TF: total fertility. Total fertility is, by definition, the average number of children born to women who survive the entire reproductive life span; it can be either a period or a cohort measure. When total fertility is measured on the basis of period data, it is an estimate of the average number of children born to members of a hypothetical, mortality-free cohort that is subject, during its entire reproductive life, to the age-specific fertility rates for the period. Thus, if the age limits of female reproduction are taken as being 15 and 50,

$$TF = 5\sum_{i=1}^{7} f(i);$$

Exp and ln: the exponential and the natural logarithm, respectively. The symbol exp is used to denote the value of e, an irrational number whose first seven digits are 2.718282. This number is defined mathematically as the base of natural logarithms (denoted in this *Manual* by ln, though also commonly denoted by $\log_e$). The use of exponential and logarithmic functions is common in the specification of demographic models and the reader should be acquainted with their mathematical properties.

Even though this list of consistently used symbols is not exhaustive, it does highlight the most important characteristics of the relevant ones. The reader may find it useful to consult this section whenever any of the symbols listed here appears in later chapters.

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Chapter I

DEMOGRAPHIC MODELS

A. General background

Demographic models are an attempt to represent demographic processes in the form of a mathematical function or set of functions relating two or more measurable demographic variables. The primary purpose of modelling is simplification, to reduce a confusing mass of numbers to a few, intelligible basic parameters, or to make possible an approximate representation of reality without its complexity.

Because all demographic models attempt to represent reality, they are based to a greater or lesser extent on actual data. Yet, according to their degree of dependence upon observed data, two broad categories can be distinguished. On the one hand, there are models that can be derived solely from a set of simple assumptions or postulates. An example is the stable population model which arises from assuming that fertility and mortality have remained constant for a sufficiently long period. The proof of the convergence of almost any initial age distribution to a stable state after being subject for a long time to constant fertility and mortality can be carried out mathematically without recourse to any type of observed data.[1] Yet, a model of this sort is powerful only to the extent to which it reflects an actual process. Therefore, even though the stable population model is not derived from actual observations, only the good fit it provides to the age structures of populations that might reasonably be considered "stable" (i.e., having been subject to unchanging fertility and mortality for many years) establishes its value.

On the other hand, there are models that could not be derived at all if suitable data were not available. In this category are most of the model life tables that have been proposed. These models arise from the systematic analysis of observed mortality patterns and from the discovery and exploitation of common patterns present in them.

In general, most models fall between these two extremes; that is, their basis is neither purely empirical nor purely theoretical. In fact, some have evolved from a purely empirical to a purely theoretical foundation. An example of this type of evolution is the nuptiality model first presented by Coale.[2] It arose from the analysis of marriage rates among selected populations. Coale discovered that by suitably changing the spread, the location and the area under the nuptiality schedules of these populations, they could all be made to conform to the same standard shape. Further analysis showed that this shape could be closely approximated by a probability density function corresponding to the sum of a normal and several exponential random variables.

In probability theory, an exponentially distributed random variable is frequently used as a model of waiting-time periods. Hence, the density that approximates the standard nuptiality schedule suggests an appealing description of the process by which marriage takes place: women enter the "marriage market" (they become socially acceptable candidates for marriage) according to a normal distribution; and, once they reach this state, their actual time of entrance into marriage is just the sum of several waiting times (the time to find the right partner, the time to arrange a wedding etc.).

The description of this particular process of model creation seems interesting because it clearly illustrates how profitable the interaction between fact and theory may be. Besides, this model is especially appealing because it achieves an almost perfect amalgamation of reality and mathematical theory. Not only does the final equation arrived at describe the observed data with high accuracy, but once discovered, its interpretation appears most reasonable. The model proposed serves at the same time as a summary and as an explanation of reality. Ideally, every model should fulfil these two purposes.

Unfortunately, the modelling of other demographic phenomena has not been equally successful. Yet, undoubtedly, the aim of researchers in this field is to arrive at models that, while being as economical as possible in the number of parameters they incorporate, are also flexible enough to approximate all the relevant variations observable in real data, and whose form and parameters have meaningful interpretations in reality.

The next sections describe several useful demographic models in the areas of mortality, nuptiality, fertility and population age structure. Attention is focused on those models which have been relevant in the development of indirect estimation techniques, mainly because of the important part such models have played in allowing the simulation of demographic data. These simulated data have been used as a basis for the investigation of the relations existing between, on the one hand, quantities that are measured without difficulty and, on the other hand, more useful demographic parameters whose values are not easily determined directly.

[1] Alfred J. Lotka and F. R. Sharpe, "A problem in age-distribution", *Philosophical Magazine*, vol. 21, No. 124 (April 1911), pp. 435-438; and Ansley J. Coale, *The Growth and Structure of Human Populations. A Mathematical Investigation* (Princeton, New Jersey, Princeton University Press, 1972).

[2] Ansley J. Coale, "Age patterns of marriage", *Population Studies*, vol. XXV, No. 2 (July 1971), pp. 193-214.

B. Mortality models: model life tables

A life table provides a summary description of the effects of age-specific mortality rates upon a birth cohort. The very earliest demographic models attempted to describe in mathematical form the variations of mortality with age, particularly the increase in the risk of dying after childhood. Attempts to describe by a single mathematical function mortality experience throughout life have found it difficult to reproduce the characteristic U or J shape of mortality rates by age. This difficulty led to a new approach in creating mortality models or model life tables. Instead of trying to relate the risk of dying solely to age, risks at a given age were related to the risks observed at other ages or to risks observed in other populations at similar ages. Because the relations explored have, in general, shed no light on a plausible theoretical interpretation of how the process of mortality occurs, most of the model life tables existing to date depend heavily upon empirical data. At least four systems of model life tables have been developed on the principle of narrowing the choice of a life table to those deemed feasible on the basis of examination of mortality risks calculated for actual populations. These systems vary in the range of human experience they encompass, so that one may be more appropriate than another for a particular case. Each of them is described in detail below.

1. *United Nations model life tables*

The first set of model life tables was developed by the Population Division of the United Nations Secretariat during the 1950s.[3] This set, subsequently published in a revised form,[4] is based on a collection of 158 observed life tables for each sex. The model tables were constructed by assuming that the value of each ${}_5q_x$ (the probability of dying between age x and $x+5$ in a life table) is a quadratic function of the previous q value, namely, ${}_5q_{x-5}$ (except for the first two age groups, ${}_1q_0$ and ${}_4q_1$, all the other groups considered are five years in length). This assumption implies that the knowledge of only one mortality parameter (${}_1q_0$ or an equivalent "level" that indexes the ${}_1q_0$ values used) would determine a complete life table. The United Nations model life tables are thus a one-parameter system.

Since the coefficients of the quadratic equations relating each ${}_5q_x$ value to its predecessor were not known a priori, they had to be estimated on the basis of observed data. Regression was used to estimate these coefficients from the 158 mortality schedules available for each sex. Once they were estimated, calculation of the actual model life tables was straightforward: a convenient value of ${}_1q_0$ would be chosen arbitrarily, it would then be substituted in the equation relating ${}_1q_0$ to ${}_4q_1$ so that a value of ${}_4q_1$ would be obtained which, in turn, would be used to generate a value of ${}_5q_5$ through the equation relating ${}_5q_5$ to ${}_4q_1$, and so on.

There are inherent disadvantages in the "chaining" technique used in calculating the United Nations model tables, especially when, as in this case, the quadratic equations relating one parameter to the next are not exact. Although the regression method of fitting equations to data does make some allowance for the existence of errors, it assumes that the distribution of these errors is known and that their mean is zero. The presence of systematic errors with non-zero mean (biases) may severely affect the estimates yielded by fitted regression equations, and their adverse effects are very likely to be augmented by the chaining technique described above whereby errors in one estimate would only contribute to accentuated errors in the next.

Furthermore, the 158 tables used as the data base from which the regression coefficients of the fitted equations were estimated were not all of the same quality. Because many tables from developing countries were used in the data base, mortality data with numerous deficiencies were included; and since large areas of the world did not possess any mortality statistics, life tables for those areas could not be included in the data base.

Owing to these shortcomings and to the fact that a one-parameter system lacks the flexibility required to approximate adequately the variety of cases encountered in reality, the United Nations life tables were soon superseded by other model sets which, though based on a similar approach, tried to avoid the pitfalls encountered by the creators of the first system. However, the United Nations set established the usefulness of a model life-table system.

2. *Coale and Demeny regional model life tables*

The Coale and Demeny[5] regional model life tables were published in 1966. They were derived from a set of 192 life tables by sex recorded for actual populations. That set included life tables from several time periods (39 of them relate to the period before 1900 and 69 refer to the period after the Second World War) and was heavily weighted towards Western experiences. Europe, Northern America, Australia and New Zealand contributed a total of 176 tables. The remaining 16 tables were from Africa and Asia, 3 from Israel; 6 from Japan; 3 from Taiwan Province; and 4 from the white populations of South Africa. The 192 life tables were chosen for inclusion from an original set of 326 tables. Life tables in which age patterns exhibited large deviations from the norm were excluded. All of the 192 life tables selected were derived from the registration data and from the complete enumeration of the populations to which they refer. Most tables covered entire countries, but a few representing the mortality experience of subregions also were included, especially when that experience showed distinctive characteristics that persisted over time.

A preliminary analysis of the tables revealed that four different mortality patterns could be distinguished

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[3] *Age and Sex Patterns of Mortality: Model Life Tables for Under Developed Countries* (United Nations publication, Sales No. 55.XIII.9).

[4] *Manual III: Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.56.XIII.3).

[5] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966).

among them. Those patterns were labelled "North", "South", "East" and "West" because of the predominance of European countries belonging to the various regions in each category. Hence, the adjective "regional" was applied to the entire set.

The countries whose life tables underlie each of the patterns identified and the outstanding characteristics of the patterns are discussed below:

(a) *East model.* The life tables underlying the East model come mainly from Austria, Germany (before 1900), the Federal Republic of Germany (after the Second World War), and northern and central Italy, although some from Czechoslovakia and Poland are also included. When the pattern of these tables is compared with the "standard pattern" (that exhibited by the majority of tables), their deviations from the standard follow a U shape, revealing their relatively high mortality rates in infancy and at older ages (over age 50). A total of 31 tables was used to estimate this model. The life expectancy in those tables ranges from a low of 36.6 years for Bavaria in 1878 to a high of 72.3 years for Czechoslovakia in 1958;

(b) *North model.* The observed life tables underlying the North model come from Iceland (1941-1950), Norway (1856-1880 and 1946-1955) and Sweden (1851-1890). Nine tables were used to derive this pattern of mortality, characterized by comparatively low infant mortality coupled with relatively high child mortality and by mortality rates above age 50 that fall increasingly below those of the standard. The populations displaying this mortality pattern were very probably subject to endemic tuberculosis (positive deviations from the standard pattern in the middle age range, from age 10 to 40, suggest this fact). Therefore, this model is recommended as an adequate representation of mortality in populations where the incidence of this disease is high. Life expectancy in these tables ranges from 44.4 years (Sweden, 1851-1860) to 74.7 years (Norway, 1951-1955);

(c) *South model.* The South model is based on life tables for Spain, Portugal, Italy, southern Italy and the region of Sicily, covering a period from 1876 to 1957. The levels of life expectancy range from 35.7 years (Spain, 1900) to 68.8 years (southern Italy, 1954-1957). A total of 22 tables was used in deriving this model. Their mortality pattern is characterized by high mortality under age 5, low mortality from about age 40 to age 60, and high mortality over age 65 in relation to the standard;

(d) *West model.* The West model is based on the residual tables, that is, those not used in the derivation of the other regional sets. Their mortality patterns do not deviate systematically from the standard pattern obtained when all the available life tables are put together; and, in this sense, they are closer to the standard than those on which other regional sets are based. Furthermore, because this model is derived from the largest number and broadest variety of cases, it is believed to represent the most general mortality pattern. For this reason, the West model is often recommended as a first choice to represent mortality in countries where lack of evidence prevents a more appropriate choice of model. It is of interest that the age pattern exhibited by the West model is very similar to that of the earlier United Nations life tables. Life expectancy in these tables ranges from 38.6 years (Taiwan Province, 1921) to 75.2 years (Sweden, 1959).

Having identified each of the four patterns present in the observed life tables, the coefficients of linear equations relating the values of ${}_nq_x$ to e_{10}, the expectation of life at age 10, and those relating the values of $\log_{10}({}_nq_x)$ to e_{10} were estimated by using least-squares regression. From the equations thus established, it was simple to derive a complete set of ${}_nq_x$ values, and, therefore, a model life table, from any given value of e_{10}. The exact equation used (whether on ${}_nq_x$ or on $\log_{10}({}_nq_x)$) for each section of the age range changed according to some simple criteria;[6] but, essentially, the models derived from these equations depend, within each region, upon only one parameter, namely, e_{10}. Twenty-four values of e_{10} for females were selected so that they would produce e_0 values ranging from 20 to 77.5 years, increasing in steps of 2.5 years.

The female e_{10} values were then used to estimate e_{10} values for males on the basis of the mortality differentials by sex present in the actual data. In this way, pairs of life tables (one for each sex) were generated for each level of mortality considered. For simplicity, the pair of life tables with a female e_0 of 20 was identified as level 1 and that with a female e_0 of 77.5 became level 24. An explanation of how the different functions that constitute a life table were calculated from the ${}_nq_x$ values can be found in the life tables themselves.[7]

In order to give the user a better idea of the relationships between the four different mortality patterns embodied by these models, figures 1-4 show plots of the proportional deviations of the ${}_nq_x$ values of models North, South and East from those of West. In all cases, the ${}_nq_x$ values compared refer to female level 9 (with an e_0 of 40 years). The exact function plotted in figures 1-4 is

$$Z(x) = \left[{}_nq_x / {}_nq_x^W \right] - 1.0$$

where the index W indicates model West.

As figure 1 shows, model North is characterized by a somewhat lower infant mortality than that of West, by significantly higher child mortality (over the age range from 1 to 10) and by adult mortality (over the range from 20 to 50) that, although lower than that of West, is higher than the values associated with either South or East (see figure 4). It is also worth noting that at older ages (over 65) North is the model with the lowest mortality.

Model South (shown in figure 2) exhibits higher child mortality than West (over the range from 0 to 5), the

[6] *Ibid.*

[7] *Ibid.*

Figure 1. Relative deviations of North model values for the probability of dying, ${}_nq_x$, from those of the West model for females, level 9

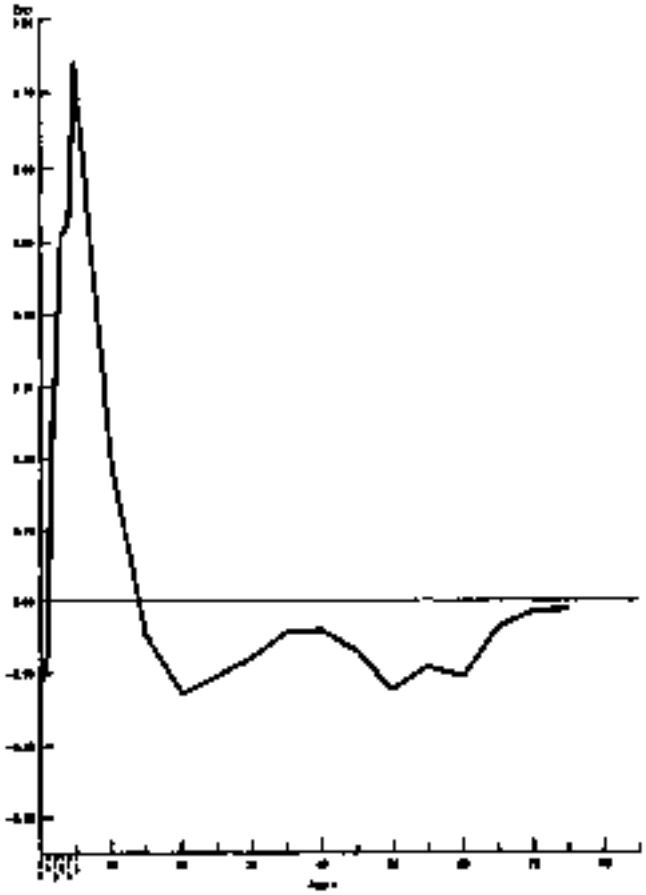
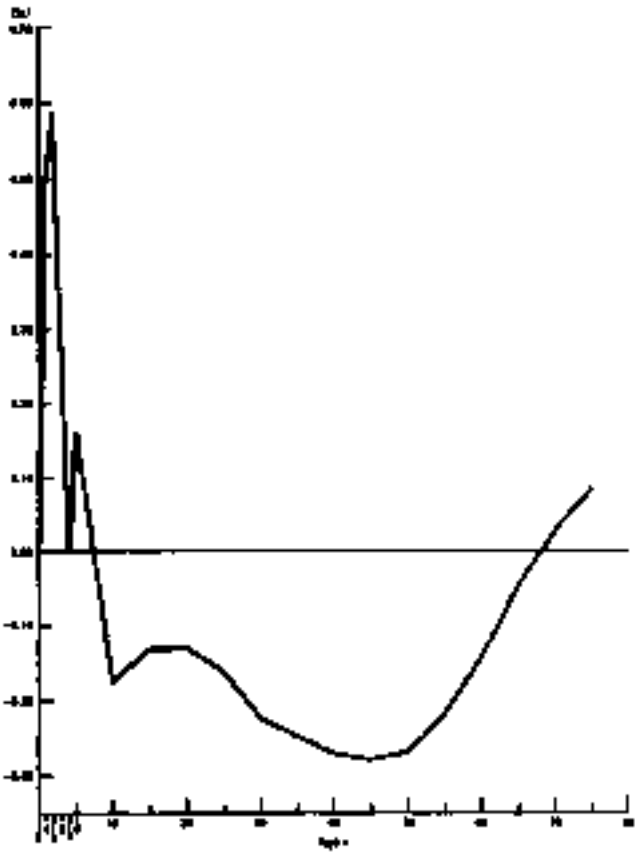


Figure 2. Relative deviations of South model values for the probability of dying, ${}_nq_x$, from those of the West model for females, level 9



lowest adult mortality among all models (see figure 4) and somewhat high mortality over age 65.

Model East (shown in figure 3) displays a relatively high infant mortality, coupled with the lowest child mortality in all models (see figure 4) and an adult mortality that falls between that of South and North. It is also characterized by having the highest mortality at older ages (over age 65).

Note should be taken that, as shown in figure 4, the model having the highest probabilities of dying at adult ages (20-50 years) is the West model (although when mortality is very low, the North model shows higher adult mortality); and only the East model is lower than the West model in terms of child mortality (probabilities of dying between ages 3 and 10).

At the time of its creation, this set of model life tables was probably the most general and flexible model of mortality available, and its widespread use has helped to demonstrate both its strengths and weaknesses. Among the latter, the most important is that, even though more varied than other sets of tables, the patterns embodied by the regional model life tables do not cover the entire range of human experience. For example, Demeny and Shorter[4] found that none of the four model patterns contained in the model life tables adequately reflects the Turkish experience. In general, because the data upon which the regional model life tables are based include very few examples from developing countries, it is possible that the patterns of mortality characteristic of these countries may be different from those represented by the models (see subsection B.5 for a discussion of the new United Nations model life tables).

However, owing to their long-standing use and acceptance, the Coale-Demeny life tables have become a necessary tool in indirect estimation; and they are consistently used in the worked examples presented in this *Manual*.

When employing these models, a problem that always arises is the selection of the mortality pattern that best represents the mortality prevalent in the country or region being studied. As described above, the regional model life tables contain four families or patterns of mortality. The most striking differences between these families are well known; and because the tables themselves are available, they can be investigated further by any analyst. Of course, if there are reasonable estimates of the mortality pattern for a given country, the best model can be selected by comparing the observed pattern to those embodied by the model tables. But when almost no reliable information on the incidence of mortality by age is available, the analyst can do little more than guess which pattern would be most appropriate.

[4] Paul Demeny and Frederic C. Shorter, *Estimating Turkish Mortality, Fertility and Age Structure* (Istanbul, Istanbul University, Statistics Institute, 1968).

Figure 3. Relative deviations of East model values for the probability of dying, ${}_nq_x$, from those of the West model for females, level 9

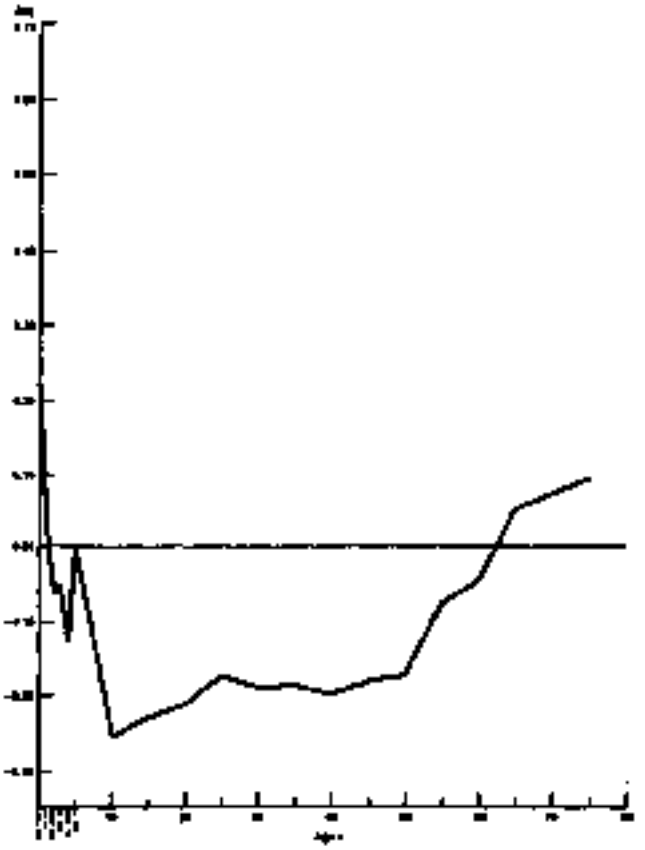
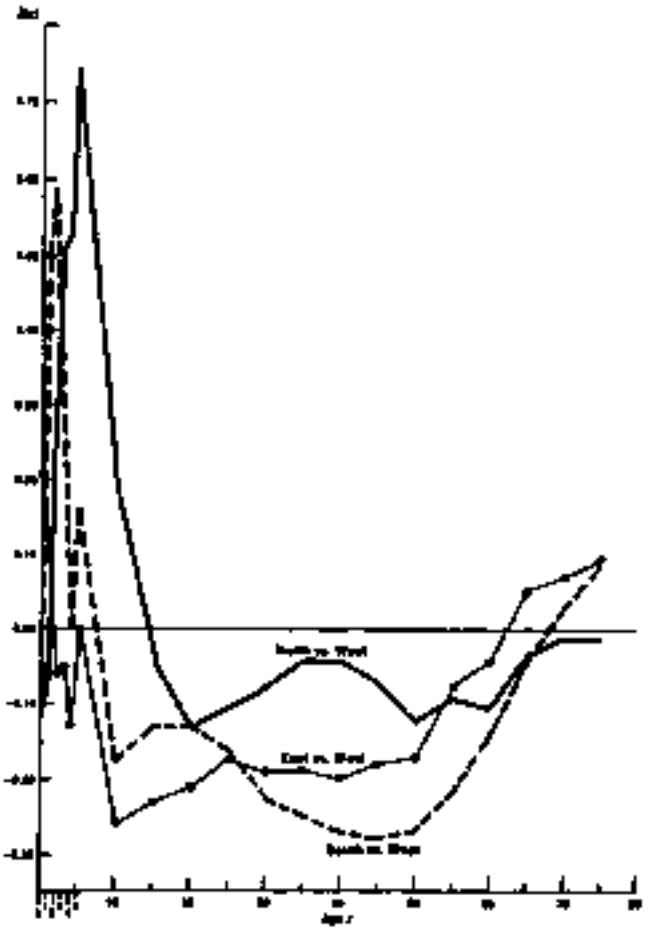


Figure 4. Relative deviations of the North, South and East values for the probability of dying, ${}_nq_x$, with respect to those of the West model for females, level 9



In situations where all extra information is lacking, the use of the West family is recommended because of the relatively wider data base from which it was derived. As for the "extra information" that would permit a more enlightened selection, it can vary considerably in type and quality, ranging, for example, from estimates of age-specific mortality rates derived from vital registration to knowledge of some fairly general facts, such as the extent to which breast-feeding is practised or the probable incidence of tuberculosis. When a set of observed age-specific mortality rates is available (preferably a set adjusted according to the methods described in chapter V), a model mortality pattern may be chosen by comparing the observed ${}_nm_x$ values to those corresponding to model life tables belonging to different families and whose levels (i.e., their expectation of life) roughly correspond to those of the observed rates. This comparison may be carried out by dividing the observed ${}_nm_x$ values by those of the model. Exact agreement between the two values would yield a quotient value of one. Therefore, the family whose quotient values are, on the average, closest to one could be selected as an appropriate representation of observed mortality. A slightly more sophisticated procedure would be to select the model set that minimizes the sum of the squared deviations from 1.0 of the ratios of observed ${}_nm_x$ over model ${}_nM_x$ values.

The selection of an adequate family is especially important when the indirect estimation of infant and child mortality is to be attempted. According to the description given above of the four mortality patterns contained in the regional model life tables, it is evident that they differ most markedly in their values at early ages and in the relation between infant ($_1q_0$) and child ($_4q_1$) mortality. It follows that quite different child mortality estimates may be obtained from the same information according to which family is selected as representative. Furthermore, in this case, reasonably sound external evidence that would permit a solid selection is very hard to obtain, mainly because infant deaths are very often grossly underreported. Currently, in the absence of adequate empirical data for selecting a family of model life tables, only the few general guide-lines given below may be proposed to narrow the possibilities and lead to a reasonable choice:

(*a*) In a population where breast-feeding is common practice and where weaning occurs at a relatively late age (12 months or over), one may reasonably expect child mortality ($_4q_1$) to be relatively higher than infant mortality ($_1q_0$) since breast-feeding may successfully prevent deaths due to malnutrition and infectious diseases among young infants. When weaning takes place, however, the child is less protected from these perils and is more likely to die. In these cases, mortality

in childhood is likely to be well represented by the North family. Yet, it cannot be inferred from these observations that the North family will also provide an appropriate mortality model for other sections of the age range. More information on the incidence of mortality in adulthood is necessary to establish this fact;

(*b*) In some populations today, breast-feeding has been abandoned by a high proportion of the female population; and, from a very early age, infants are fed unsterilized and often inadequate rations of "milk formula". When this practice is adopted by women living in relatively unhealthy conditions, a higher degree of malnutrition and an increase of the incidence of infectious diseases among infants are observed. Under these conditions, a pattern of mortality similar to that of the South family may be a good representation of mortality in childhood;

(*c*) Early weaning may not be the only cause of malnutrition which results in a high child-to-infant mortality ratio. In some populations, breast-feeding is nearly universal but nutritional levels are low and both infant and child mortality are high. For such least developed countries, either the South or the North families of model life tables may be the most appropriate;

(*d*) If it is known that infant mortality is very high in relation to child mortality because of the prevalence of neonatal tetanus or some other cause, the East family may best reflect the actual age pattern of mortality;

(*e*) In the absence of data adequate to determine the most suitable family of model life tables to use for a particular country, one may select the same family as that employed for a neighbouring country with similar cultural and socio-economic characteristics;

(*f*) If little is known about the population under study, the West model is recommended, simply on the grounds of generality.

From these remarks, it is clear that the knowledge about mortality patterns is still fairly limited and that, certainly, better information concerning the mortality experience of populations in developing countries is needed to assess the adequacy of the models now available.

3. *Ledermann's system of model life tables*

Ledermann and Breas[9] used factor analysis to identify the most important variables or factors explaining the variation among a set of 154 observed life tables. The data base was nearly identical to that used for the earlier United Nations tables (see subsection B.1) and therefore had the same advantages and shortcomings.

Five factors were found to explain most of the variability among the observed tables. The first and largest is associated with a general mortality level; the second refers to the relation between child and adult mortality; the third is related to the pattern of mortality at older ages, while the fourth is related to the pattern of mortality under age 5; and lastly, the fifth reflects the differential between male and female mortality in the age range from 5 to 70.

At a later date, Ledermann[10] developed a series of one-parameter and two-parameter model life tables based on the regression analysis of the 154 actual life tables used in his first study of mortality patterns. The model life tables were obtained by estimating the probabilities of dying between ages x and $x+5$, $_5q_x$, for males, females and both sexes combined, through logarithmic regression equations of the following type:

$$\ln {}_5q_x = a_0(x) + a_1(x) \ln Q \qquad \text{(B.1)}$$

for the one-parameter models, and

$$\ln {}_5q_x = b_0(x) + b_1(x) \ln Q_1 + b_2(x) \ln Q_2 \qquad \text{(B.2)}$$

for the two-parameter models, where Q, Q_1 and Q_2 are the independent variables used in each case, and $a_i(x)$ and $b_i(x)$ represent the estimated regression coefficients for the age group from x to $x+5$.

Ledermann's models form a flexible system. While the early United Nations and the four Coale-Demeny regression models are based on just one independent variable ($_1q_0$ and e_{10}, respectively), Ledermann estimated different sets of regression coefficients for equations (B.1) and (B.2), each based on a different independent variable or pair of variables. In the case of the one-parameter models, seven independent variables were used, namely: e_0, $_1q_0$, $_5q_0$, $_{15}q_0$, $_{20}q_{30}$, $_{20}q_{45}$ and m_{50+} (the central mortality rate for ages 50 and over). The two-parameter models were obtained by using the following pairs of independent variables: $_5q_0$ and $_{20}q_{45}$; $_{15}q_0$ and $_{20}q_{30}$; and $_1q_0$ and m_{50+}. Every parameter refers to both sexes, except for $_{20}q_{30}$, which refers only to females. The use of different independent variables to generate each set of model life tables makes it easier for the user to avoid the bias introduced when a model life table is identified by way of an observed value that is not the independent variable used to generate the model. For example, this type of bias is introduced when, in the Coale-Demeny system, a life table is identified on the basis of the observed l_2 value rather than on the basis of the observed e_{10}. However, even though the Ledermann set does provide a wider variety of entry values that minimize the bias in identifying an appropriate model, in practice most of these values are not easily estimated with an adequate degree of accuracy for developing countries, so that the introduction of some bias cannot be avoided.

The Ledermann models also incorporate a feature absent in other tables. They provide not only the estimated values of the probabilities of dying but a measure of the dispersion of the observed values around the estimated values ($2\sigma_x$, where σ_x is the standard error of the $_5q_x$ values estimated through a regression equa-

[9] Sully Ledermann and Jean Breas, "Les dimensions de la mortalité", *Population* (Paris), vol. 14, No. 4 (October-December 1959), pp. 637-682.

[10] Sully Ledermann, *Nouvelles tables-types de mortalité*, Travaux et Documents, Cahier No. 53 (Paris, Institut national d'études démographiques, 1969).

tion). Obviously, this measure refers only to the particular life tables from which the regression coefficients were calculated, and the former do not necessarily cover all possible situations. Nevertheless, the measures of dispersion presented do indicate the possible magnitude of the discrepancies between estimated and actual values.

In addition, the Ledermann tables reflect the sex differentials in age patterns of mortality and the way in which these differentials vary with respect to the overall level of mortality in actual life tables. Thus, for example, the effects of maternal mortality at high mortality levels are translated into a marked excess of female mortality in the early reproductive ages, but such excess disappears in tables of the same model corresponding to lower mortality levels.

This characteristic, however, may be a potential shortcoming of the system, because even though regression coefficients are given for the calculation of separate life tables for each sex, the independent variables used refer, with only one exception, to parameters obtained from data on both sexes combined. Thus, the user is forced to accept the relationships between male and female mortality that the model embodies, relationships that may not always be satisfactory. For instance, it is almost impossible to estimate from Ledermann's models a life table in which the male expectation of life exceeds that for females. When little is known about the sex differentials prevailing in a population, it is highly desirable to analyse data for each sex separately. On these grounds, the Ledermann tables are of limited value for the study of such populations. It may also be noted that, for applications to developing countries, the Ledermann system is not easy to use, as its independent variables, or points of entry into the tables, cannot be readily estimated by the indirect techniques currently available.

4. *Brass logit life-table system*

The main shortcoming of the model life-table systems described above is their dependence upon the type of data that generated them. The rather restricted data base used for this purpose and the fact that the model systems themselves consist of only a finite number of cases which cannot be expected to represent all possible human experience make them less than ideal. Another type of model is needed. Naturally, this model should adequately reflect the patterns found in empirical mortality data. However, it should not be constrained to represent exclusively the patterns these data embody for, as pointed out earlier, the true mortality experience of many populations has not yet been ascertained with any degree of accuracy, and it may or may not strictly conform to patterns observed in countries where accurate measurement has been possible.

A model that provides a greater degree of flexibility is that proposed by Brass and colleagues,[11] better known as the "logit system". Brass attempted to relate mathematically two different life tables. He discovered that a certain transformation of the probabilities of survival to age x ($l(x)$ values in life-table terms) made the relationship between corresponding probabilities for different life tables approximately linear. In other words, if one lets $\lambda(l(x))$ represent some transformation of the value $l(x)$, for empirical data, the linear relationship:

$$\lambda(l^*(x)) = \alpha + \beta\lambda(l(x)) \qquad \text{(B.3)}$$

where $l^*(x)$ and $l(x)$ are two different life tables, and α and β are constants, is approximately true for all values of x if λ is defined specifically as

$$\lambda(l(x)) = \text{logit}\,(1.0 - l(x)) =$$

$$0.5 \ln((1.0 - l(x))/l(x)). \qquad \text{(B.4)}$$

Those familiar with the logit as defined in statistics will notice that λ is just a special case of this function, being calculated for the probabilistic complement of $l(x)$ rather than for $l(x)$ itself, as would be usual practice in statistics where the logit of a probability p is:

$$\text{logit}\,(p) = 0.5 \ln(p/(1.0 - p)). \qquad \text{(B.5)}$$

Assuming that, for any pair of life tables, values of α and β can be found such that equation (B.3) is satisfied, it can be proved that the λ transformation of any life table can be expressed as a linear function of the λ transformation of some "standard" life table. That is, if equation (B.3) holds for every pair of life tables, all life tables can be generated from a single life table by changing the pairs of (α, β) values used. In fact, this proposition is not strictly true because the assumption made, namely, that (B.3) represents an exact relationship between life tables, is not entirely true. Equation (B.3) is only approximately satisfied by pairs of actual life tables, but the approximation is close enough to warrant the use of this relation to study and fit observed mortality schedules.

Before describing how equation (B.3) is used to generate model life tables, a word about the meaning of the parameters α and β is in order. Consider the life tables $l^*(x)$ that can be generated by selecting a specific life table $l(x)$ and calculating $\lambda(l^*(x))$ for different values of α and β. If β were to remain constant and equal to one, for instance, different values of α would produce life tables $l^*(x)$ whose shapes would essentially be the same as the $l(x)$ table used to generate them, but whose overall levels would change (see figure 5). If, on the other hand, α remains fixed and β is allowed to vary, the resulting $l^*(x)$ life tables will no longer display the same shape as $l(x)$. All of the $l^*(x)$ tables will intersect at a single point located somewhere in the central portion of the age range. Therefore, their probabilities of survival will be either lower at younger ages and higher at older ages or lower at younger ages and higher at the older than the standard survival probabilities $l(x)$ from which they are generated (see figure 6). Hence, a changing

11 William Brass and others. *The Demography of Tropical Africa* (Princeton, New Jersey, Princeton University Press, 1968).

Figure 5. Life tables derived through the logit system, letting $\beta = 1.0$ and using the Brass general standard

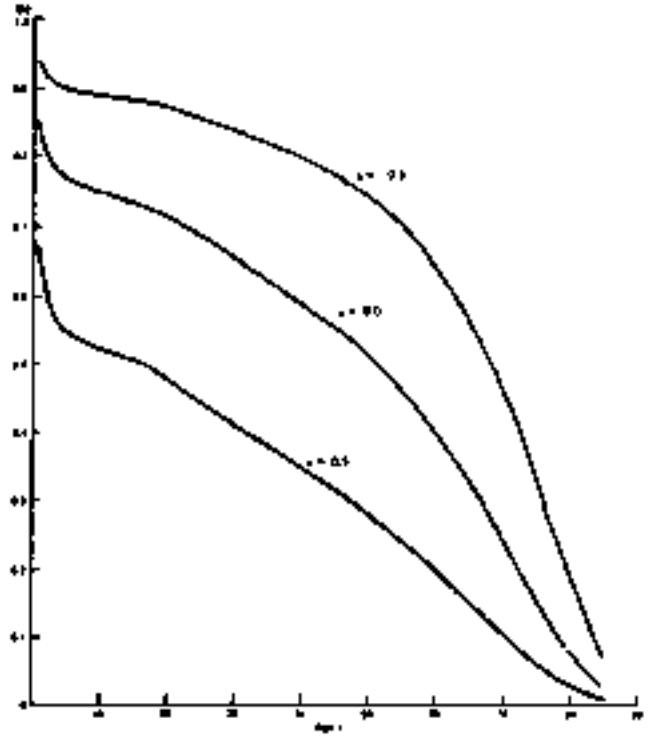
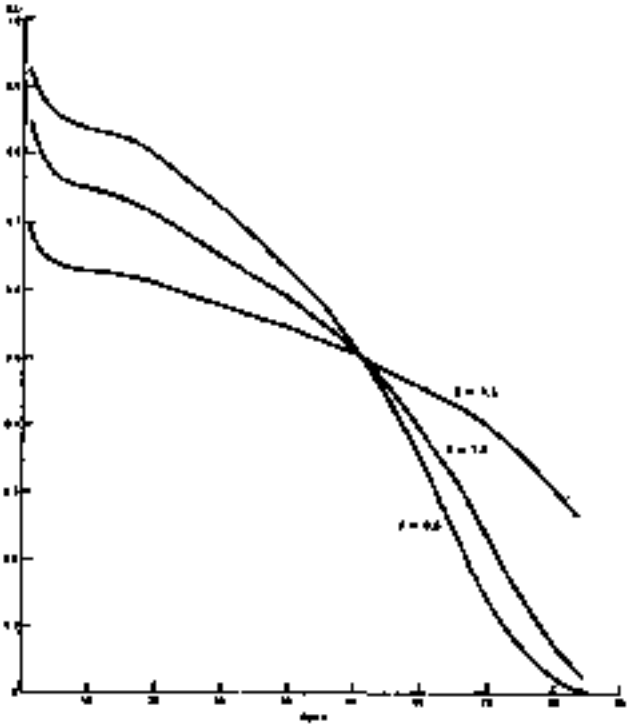


Figure 6. Life tables derived through the logit system, letting $\alpha = 0.0$ and using the Brass general standard



value of β modifies the shape of the generated mortality schedule rather than its level. Naturally, simultaneous changes of α and β will bring about changes in both the level and shape of the mortality schedule being generated.

From equations (B.3) and (B.4), the following expression can be derived:

$$l^*(x) = (1.0 + \exp(2\alpha + 2\beta\lambda(l(x))))^{-1} \qquad (B.6)$$

so that, for any set of $l(x)$ values defining a life table, another set $l^*(x)$ can be obtained by using any pair of α and β values. (Note should be taken that at the endpoints of the age range, where $l(x)$ is either 0 or 1, equation (B.6) cannot be used to calculate $l^*(x)$. Rather, $l^*(0)$ and $l^*(\omega)$ should be arbitrarily set equal to one and to zero, respectively.) Equation (B.6) can be used to generate model life tables simply by selecting an adequate standard. Potentially, any life table can be used as a standard, but for simulation and fitting purposes the standard proposed by Brass[12] is very frequently used. This "general" standard (presented in table 2) is different from the so-called "African" standard also proposed by Brass.[13] The latter is characterized by lower infant mortality and higher child mortality. In this *Manual*, only the Brass "general" standard is used.

Because of the mathematical simplicity of equations (B.4) and (B.6), the use of model life tables generated by means of the λ transformation (also call the "logit" from here on) does not require that the resulting values be available in printed form. However, some such values have been printed. For example, Carrier and Hobcraft[14] produced a set of model tables from the African standard by fixing the value of β at one. This set represents, therefore, a one-parameter system of model life tables.

The simple mathematical form of equation (B.6) also simplifies its use in computer applications. For this reason, life tables generated by the logit system are very often used for simulation purposes. Furthermore, the logit system is particularly appropriate for projecting mortality. If the past and current mortality schedules of a population are known, trends in the α and β parameters can be determined by using the logit model life-table system to fit each mortality schedule, and with some caution these trends can be projected to generate estimates of future mortality.

In this *Manual*, the logit system is used to fit the adjusted mortality rates of a population and to synthesize independent estimates of child and adult mortality into coherent mortality schedules.

5. *United Nations model life tables for developing countries*

As more data of better quality have become available

[12] See K. Hill and J. Trussell, "Further developments in indirect mortality estimation", *Population Studies*, vol. XXXI, No. 2 (July 1977), pp. 313-333.

[13] W. Brass and others, *op. cit.*

[14] Norman H. Carrier and John Hobcraft, *Demographic Estimation for Developing Societies: A Manual of Techniques for the Detection and Reduction of Errors in Demographic Data* (London, London School of Economics, Population Investigation Committee, 1971).

TABLE 2. LOGIT VALUES FOR THE BRASS GENERAL STANDARD LIFE TABLE

Age x (1)	Logit $\frac{1}{2}l_x$ (2)	Age x (3)	Logit $\frac{1}{2}l_x$ (4)	Age x (5)	Logit $\frac{1}{2}l_x$ (6)	Age x (7)	Logit $\frac{1}{2}l_x$ (8)	Age x (9)	Logit $\frac{1}{2}l_x$ (10)
0	–	20	−0.4551	40	−0.1816	60	0.2100	80	1.2375
1	−0.8670	21	−0.4601	41	−0.1674	61	0.2394	81	1.3296
2	−0.7152	22	−0.4248	42	−0.1530	62	0.2701	82	1.4284
3	−0.6552	23	−0.4103	43	−0.1381	63	0.3204	83	1.5346
4	−0.6219	24	−0.3963	44	−0.1229	64	0.3364	84	1.6489
5	−0.6015	25	−0.3829	45	−0.1073	65	0.3721	85	1.7722
6	−0.5879	26	−0.3686	46	−0.0911	66	0.4097	86	1.9053
7	−0.5766	27	−0.3549	47	−0.0745	67	0.4494	87	2.0493
8	−0.5666	28	−0.3413	48	−0.0574	68	0.4912	88	2.2051
9	−0.5578	29	−0.3280	49	−0.0396	69	0.5353	89	2.3740
10	−0.5498	30	−0.3150	50	−0.0212	70	0.5818	90	2.5573
11	−0.5431	31	−0.3020	51	−0.0021	71	0.6311	91	2.7564
12	−0.5365	32	−0.2889	52	0.0177	72	0.6832	92	2.9727
13	−0.5296	33	−0.2759	53	0.0383	73	0.7385	93	3.2079
14	−0.5220	34	−0.2627	54	0.0598	74	0.7971	94	3.4639
15	−0.5134	35	−0.2496	55	0.0821	75	0.8593	95	3.7424
16	−0.5043	36	−0.2364	56	0.1055	76	0.9255	96	4.0456
17	−0.4941	37	−0.2230	57	0.1299	77	0.9960	97	4.3758
18	−0.4824	38	−0.2094	58	0.1554	78	1.0712	98	4.7353
19	−0.4694	39	−0.1956	59	0.1821	79	1.1516	99	5.1270

for less developed countries it has become evident that the age patterns of mortality exhibited by their populations often differ from those recorded in the developed countries during the period 1850-1960, and consequently from those embodied in the models described above. For this reason, the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat prepared and recently published a set of model life tables based on data from developing countries.[15]

Although the availability and reliability of data from the less developed regions have increased remarkably since the publication of the early United Nations life tables in 1955 and the Coale-Demeny tables in 1966, such data remain essentially poor. As a result, careful evaluation, selection and adjustment procedures were used to construct the data base upon which the new United Nations models were constructed. This data base consists of 36 life tables by sex (72 in total), covering a wide range of mortality levels (for 10 life tables, e_0 is below 50 years; and for another 10, e_0 is 70 years or greater). Geographically, 16 pairs of male/female life tables were obtained from 10 countries of Latin America, 19 from 11 countries of Asia, and one from Africa. The meagre contribution by Africa reflects both a shortcoming and a strength of this system—a shortcoming in that the lack of data from Africa casts doubts about the use of the resulting models to represent the experience of this major area, a strength in that the rejection of whatever data were available for sub-Saharan Africa, usually of extremely low quality, is to some extent a validation of the evaluation procedures used to select the data base and a partial assurance that the empirical tables underlying these models are of generally high quality so that the models themselves reflect distinctive patterns of mortality in developing countries rather than typical patterns of data flaws.

The new United Nations model life tables are similar to the Coale and Demeny set in that distinct patterns of age-specific mortality schedules have been identified and are published in detail. In addition, the new models incorporate a greater degree of in-built flexibility, allowing the user to construct mortality patterns different from those actually published. In this sense, they are more akin to the logit system proposed by Brass.

Four distinct patterns of mortality were identified on the basis of the data available. Because of the predominance of these patterns in certain geographical regions, they are identified in regional terms as the "Latin American", the "Chilean", the "South Asian" and the "Far Eastern" patterns.[16] A fifth pattern, called the "general" pattern, was constructed as the overall average of those listed above.

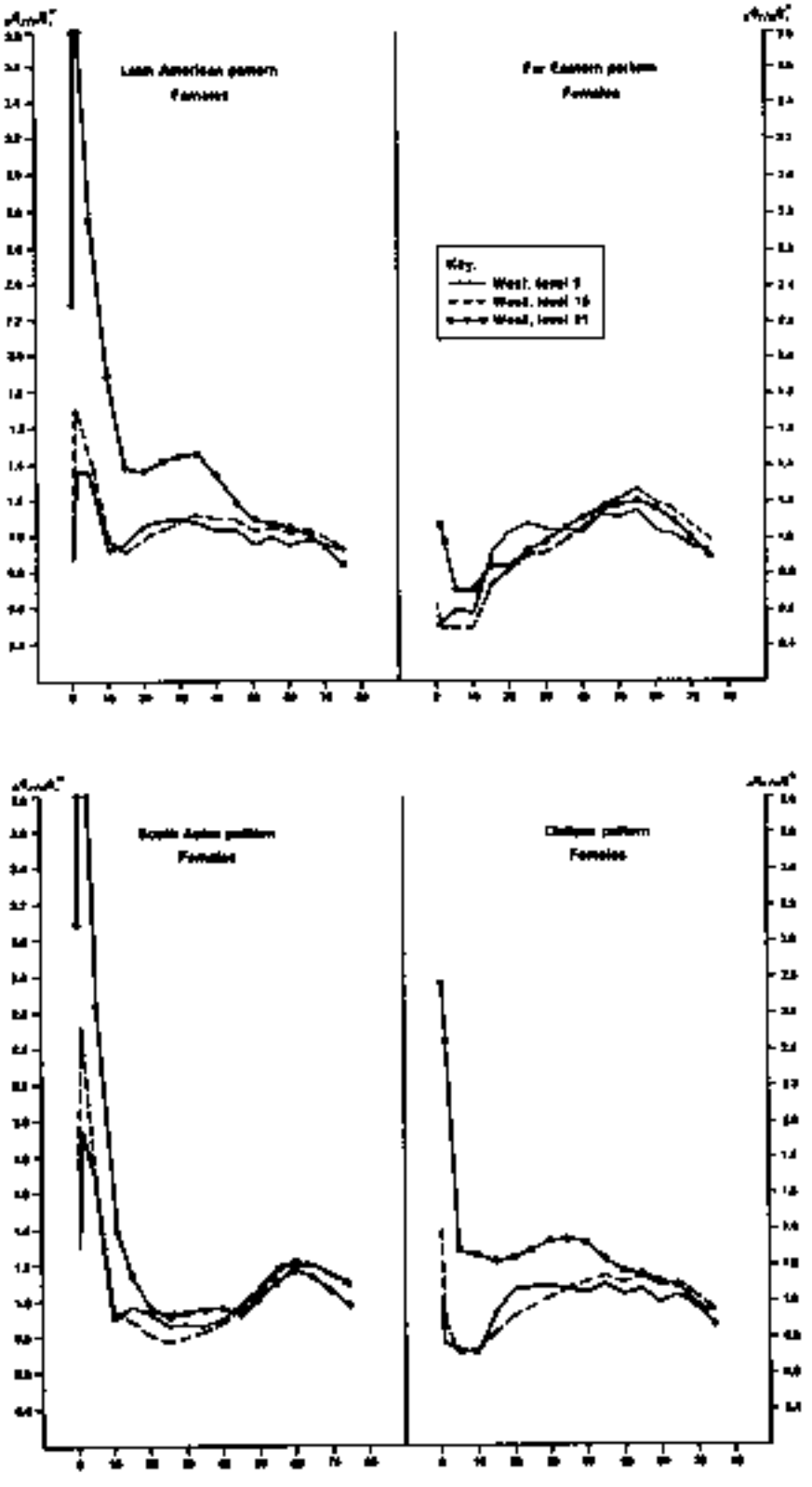
In order to illustrate the differences between the Coale and Demeny models and the United Nations patterns, figure 7 shows plots of the ratios ${}_nq_x / {}_nq_x^W$, where the superscript W indicates the West model in the Coale-Demeny set. Comparisons are made between ${}_nq_x$ values corresponding to life tables with the same life expectancy at age 10. Levels 9, 15 and 21 of the Coale-Demeny tables for females are used as denominators. These comparisons highlight the most distinctive characteristics of the new United Nations patterns, which are discussed below.

The Latin American model, when compared with the West family of the Coale and Demeny models, exhibits high mortality during the infant and childhood years (due mainly to excess diarrhoeal and parasitic diseases), and again during the young adult ages (largely due to

[15] *Model Life Tables for Developing Countries* (United Nations publication, Sales No. E.81.XIII.7).

[16] The labelling used for these patterns does not relate to the geographical divisions established by the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat.

Figure 7. Comparison of the mortality patterns in the United Nations model life tables for developing countries with the West family of the Coale and Demeny set



Note: The labelling used for these patterns does not relate to the geographical divisions established by the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat.

accidents). It also exhibits relatively low levels at older ages, apparently because of comparatively low death rates due to cardio-vascular diseases.

The Chilean pattern is characterized mainly by extremely high infant mortality in relation to both the West family and its own child mortality. This excessive infant mortality appears to be due mainly to deaths from respiratory diseases and may also be related to early weaning.

The South Asian pattern is typified by extremely high mortality under age 15 and relatively high mortality at older ages (over age 55 approximately). Correspondingly, mortality during the prime adult ages is relatively low. Data about causes of death are scarce in this region; however, those gathered by the International Diarrhoeal Disease Research Centre in Matlab, Bangladesh, and by the Indian Model Registration Project reveal high rates of diarrhoeal and parasitic diseases at younger ages and high mortality from diarrhoeal and respiratory diseases at older ages.

The Far Eastern pattern exhibits very high death rates at the older ages compared with those at younger ages. There is some evidence that this distinctive pattern may be due to a past history of tuberculosis.

The general pattern (not shown), which can be considered an average of the four regional patterns described above, is very similar to the West family of the Coale-Demeny set.

As mentioned earlier, these tables combine the advantages of the Coale and Demeny regional system and its detailed publication format with the type of flexibility inherent in the Brass logit system. Such characteristics were achieved by using principal component analysis to construct each model, after preliminary clustering of the data had been carried out. Clustering allowed the identification of the four distinctive patterns described above. Within each cluster equations of the form

$$\text{logit}({}_nq_x) = U^c_{0x} + \sum_{i=1}^{k} a_i U^c_{ix} \qquad \text{(B.7)}$$

were fitted, where ${}_nq_x$ is the observed probability of dying between ages x and $x+n$; U^c_{0x} is the overall average (in logit terms) for cluster c; U^c_{ix} represents the characteristic deviation of the observed from the average; and the coefficient a_i indicates the size of such deviations. Because the fitting procedure used identifies the U_{0x} values as the principal components of the observed ${}_nq_x$ vectors (with x ranging from 0 to ω), the U_{0x} vector can be interpreted as a measure of the average mortality model within each cluster, while U_{1x} may be interpreted as a measure of the typical deviations from that average as mortality levels change. Deviations from the overall average not due purely to changes in level are embodied by the second and third principal components, U_{2x} and U_{3x}, respectively. Hence, in constructing the one-parameter models printed in the tables, equation (B.7) with $k = 1$ was used, but setting $k = 2$ or $k = 3$ and selecting a_i values judiciously allows the user to derive mortality schedules whose pattern deviates from that of the printed tables. In this way, the flexibility of the models is enhanced.

In spite of their qualities, the United Nations life tables for developing countries are not used in this *Manual*, mainly because they were not yet available at the time of its preparation. In addition, because some of the methods described in the remainder of this volume have been developed on the basis of the Coale-Demeny models, it would be unfair to judge their performance on the basis of new models. The development of variations of these methods specifically designed for use with the new United Nations models is currently being undertaken and will soon add to the tools available for the analysis of demographic data in the developing countries.

C. Model stable populations

The concept of a stable population was first formulated by Lotka,[17] who proved that almost any population that is subject to constant fertility and mortality for a sufficiently long time acquires ultimately an unchanging age distribution which is characteristic of the prevailing fertility and mortality rates, and which is independent of the initial age distribution. He called the end-product of such constant fertility and mortality conditions a stable population and established that the stable age distribution has the following form:

$$c(x) = b \exp(-rx)l(x) \qquad \text{(C.1)}$$

where $c(x)$ is the infinitesimal proportion of the stable population at exact age x ($c(x)$ is actually a density function), b is the constant birth rate, r is the constant rate of natural increase and $l(x)$ is the probability of surviving from birth to age x (the usual life-table function).

Using equation (C.1) and recalling that an integral can be interpreted as a sum of infinitesimals, one can deduce that the proportion under age y, $C(y)$, is

$$C(y) = \int_0^y b \exp(-rx)l(x)dx \qquad \text{(C.2)}$$

and, since for the highest age possible, denoted by ω, $C(\omega)$ must equal one (that is, the portion of the population under the highest age attainable must be the entire population), it follows that

$$\int_0^\omega b \exp(-rx)l(x)dx = 1.0 \qquad \text{(C.3)}$$

or

$$b = \left[\int_0^\omega \exp(-rx)l(x)dx \right]^{-1} \qquad \text{(C.4)}$$

Equations (C.2) and (C.4) allow the computation of the age distribution of a stable population whenever its mortality schedule ($l(x)$) and its growth rate (r) are known.

[17] A. J. Lotka and F. R. Sharpe, *loc. cit.*

Model stable populations arise from the use of model mortality schedules or life tables to generate, through equations (C.2) and (C.4), stable age distributions for selected values of r. Among the five types of model life tables discussed above, at least four have been used to generate model stable populations. The Coale-Demeny set has probably been the most widely used for the purpose of demographic estimation (cf. chapter VII). Four families of model stable populations are included in this set: one for each of the patterns of mortality it contains. Stable age distributions are printed for each sex separately, for values of r ranging from −0.01 to 0.05 (increasing in steps of 0.005 at a time), for values of the gross reproduction rate ranging from 1.0 to 6.0 (increasing in steps of 0.25) and for the 24 mortality levels of each family. Each age structure is accompanied by a series of other parameter values corresponding to the stable population it represents. These values include the death and birth rates and the gross reproduction rate. The degree of detail in which this set is tabulated makes it rather simple to use.

Another set of stable populations was published by Carrier and Hobcraft,[18] who computed stable age distributions based on life tables calculated by using the logit system with the African standard. Two types of life tables are presented: one in which β is held constant with a value of one, and another where both α and β are allowed to change. This set of stable populations is far less detailed than the Coale-Demeny set, but its use for estimation purposes may be required in situations where none of the Coale-Demeny mortality patterns is judged to approximate adequately the mortality experienced by an actual population.

The United Nations also published a set of model stable populations that can be used for demographic estimation, especially in cases where the other sets are not able to provide an acceptable fit.[19] Lastly, the United Nations has recently published a new set of model stable populations corresponding to its new model life tables.[20] Stable populations are presented for each of the five United Nations mortality patterns, for growth rates from 0.0 to 4.0 per cent by 0.5 per cent increments and for life expectancies at birth from 35 to 75 years by one-year increments. Intrinsic birth and death rates are also presented.

D. Nuptiality models

As mentioned in section A of this chapter, while studying first-marriage frequencies (the number of first marriages taking place in a short age interval divided by the number of persons in that interval) in different female populations (or, more precisely, in different female cohorts), Coale[21] discovered that they could all be made to conform to a common standard pattern. The observed curves of first-marriage frequencies by age differed from one another only in the age at which marriage began, the rate at which marriages took place and the ultimate proportion that ever married. Therefore, a transformation of their origin, and of the horizontal and vertical scales, was all that was required to make them conform to a standard. Once a common pattern was discovered among the transformed distributions, a standard was calculated on the basis of period data from Sweden (1865-1869). The availability of this standard permitted the calculation of an empirical risk function $R(x)$, whose values quantify the risk of marrying for the first time at each age. By trial and error, Coale discovered that a good fit to $R(x)$ was obtained by the double exponential function

$$R_s(x) = 0.174 \exp(-4.411 \exp(-0.309x)). \quad \text{(D.1)}$$

It was later proved[22] that this standard risk function is closely approximated by the density function associated with an infinite sum of independent, exponentially distributed random variables. The exact form of this standard density function, $g_s(x)$, is

$$g_s(x) = 0.1946 \exp(-0.174(x - 6.06) - \exp(-0.288(x - 6.06))). \quad \text{(D.2)}$$

The fit of this function to the empirical standard is just as good as that of a density corresponding to the sum of a normally distributed random variable and several independent, exponential delays, and equation (D.2) was adopted as a model because it is easier to handle mathematically.

If the effect of differential mortality and migration by marital status on the proportion of women who have ever been married is neglected, the existence of a standard curve of first-marriage frequencies implies the existence of a standard curve describing the proportions ever married by age for any given cohort. The shape of this curve is standard but, naturally, there are differences in the beginning age at marriage (the age at which marriages begin taking place among members of a cohort), in the pace at which the curve rises and in the ultimate proportion getting married (the proportion ever married by the age at which first-marriage rates have fallen essentially to zero). If one denotes by $G_s(x)$ the standard proportion ever married x years after marriages begin, then $G(a)$, the proportion married by age a in some true cohort, can be expressed as

$$G(a) = \Theta G_s((a - a_0)/\gamma) \quad \text{(D.3)}$$

where Θ is the ultimate proportion ever married, a_0 is the age at which first marriages begin and γ is the scale

[18] N. H. Carrier and J. Hobcraft, *op. cit.*

[19] *The Concept of a Stable Population: Application to the Study of Populations of Countries with Incomplete Demographic Statistics*, Population Studies No. 39 (United Nations publication, Sales No. 65.XIII.1).

[20] *Stable Populations Corresponding to the New United Nations Model Life Tables for Developing Countries* (ST/ESA/SER.R/44).

[21] A. J. Coale, "Age patterns of marriage"

[22] Ansley J. Coale and Donald R. McNeil, "The distribution by age of the frequency of first marriage in a female cohort", *Journal of the American Statistical Association*, vol. 67, No. 340 (December 1972), pp. 743-749

factor expressing the number of years of nuptiality in the given population which are equivalent to one year in the standard population.

Since, mathematically, $G_1(x)$ is just the integral of $g_1(x)$,

$$G_1(x) = \int_0^x g_1(y)dy, \qquad (D.4)$$

the density $g(a)$ associated with the cohort described above can be obtained from equations (D.3) and (D.4) by making a change of variable in the latter and substituting in the former. The result implies that $g(a)$ has the form

$$g(a) = (0.1946\Theta/\gamma)\exp[(-0.174/\gamma)(a - a_0 - 6.06\gamma) - \exp[(-0.288/\gamma)(a - a_0 - 6.06\gamma)]]. \qquad (D.5)$$

No analytical expression for $G(a)$ has been found, but its value for any age a can be calculated by the numerical integration of $g(a)$, since

$$G(a) = \int_{a_0}^{a} g(y)dy. \qquad (D.6)$$

The problem of fitting a model of the type defined by equation (D.5) to an actual population consists of identifying values of the parameters Θ, a_0 and γ that adequately reflect the experience of the population in question. Approximate values for these parameters may be estimated from the knowledge of the proportions single classified by age. Normally, the proportion single in age group 50-54 may be considered an estimate of $U(\omega)$, the proportion who never marry. Therefore, the proportion who will eventually marry, Θ, may be estimated by:

$$\Theta = 1 - U(\omega). \qquad (D.7)$$

Furthermore, it is known that the mean of the fitted first-marriage schedule is $a_0 + 11.37\gamma$. This mean, also known as the "singulate mean age of marriage", *SMAM*, can be estimated from the proportions single classified by age by a simple procedure first proposed by Hajnal[23] and described in detail in annex I. Since

$$SMAM = a_0 + 11.37\gamma \qquad (D.8)$$

then

$$\gamma = (SMAM - a_0)/11.37 \qquad (D.9)$$

so that if both *SMAM* and a_0 are known, γ can be easily estimated. The estimation of a_0 is usually carried out in a very rough way. It is assumed that marriages in most developing countries begin early, and unless there is evidence to the contrary, a_0 is selected to be 13 or 14 years. Values of 12 or 15 may also be used. The exact value of a_0 is generally not crucial. Equation (D.9) can then be used to estimate γ. Once values for Θ, a_0 and γ have been estimated, Coale[24] provides tables that make the calculation of first-marriage frequencies and of the proportion ever married by age a simple matter.

To conclude, it may be mentioned that although, strictly speaking, model (D.5) only describes the first-marriage experience of a cohort, in practice, the proportions of women who have ever been married observed during a given period may also be closely approximated by this model, particularly if marriage patterns have remained constant, but also, and more surprisingly, in cases where marriage patterns have been changing.

E. Fertility models

1. *Coale and Trussell model*

Louis Henry[25] discovered that in populations where there is little or no voluntary control of fertility the age pattern of fertility within marriage is approximately constant. According to Henry, voluntary control is any behaviour affecting fertility that is modified as parity increases. He called "natural fertility", $h(x)$, the fertility observed in the absence of control and was able to infer its general pattern from the study of several populations where voluntary fertility control was presumed absent. In these populations, the level of natural fertility varied, but its age pattern remained the same. Henry attributed level variations between populations to differences in overall health, the practice of breast-feeding and any other physical or social factors that might affect the marital fertility experience of women irrespective of their parity.

In 1974, Coale and Trussell[26] proposed a model that, by generalizing the pattern of natural fertility, was able to represent the fertility experience of populations where voluntary fertility control was exercised. This model is based on the following assumption: marital fertility either follows natural fertility (if deliberate birth control is not practised); or it departs from natural fertility in a way that increases with age according to a typical pattern. Therefore, if one denotes by $\phi(x)$ marital fertility at age x and by $h(x)$ natural fertility at the same age, in a population where fertility is controlled voluntarily,

$$\phi(x) = M h(x) e^{m(x)} \qquad (E.1)$$

where M is a parameter indicating the level of natural fertility that the population would experience in the absence of all voluntary control and $m(x)$ is a function of age indicating the typical pattern of departure from natural fertility when voluntary control is exercised.

By examining the function $m(x)$ calculated for several

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[23] John Hajnal, "Age at marriage and proportions marrying", *Population Studies*, vol. VII, No. 2 (November 1953), pp. 111-136.

[24] A. J. Coale, "Age patterns of marriage".

[25] Louis Henry, "Some data on natural fertility", *Eugenics Quarterly*, vol. VIII, No. 2 (June 1961), pp. 81-91.

[26] Ansley J. Coale and T. James Trussell, "Model fertility schedules: variations in the age structure of childbearing in human populations", *Population Index*, vol. 40, No. 2 (April 1974), pp. 185-258.

populations, Coale and Trussell[27] found that it could be represented by

$$f(x) = \exp(m\,v(x)) \qquad \text{(E.2)}$$

where the function of $v(x)$ was very nearly the same for different populations and the parameter m changed from population to population. They interpreted these results as meaning that $v(x)$ represents the typical pattern of deviation from natural fertility when deliberate birth control is practised, while m measures the degree to which this control is practised. The final model of marital fertility arrived at is appealing both mathematically and theoretically. It can easily be derived from equations (E.1) and (E.2) and has the form

$$\phi(i) = Mh(i)\exp(m\,v(i)) \qquad \text{(E.3)}$$

where the index i is used in place of age x to indicate that, in general, only data referring to five-year age groups are used. Values of the two functions $h(i)$ and $v(i)$ have been estimated, and are shown in table 3.

TABLE 3. STANDARD PATTERN OF NATURAL FERTILITY AND OF DEVIATIONS FROM NATURAL FERTILITY BY AGE GROUP, FOR THE COALE AND TRUSSELL FERTILITY MODEL

Age group	Index i	Natural fertility $h(i)$	Deviation from natural fertility $v(i)$
15-19	1	0.411	0.000
20-24	2	0.460	0.000
25-29	3	0.431	−0.279
30-34	4	0.395	−0.667
35-39	5	0.322	−1.042
40-44	6	0.167	−1.424
45-49	7	0.024	−1.671

Therefore, this model can be fitted to any population whose marital fertility rates are known by just identifying the values of the parameters M and m. Coale and Trussell[28] suggest two possible ways of estimating M and m. According to the first and simplest procedure,

$$M = \phi(2)/h(2) \qquad \text{(E.4)}$$

and

$$m = 0.2\sum_{i=3}^{7} \ln[\phi(i)/Mh(i)]/v(i). \qquad \text{(E.5)}$$

That is, the level of marital fertility M is defined by the relation between the observed marital and natural fertility rates at ages 20-24, at which ages voluntary control of fertility is deemed to have no effect on pattern; while m is just the mean of the $m(i)$ values that can be estimated directly from the observed $\phi(i)$ once M is known.

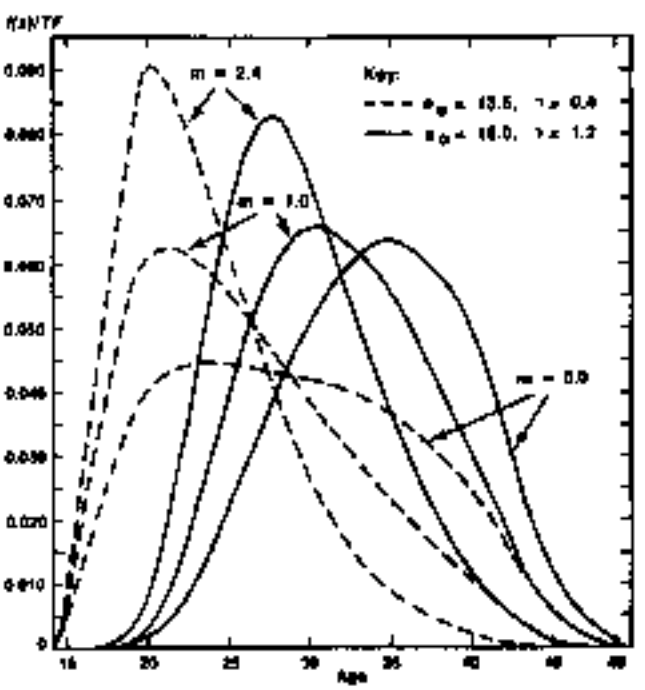
The second approach makes use of the following re-expression of (E.3):

$$\ln(\phi(i)/h(i)) = \ln(M) + m\,v(i) \qquad \text{(E.6)}$$

which clearly shows that the quantities $\ln(\phi(i)/h(i))$ and $v(i)$ are linearly related. Therefore, $\ln(M)$ and m can be estimated by identifying the line that best fits the observed $[\ln(\phi(i)/h(i)),\ v(i)]$ points. Coale and Trussell[29] recommend the use of the least-squares method to fit this line and suggest that only the points corresponding to age groups from 20-24 to 40-44 (i ranging from 2 to 6) should be considered.

Note that the values of the standard function $v(i)$ indicating the pattern of deviations from natural fertility (cf. table 3) are all either negative or zero. Furthermore, the absolute value of $v(i)$ increases as age increases. Therefore, a positive value of m indicates that in the population being studied marital fertility falls increasingly short of natural fertility as age increases (see figure 8). A negative value of m would indicate, on the contrary, that marital fertility is increasingly higher than natural fertility as age advances. If $m = 0$, natural fertility and the marital fertility under observation follow the same pattern.

Figure 8. Coale and Trussell fertility schedules: combinations of early marriage and various degrees of fertility control; and late marriage with the same degrees of fertility control



Model (E.3) can be used not only to investigate the pattern of marital fertility that a population experiences but to generate model marital fertility schedules that can be very useful for simulation purposes. Yet, since in

[27] *Ibid.*

[28] *Ibid.*, and Ansley J. Coale and T. James Trussell, "Technical note: finding the two parameters that specify a model schedule of marital fertility", *Population Index*, vol. 44, No. 2 (April 1978), pp. 203-213.

[29] A. J. Coale and T. J. Trussell, "Technical note: finding the two parameters that specify a model schedule of marital fertility".

many instances the function of interest is not marital fertility but overall fertility, the marital fertility model (E.3) may be combined with the nuptiality model described earlier (in section D) so that age-specific fertility, $f(x)$, can be obtained as

$$f(x) = G(x)\eta(x) \qquad \text{(E.7)}$$

where $G(x)$ is the proportion married by age x defined by equations (D.5) and (D.6).

Model (E.7) is a five-parameter model of overall fertility. Its parameters are: Θ, the ultimate proportion that ever marries; a_0, the age at which marriage begins; γ, the pace at which marriage takes place; M, the overall level of marital fertility; and m, the degree of departure from natural fertility. Since Θ and M appear only as constant multipliers in $G(x)$ and $\eta(x)$, respectively, they determine the level of $f(x)$ and not its shape. The latter aspect is influenced only by the values of the three other parameters present. Coale and Trussell[30] constructed a set of model patterns of age-specific fertility $f(x)$ by evaluating equation (E.7) for different values of a_0, γ and m. The model fertility schedules generated in this way fit a wide range of observed fertility experience and permit the investigation of extreme patterns which have never or only rarely been accurately measured in practice.

2. *Brass relational Gompertz fertility model*

Brass[31] has sought to reduce the number of parameters determining the shape of age-specific fertility from the three required by the Coale-Trussell models to two by postulating, once more, a relational scheme between a "standard" fertility schedule and any other schedule. Specifically, denoting by $F(x)$ cumulated fertility up to age x and by TF total fertility, the ratio $F(x)/TF$, the proportion of total fertility experienced up to age x, is assumed to follow a Gompertz distribution function, whose form is

$$F(x)/TF = \exp(A\exp(Bx)) \qquad \text{(E.8)}$$

where A and B are constants, $A < 0$. This expression can be reduced to a linear function of x by taking logarithms (ln) twice. The two steps necessary to carry out this transformation are

$$\ln(F(x)/TF) = A\exp(Bx) \qquad \text{(E.9)}$$

and

$$\ln(-\ln(F(x)/TF)) = \ln(-A) + Bx. \qquad \text{(E.10)}$$

A minus sign must be introduced when transforming equation (E.9) into (E.10) because the quantity $F(x)/TF$ is smaller than one; hence, $\ln(F(x)/TF)$ is negative and the logarithm of a negative number is not defined. To simplify notation, one may denote the $\ln(-\ln(F(x)/TF))$ transformation of $F(x)$ by $\eta(F(x))$. Then equation (E.10) becomes

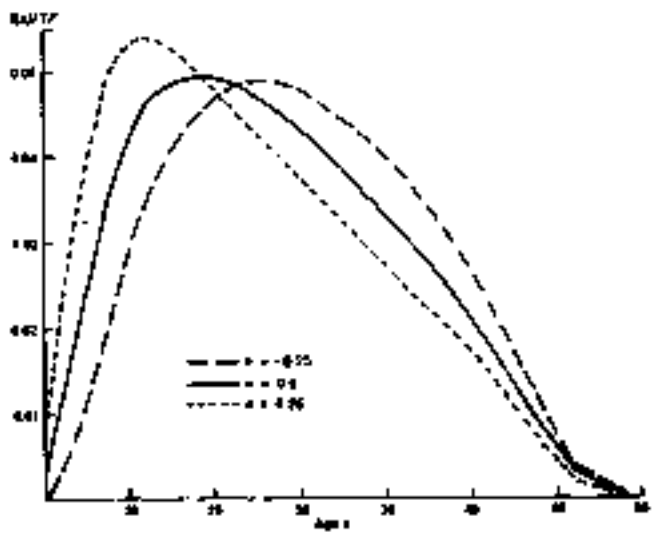
$$\eta(F(x)) = \ln(-A) + Bx \qquad \text{(E.11)}$$

This model, in which $\eta(F(x))$ is linear function of age, approximates the observed $F(x)/TF$ ratios fairly well over the central ages of childbearing, but its fit deteriorates at the extremes. Brass discovered that a better fit can be obtained by substituting for the age variable x a function of x that can be interpreted as an η transformation of a specific, standard fertility schedule. According to this finding, the relation expressed in equation (E.11) can be transformed into

$$\eta(F(x)) = \alpha + \beta\eta(F_s(x)). \qquad \text{(E.12)}$$

That is, the η transformation of the observed fertility schedule is a linear function of the η transformation of the standard. The parallel with the Brass logit life-table system is obvious; in both cases, a transformation that tends to linearize the distribution under consideration is used to relate any observed schedule to a standard pattern by the use of two constants. In the case of model life tables, the two parameters can be interpreted as determining the general level of mortality and the relationship between mortality early in life and that late in life. In the case of the relational fertility model, α in equation (E.12) can be taken as determining the age location of the fertility schedule or, more specifically, the age by which half of the total childbearing has occurred, while β may be interpreted as determining the spread or degree of concentration of the schedule. (To see the effects that changes in α and β have on the shape of fertility schedules, refer to figures 9 and 10.)

Figure 9. Fertility schedules generated through the Gompertz relational model with $\beta = 1.0$



[30] A. J. Coale and T. J. Trussell, "Model fertility schedules: variations in the age structure of child bearing in human populations".

[31] William Brass, "The relational Gompertz model of fertility by age of woman", London School of Hygiene and Tropical Medicine, 1978 (mimeographed).

Brass derived an appropriate standard from the Coale-Trussell model schedules. Values of its η transformation for each age within the childbearing span are shown in table 4. The Brass model is definitely easier to use than the one developed by Coale and Trussell, and may prove very useful for simulation and projection purposes. However, its development is still fairly recent and experience with its use is limited.

Figure 10. Fertility schedules generated through the Gompertz relational model with $\alpha = 0.0$

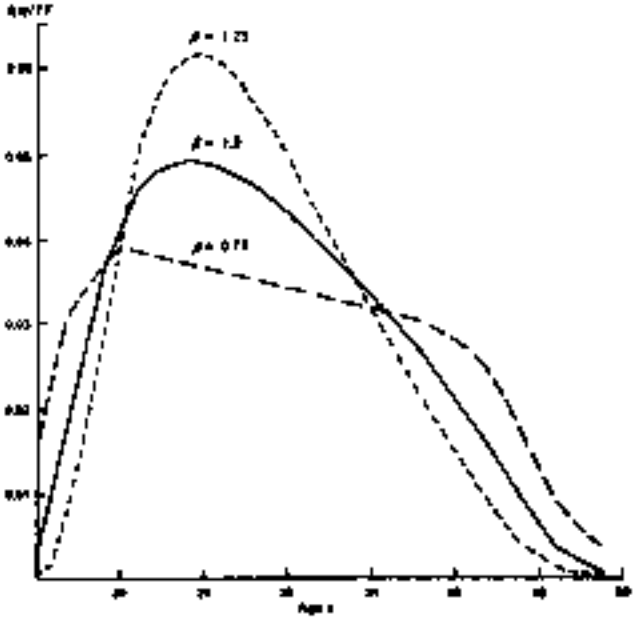


TABLE 4. VALUES OF THE η TRANSFORMATION OF A STANDARD FERTILITY SCHEDULE, $\eta(F(x))$

Age x (x)	η Transformation $\eta(F(x))$ (Yx)	Age x (x)	η Transformation $\eta(F(x))$ (Yx)
11	3.18852	31	−0.84272
12	2.70000	32	−0.99014
13	2.37295	33	−1.14407
14	2.07262	34	−1.30627
15	1.77306	35	−1.47872
16	1.49286	36	−1.66426
17	1.25061	37	−1.86597
18	1.04479	38	−2.08894
19	0.85927	39	−2.33192
20	0.69130	40	−2.62602
21	0.53325	41	−2.95500
22	0.38524	42	−3.32873
23	0.24423	43	−3.75984
24	0.10783	44	−4.25499
25	−0.02564	45	−4.80970
26	−0.15853	46	−5.41311
27	−0.29147	47	−6.12864
28	−0.42515	48	−7.00022
29	−0.56101	49	−8.04839
30	−0.70000		

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Chapter II

ESTIMATION OF FERTILITY BASED ON INFORMATION ABOUT CHILDREN EVER BORN

A. BACKGROUND OF METHODS

1. *Nature of information on children ever born*

This chapter describes methods of fertility estimation based on data about children ever born. The number of children ever born to a particular woman is an aggregate measure of her lifetime fertility experience up to the moment at which the data are collected. This number conveys no information about timing, whether on a personal scale, such as age or duration of marriage, or on an external scale, such as calendar years. When women are grouped according to some other variable, such as age or duration of marriage, the average number of children ever borne by the group, also known as their average parity, can be computed by dividing the total number of children borne by the women in the group by the total number of women in the group. The result is a measure of the average lifetime fertility experience of the survivors of a birth or marriage cohort, but as before it conveys no information about the timing of the births. When average-parity data are used for analytical purposes, additional information about timing, either personal or external, is introduced from other sources or inferred from additional assumptions. Strictly speaking, the average parities refer to the survivors of particular cohorts, though it is generally assumed that the distortion introduced by female mortality may be disregarded.

The foregoing general account gives some indication of the strengths and weaknesses of data on children ever born. Their main strength is that no dating is involved, so that the data cannot be distorted by dating errors. This strength is also, of course, a weakness, as by themselves the data refer to no clearly defined time period, providing no information about age patterns or time trends of fertility. Another weakness is that the information must generally be collected in the form of numbers, which may be subject to relatively greater errors than is information derived from questions with a simple "yes" or "no" response.

The simplest way of collecting information on children ever born is by the question "How many children have you ever borne alive?", the expected answer being a number. Note that this question concerns only children born alive, excluding stillbirths and other foetal deaths. It is important to adhere to this definition because all the methods of analysis currently available are based strictly on live births, and because it is likely that the completeness of reporting of stillbirths may vary widely from one society to another. The simple question quoted above has been extensively used, but examination of the results it yields suggests that children are sometimes omitted, particularly by women aged 35 and over. Children who have left home or who have died are particularly likely to be omitted, so that questions focusing on these two groups may be expected to produce better results (practice seems to support this expectation, although it is difficult to disentangle the effects of improved education and improved phrasings of the basic question). It is therefore recommended that when the time allowed to each interview and the questionnaire space permit, the basic question be broken down into three parts, becoming "Of the children you have ever borne alive, how many: 1. Are still living at home with you?; 2. Are still living, but elsewhere, in some other household?; 3. Have died?". This form of the basic question has the advantage of providing the additional information necessary for the estimation of child mortality (see chapter III). A further refinement is to collect information on children ever born classified by sex, asking the three questions presented above for children of each sex. Data classified by sex provide a basis for the estimation of child mortality by sex and prove useful in the assessment of data quality.

It is also necessary to decide from which women the information should be collected. In general, the best procedure is to put the necessary questions to all women over the age of 15, regardless of their marital status. In some societies, however, it is culturally impossible to ask women who have never been married whether they have had any children; and in these cases, the questions can only be put to women who have been married at least once (ever-married women). It is important to recognize, however, that in this case the value of the information collected will vary inversely with the incidence of premarital fertility.

Data on children ever born are generally tabulated by five-year age groups of women, or by five-year duration of marriage groups if the necessary data are available. A simple tabulation of number of women, number of children ever born, and average parity for each group should be supplemented by a tabulation of women classified by both age or duration group and by the number of children ever born (0, 1, 2 ... 10, 11+, not stated). When data by sex are available, all these tabulations should be repeated by sex of children.

A special form of data on children ever born is collected by inquiries about fertility histories or pregnancy histories which obtain information about the date of occurrence of each birth or about the termination of each pregnancy, and often inquire as well about the sub-

sequent fate of the child, recording the date of death when appropriate. The collection of a fertility history is time-consuming and hence fairly expensive, so that it has been limited in practice to relatively small-scale sample surveys. It is clear that this form of collection of data on children ever born also provides information about the timing of fertility, but at the cost of recording specific dates which are more likely to be subject to error.

A thorough analysis of the data gathered by fertility histories often requires fairly detailed tabulations of the data and access to them in machine-readable form. Thanks to the vast effort undertaken by the World Fertility Survey, the availability of both fertility-history data and methods designed specifically to analyse them has been greatly enhanced. Because the detailed description of these methods is beyond the scope of this *Manual*, the reader is referred to the series of publications issued as part of the World Fertility Survey. Of special interest is the collection of scientific reports which present detailed examples of the analysis and evaluation of data on fertility histories. Some of these analyses are cited in this chapter and others are listed at the end of the chapter.

2. *Typical errors in data on children ever born*

Average parities for groups of women calculated from data on children ever born can be distorted either by errors in the number of children reported or by errors in the classification of women in particular groups.

The most important error in the number of children reported is due to omission. Women tend to omit some of their live-born children, particularly those living in other households and those who have died, with the result that the proportion omitted tends to increase with age of mother. The symptoms of such omission are average parities that fail to increase rapidly enough as age increases; and, in some cases, average parities for women aged 40-44 and 45-49 may actually fall below that for women aged 35-39 even when there is *no reason* to suppose that fertility has been rising. Similar biases may be observed for duration-of-marriage groups 15-19 and higher. The effects of omission, if limited to women over 35, are not very serious because most methods of analysis make little use of the data referring to these women. However, in order to minimize omission at younger ages (under 35), it is advisable to stress at the data-collection stage the importance of obtaining accurate information from all women, regardless of age.

Another error in the reported children ever born arises from the inclusion of stillbirths or late foetal deaths among live-born children. The possible upward effect of this error on average parity is small, but stress should be laid during the data-collection process on including only live-born children. (A live-born child is generally defined as one who cries after birth.)

A third error affecting the recorded number of children ever born is introduced when the parity of a sizeable proportion of women is not recorded, that is, when there is a non-trivial proportion of women whose parity is not stated. If these women have in fact borne some children, their inclusion in the denominator of average parity, but the exclusion of their children from the numerator, will bias average parity downward. If it can be assumed that the women who provided information are representative, in parity terms, of those who did not, an unbiased estimate of average parity can be obtained by dividing the reported number of children by the number of women reporting. However, in many surveys there seems to be a disproportionate tendency for childless women to be tabulated as "not stated". A plausible explanation for this tendency has been suggested by El-Badry,[1] who argues that interviewers tend to leave blank the space for recording children ever born in cases where the true number is zero; at the coding stage, such blanks are interpreted as non-response, thus causing a net transfer of women from the zero category to the non-stated category. In such circumstances, average parity would be inflated if women tabulated as "not stated" were subtracted from the denominator. El-Badry also proposed a method by which the true incidence of non-response may be estimated by considering the relationship between the proportion of women with no children and the proportion of women whose parity was not stated; his method of adjustment is fully described in annex II. Its use is recommended when the relationship mentioned is linear in nature; otherwise, it is recommended that women whose parity is not stated should be included in the denominator when calculating average parities.

Misclassification errors arise from misreporting the age or duration of marriage of the women considered. When grouped data are used, errors arise only in so far as a woman is transferred from one group to another. The effects of such errors are complex, but certain principles can be outlined. Random errors in reporting age or duration are likely to have a slight equalizing effect on average parities, since a transfer upward that probably reduces the average parity of the higher group is likely to be matched by a transfer downward that probably increases the average parity of the lower group. However, the overall effect is expected to be small, since the misclassified women are likely to be close to the boundary of adjacent groups, with those transferred upward having parities above the average and those transferred downward having parities below the average for women of their true ages.

In contrast, a systematic transfer upward, in age or duration terms, is likely to reduce average parities for all groups, with the effect declining as age or duration increases, until it disappears in the age or duration group into which only women of completed fertility are shifted. Similarly, a systematic transfer downward will increase average parities, the effect again declining as age or duration increases, until it disappears for that age or duration group containing the point at which all childbearing has ceased. A systematic tendency to shift

[1] M. A. El-Badry, "Failure of enumerators to make entries of zero: errors in recording childless cases in population censuses", *Journal of the American Statistical Association*, vol. 56, No. 296 (December 1961), pp. 909-924.

age or duration upward until reaching some boundary, followed thereafter by a tendency to shift downward, concentrates respondents in a central group at the expense of the extremes and may result in relatively low average parities for groups below the boundary, an approximately correct average parity for the group that contains the boundary and relatively high average parities for the groups that lie above the boundary. In this case, groups lying above the upper limit of childbearing would exhibit correct average parities if age misstatement were the only reporting problem.

Because marriage is a more recent and more personally memorable event than one's own birth, it would seem that data classified by duration of marriage might be less distorted by dating errors than data classified by age, if duration is measured from a reported date of marriage. However, such data suffer from possible ambiguity about the date of marriage. The analyst is interested in the length of time since sexual relations began; but in some societies the onset of intercourse may predate formal marriage, and in others it may not occur immediately upon formal marriage. Further confusion exists in the case of remarriages, since the date reported may be that of the second or most recent marriage. This problem may be minimized by asking a question that refers specifically to the first marriage. Hence, it is important that interviewers and survey planners be aware of the conceptual problems related to this topic in order to devise the best data-gathering mechanisms to capture the information required.

It is generally assumed, when analysing data on children ever born, that the effects of mortality among women are negligible. The issue is important if one is comparing the average parity of a cohort at two different times, since the normal assumption is that the change in average parity between the two points is accounted for entirely by fertility in the intervening period. Ignoring mortality effects, one thus assumes that those members of the cohort who did not survive the period experienced similar fertility levels up to the time of their deaths as did the survivors, an assumption that is not likely to hold strictly in practice. It is not clear, however, in what direction the effects of mortality on average parity would be: if high-parity women experience above-average mortality risks, then the average parity reported by older women will underestimate the true level of cohort fertility; on the other hand, in developed countries, unmarried women experience higher mortality than married women, suggesting that low-parity women may be subject to higher mortality risks, in which case reported average parity will overestimate cohort fertility. However, the effects of mortality on average parity are likely to be very small, since in most countries today the mortality risks experienced by women in their childbearing years are fairly low.

Migration poses a problem similar to that introduced by mortality, though potentially more serious, particularly at the subnational level. Average parities for particular areas may be distorted by the migration of women not typical of the area, and changes between two points in time may also be distorted by migration. Thus, for example, the average parity for a city experiencing an influx of migrants may be inflated by the arrival of high-parity rural women. The problem arises, of course, from the timeless nature of the data on average parity, compounded, in this case, by a lack of information on place of previous residence. There is no way to resolve this problem other than by using areas that are not much affected by net migration or by resorting to other types of data. It would seem attractive to tabulate women by birthplace and children ever born by birthplace of mother, and to limit analysis to women born in the area being considered; but the estimates obtained would still not necessarily represent fertility in the area considered because immigrants may bring with them not only high historical fertility—and hence above-average parity—but above-average current fertility. Furthermore, if fertility behaviour is affected by place of residence, fertility estimates derived from data classified by place of birth would not represent adequately the current regional fertility differentials that are of greatest interest.

3. *Organization of this chapter*

All the estimation methods presented in this chapter have one characteristic in common: they use data on children ever born. However, the methods can be separated into categories according to the exact type of data they require (whether classified by age or by duration of marriage, for example). Sections B-D present the available methods divided into these categories. To aid the user in selecting the method best suited for a particular application, brief descriptions of each section are given below:

Section B. Methods of the Brass type based on comparison of cumulated age-specific fertility rates with reported average parities. This section presents several methods based on the idea, first proposed by Brass,[2] of comparing reported average parities with those estimated from period age-specific fertility rates. Their main characteristic, therefore, is that they require the availability of at least two types of information on fertility: children ever born for at least one point in time; and age-specific fertility rates referring to some period of interest. Variations of the basic method arise because of variations in the assumptions underlying it or because of the greater or lesser availability of data. In general, the methods presented in section B are ordered on the basis of their data requirements, that is, those presented earlier usually require less information than those presented later. Table 5 lists the data requirements of each method. It should be noted that all methods described in this section use data classified by age;

Section C. Estimation of age-specific fertility from the increments of cohort parities between two surveys. The method presented in section C is based exclusively on data on children ever born. Independently calculated

[2] William Brass, "Uses of census or survey data for the estimation of vital rates" (E/CN.14/CAS.4/V.57), paper prepared for the African Seminar on Vital Statistics, Addis Ababa, 14-19 December 1964.

TABLE 5. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER II

Section	Subsection and method	Type of input data	Estimated parameters
B. Methods of the Brass type based on comparison of period fertility rates with reported average parities	B.2. P/F ratio method based on data about all children	Children ever born classified by five-year age group of mother Births in a year classified by five-year age group of mother Women by five-year age group Total population	Adjusted age-specific fertility rates Adjusted total fertility Adjusted birth rate
	B.3. P_1/F_1 ratio method: first births	Women with at least one child classified by five-year age group First births in a year by five-year age group of mother Women classified by five-year age group	Adjusted first-birth age-specific fertility rates Adjusted overall proportion of mothers
	B.4. P/F ratio method for a hypothetical intersurvey cohort	Children ever born classified by five-year age group of mother from two surveys or censuses five or 10 years apart Births in the year preceding each survey or census classified by five-year age group of mother or, failing that, an estimate of intersurvey age-specific fertility rates (from vital registration data, for example) The number of women enumerated by each survey or census, classified by five-year age group	Adjusted intersurvey fertility schedule Adjusted intersurvey total fertility
	B.5. P/F ratio method for true cohorts	Children ever born classified by five-year age group of mother from a census Births registered during each of the 15 or 20 years preceding the census, classified by five-year age group of mother Women classified by five-year age group enumerated by the census gathering information on children ever born and by censuses taken during the 15 or 20 years preceding it	Estimates of completeness of birth registration Adjusted age-specific fertility rates for same period preceding the census Adjusted total fertility for the same period Adjusted birth rates for the same period
	B.6. P/F ratio method for hypothetical intercensal cohorts using registered births	Children ever born classified by five-year age group of mother from two censuses five or 10 years apart Births registered during each of the years of the intercensal period, classified by five-year age group of mother Women classified by five-year age group from the two censuses The total population according to each census	An estimate of the completeness of birth registration Adjusted intercensal age-specific fertility rates Adjusted intercensal total fertility Adjusted intercensal birth rate
C. Estimation of age-specific fertility from the increment of cohort parities between two surveys	C.2. Use of parity increments	Children ever born classified by five-year age group of mother, from two surveys or censuses five or 10 years apart Women classified by five-year age group from the two surveys or censuses	Intersurvey age-specific fertility rates Intersurvey total fertility
D. Estimation of fertility from information on children ever born classified by duration of marriage	D.2. Estimation of a natural fertility level	Children ever born classified by five-year duration of marriage group of mother Ever-married women classified by five-year duration of marriage group Total female population classified by five-year age group and by marital status (single, married, widowed and divorced) First age at which a significant number of marriages occurs The total population	Adjusted marital age-specific fertility rates Adjusted age-specific fertility rates Adjusted total fertility Adjusted birth rate

TABLE 5 (continued)

Section	Subsection and method	Type of input data	Estimated parameters
D. Estimation of fertility from information on children ever born classified by duration of marriage (continued)	D.3. *P/F* ratio method for data by duration of marriage	Children ever born classified by five-year duration of marriage group of mother Births in a year classified by five-year duration of marriage group Ever-married women classified by five-year duration of marriage group The total population	Adjusted duration-specific fertility rates Adjusted marital total fertility Adjusted birth rate

age-specific fertility rates are not necessary. All input data need to be classified by age;

Section D. Estimation of fertility from information on children ever born classified by duration of marriage. When data on children ever born classified by the duration of the mother's marriage are available, two methods of fertility estimation can be used. The simplest method requires only the data on children ever born; a more elaborate method permits a comparison of the Brass type between parity information and parity equivalent measures derived from a duration-specific fertility schedule. The latter method requires, therefore, information both on children ever born and on the number of births in a given year classified by mother's duration of marriage (see table 5).

B. METHODS OF THE BRASS TYPE BASED ON COMPARISON OF PERIOD FERTILITY RATES WITH REPORTED AVERAGE PARITIES

1. *General description of methods of the Brass type*

The total number of children ever borne by a group of women of a given age is a record of their total childbearing experience from the beginning of their reproductive life to their current age. The average number of children ever born, obtained by dividing the number of reported children by the number of women, is therefore a measure of the fertility experience of the cohort of women, though it is a measure of the level of fertility only, containing no information about its timing. If it is assumed that the fertility experience of those women who die is the same up to the age at death as that of those who survive, the average number of children born provides a mortality-free measure of cohort fertility.

A similar period measure may be obtained from age-specific fertility rates. If such rates are cumulated upward from the age at which childbearing begins, taking due account of the width of the age interval for which the rates are specific, the results obtained can be interpreted as the average number of children that would have been borne by women experiencing those fertility rates from the beginning of childbearing to the upper age boundary of the highest age group included in the cumulation.

The availability of information about both lifetime fertility, from a survey question about number of children ever born, and current fertility, from a survey question about births in the past year or date of the most recent birth, or from vital registration data, makes possible a powerful consistency check, whereby current fertility rates can be cumulated and compared with average parity. Such a comparison clearly uses both cohort rates and period rates, but it is valuable even if the two are not expected to be consistent because of changing fertility.

This comparison of lifetime with current fertility data can also provide a method of adjustment for cases where the data are distorted by typical errors. Information on children ever born is frequently distorted by omission, but this omission, perhaps of long-dead children or of those who have left the parental home, is most marked for older women; the reports of younger women, up to age 30 or 35, may be fairly reliable. Information on current fertility from a question on births occurring during the 12 months preceding a survey may be distorted by a misperception of the length of the reference period, so that the reported births correspond to an ill-defined period whose average length may be either shorter or longer than a year. If information on current fertility comes from a vital registration system, the level of the reported fertility rates may be distorted by general omission. If these errors in the information on current fertility may be assumed to be roughly constant with respect to age (an assumption that is particularly appropriate in the case of reference-period error), the age pattern of observed current fertility can be accepted as correct although its level may be distorted.

Cumulated current fertility may be compared with the reported lifetime fertility of women younger than 30 or 35 in order to obtain an adjustment factor for the level of the current fertility rates, which, once adjusted for level, provide a better estimate of actual current fertility. In order for this adjustment to be valid, it must be assumed that the fertility of younger women has not changed appreciably, for if it had changed, their lifetime fertility could not be expected to be consistent with cumulated current fertility rates. Furthermore, when dealing with data classified by five-year age group, cumulated current fertility rates provide an estimate of the average number of children ever borne by women who have reached the end of each age group, whereas parity data provide an estimate of the average number borne by women whose ages vary over the range of the age group. Therefore, a process of interpolation is required to ensure that the figures cover a comparable age range.

The essence of the Brass[3] fertility estimation procedure is the adjustment of the age pattern of fertility derived from information on recent births by the level of fertility implied by the average parity of women in age

[3] *Ibid.*

groups 20-24, 25-29, and perhaps 30-34. Several extensions of the original procedure have been proposed. First, if the reasoning outlined above may be applied to all births, it may also be applied to births of any particular birth order; and cumulated birth-order-specific fertility rates should be comparable with the proportions of women reporting at least that many children ever born. Such a comparison may be particularly revealing in the case of first births. Secondly, if data on children ever born are available from two surveys five or 10 years apart, and average fertility rates may be calculated for the same period, the assumption of constant fertility in the recent past can be relaxed, because average parity for a hypothetical intersurvey cohort can be constructed and compared with parity equivalents derived from intersurvey fertility rates. Lastly, the assumption of constant fertility may be relaxed if information on fertility rates during the past 15 or 20 years is available from a vital registration system or some other data source. The observed fertility rates for a series of true cohorts can be cumulated through the recent past for comparison with the average parities reported by women in successive age groups at the end-point. These useful extensions of the Brass method are described and illustrated below, following the description of the original technique. First, however, some general points that apply to all the versions discussed are considered.

Reported fertility rates are used to estimate the average cumulated fertility or parity equivalent, F, that women in each age interval would have if they had been subject throughout their lives to the reported rates. Two problems arise, however, in obtaining a value of F that is comparable to the average parity, P, reported by women of each age group. First, because fertility data are ordinarily tabulated by five-year age group, cumulating the reported age-specific fertility rates and multiplying by five yields estimates of the parity or cumulated fertility that women experiencing those rates would achieve by the end of each five-year age group (that is, by exact ages 20, 25, 30 etc. when conventional five-year age groups are used). These estimates are not comparable with the average parities calculated from data on children ever born, because the latter values represent the mixed experience of women of different exact ages. Hence, some procedure is required for estimating the average cumulated fertility or parity within each age group from knowledge of the values that the cumulated fertility schedule takes at the end-points of the age groups considered. Secondly, when the current fertility schedule is obtained from a survey question on births during the 12 months preceding the survey or on date of the most recent birth, the births are generally tabulated by the mother's age at the time of the survey, not at the time of the birth. If one assumes that births in a given year are uniformly distributed in time, the women who had a birth in the 12 months preceding the survey were, on average, six months younger at the time of the birth than at the time of the interview. Therefore, the age-specific fertility rates that can be calculated from data on children born during the year before the survey classified by age of mother at the time of the survey correspond to unorthodox age intervals whose limits are (14.5, 19.5), (19.5, 24.5), ..., (44.5, 49.5), rather than to the usual intervals with end-points (15, 20), (20, 25), ..., (45, 50). When the source of information on current fertility is a vital registration system, this second problem should not arise, since births are supposed to be recorded near the time of occurrence and therefore the reported age of mother is likely to be the age she had at the time of the birth. When registered births are used, however, late registrations should be excluded; otherwise, they might seriously distort the age pattern of fertility.

An interpolation procedure based on model fertility schedules has been devised to allow the estimation of parity equivalents (F) for the usual five-year age groups of women from the cumulated fertility schedule. A similar procedure that takes into account the problem of age groups displaced by six months and produces the desired estimates of F (parity equivalents) has also been developed. Hence, two variants of the procedure are available: one suited for use with fertility rates calculated from vital registration data, that is, with births tabulated by age of mother at delivery; and the other suited for use with fertility rates calculated from reported births for a 12-month period tabulated by age of mother at the end of the period.

The Brass fertility estimation method and the variants presented in this chapter are best suited for estimating fertility in countries where massive systematic age-misreporting is not apparent. Its application to populations where age is poorly reported is likely to yield biased results. Fertility estimates obtained by applying the original method to populations where either marital fertility or age at marriage has been changing rapidly in the recent past may also be subject to bias, since it would no longer be valid to assume that the "historical" pattern of fertility implied by the reported average parities is equal to that embodied by the current fertility schedule. However, when a fertility decline is due mainly to effective use of contraception at relatively older ages, the method described here may still yield valid results if the adjustment factor is selected on the basis of information pertaining to the youngest age groups (20-24 is recommended).

2. *The P/F ratio method based on data about all children*

(a) *Basis of method and its rationale*

The original P/F ratio method or Brass method seeks to adjust the level of observed age-specific fertility rates, which are assumed to represent the true age pattern of fertility, to agree with the level of fertility indicated by the average parities of women in age groups lower than ages 30 or 35, which are assumed to be accurate. Measures of average parity equivalents, F, comparable to reported average parities, P, are obtained from period fertility rates by cumulation and interpolation (these measures are effectively averages of the cumulated fertility schedule over age groups). Ratios of average parities (P) to the estimated parity equivalents (F) are calculated age group by age group, and an average of

the ratios obtained for younger women is used as an adjustment factor by which all the observed period fertility rates are multiplied. Note that P/F ratios are generally calculated for the entire age range from 15 to 49, even though not all the ratios are used for adjustment purposes. This practice is recommended because the pattern of the ratios with age may reveal data errors or fertility trends. During successful application of this method, the age pattern of the period fertility rates is combined with the level implied by the average parities of younger women to derive a set of fertility rates that is generally more reliable than either of its constituent parts.

(b) *Data required*

The following data are required for this method:

(*a*) The number of children ever born classified by five-year age group of mother;

(*b*) The number of children born during the year preceding the survey or census classified by five-year age group of mother, or the number of registered births in the year of the census, also classified by five-year age group of mother;

(*c*) The total number of women in each five-year age group (irrespective of marital status);

(*d*) The total population if the birth rate is to be estimated.

(c) *Computational procedure*

Every function in this section is indexed by a variable (i or j) the values of which represent the age groups being considered. Table 6 summarizes the relationship between the index number and the age group.

TABLE 6. CORRESPONDENCE BETWEEN INDICES AND AGE GROUPS

Index value (*i* or *j*)	Age group (*i*)
0	10-14
1	15-19
2	20-24
3	25-29
4	30-34
5	35-39
6	40-44
7	45-49

The steps in the procedure are described below.

Step 1: calculation of reported average parities. The reported average parity of women in age group i is denoted by $P(i)$. Its value is obtained by dividing the total number of children ever born to women in age group i by the total number of women in that age group (whether married or single, fertile or not). See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the use of the adjustment method proposed by El-Badry (see annex II).

Step 2: calculation of a preliminary fertility schedule from information on births in the past year or from registered births. The fertility rate of women in age group i is denoted by $f(i)$. This value is computed for each i by dividing the number of births occurring to women in age group i during the year preceding the interview by the total number of women (whether childless or not, ever married or not) in that age group. In the case of registered births, the births by age group recorded for a calendar year should be divided by an estimate of the mid-year female population of the age group (usually obtained from a census).

Step 3: calculation of cumulated fertility schedule for a period. To calculate this schedule, denoted by $\phi(i)$, the fertility rates computed in step 2 are added, beginning with $f(1)$ (or with $f(0)$ if its value is not zero) and ending with $f(i)$. The value of this sum multiplied by five is an estimate of cumulated fertility up to the upper limit of age group i. The formal definition of $\phi(i)$ is

$$\phi(i) = 5\left[\sum_{j=0}^{i} f(j)\right]. \qquad \text{(B.1)}$$

Step 4: estimation of average parity equivalents for a period. Average parity equivalents, $F(i)$, are estimated by interpolation using the period fertility rates $f(i)$ and the cumulated fertility values $\phi(i)$ calculated in previous steps. Several procedures have been proposed for this interpolation. Brass[4] uses a simple polynomial model of fertility to calculate the relationship between average parity and cumulated fertility for successive age groups for a range of age locations of the fertility model. Coale and Trussell[5] propose fitting a second-degree polynomial to three consecutive values of $\phi(i)$ and estimating the average parity of women of an age group within the range by evaluating the integral of the polynomial; in an actual application, $F(i)$ is obtained as

$$F(i) = \phi(i-1) + af(i) + bf(i+1) \qquad \text{(B.2)}$$

where a and b are constants whose values are shown in table 7 for $i = 1, 2, \ldots, 6$. $F(7)$ is obtained as

$$F(7) = \phi(6) + a^* f(6) + b^* f(7) \qquad \text{(B.3)}$$

and the values of a^* and b^* are also displayed in table 7. A somewhat more accurate procedure is based on the general principle underlying equation (B.2), but it allows the constants a and b to vary with i. The interpolation equation used is

$$F(i) = \phi(i-1) + a(i)f(i) + b(i)f(i+1) + c(i)\phi(7). \qquad \text{(B.4)}$$

Values of the parameters a, b and c were estimated by using least-squares regression to fit equation (B.4) to a large number of model cases constructed using the

[4] *Ibid.*

[5] Ansley J. Coale and T. James Trussell, "Model fertility schedule variations in the age structure of childbearing in human populations", *Population Index*, vol. 40, No. 2 (April 1974), pp. 185-258.

TABLE 7. COEFFICIENTS FOR INTERPOLATION BETWEEN CUMULATED FERTILITY RATES TO ESTIMATE PARITY EQUIVALENTS

Age group (i)	Index i (2)	Equation No. (3)	Coefficients a(i) (4)	b(i) (5)	c(i) (6)
(a) *Fertility rates calculated from births in a 12-month period by age of mother at end of period*					
15-19-40-44	1-6	B.2	3.392	−0.392	-
45-49	7	B.3	0.392	2.608	-
15-19	1	B.4	2.531	−0.188	0.0024
20-24	2	B.4	3.321	−0.754	0.0161
25-29	3	B.4	3.265	−0.627	0.0145
30-34	4	B.4	3.442	−0.563	0.0029
35-39	5	B.4	3.518	−0.763	0.0006
40-44	6	B.4	3.862	−2.481	−0.0001
45-49	7	B.4	3.828	0.016[a]	−0.0002
(b) *Fertility rates calculated from births by age of mother at delivery*					
15-19-40-44	1-6	B.2	2.917	−0.417	-
45-49	7	B.3	0.417	2.083	-
15-19	1	B.4	2.147	−0.244	0.0034
20-24	2	B.4	2.838	−0.758	0.0162
25-29	3	B.4	2.760	−0.594	0.0133
30-34	4	B.4	2.949	−0.966	0.0025
35-39	5	B.4	3.029	−0.823	0.0006
40-44	6	B.4	3.419	−2.966	−0.0001
45-49	7	B.4	3.335	−0.007[a]	−0.0002

[a] This coefficient should be applied to $f(i-1)$, not $f(i+1)$, that is, to $f(6)$ instead of $f(8)$.

Coale-Trussell[6] fertility model. Note that an additional constant term, $c(i)\phi(7)$, is introduced in equation (B.4). This term is effectively an estimated coefficient, $c(i)$, weighted by the observed total fertility rate, $\phi(7)$. In theory, the inclusion of a constant term in equation (B.4) is unsatisfactory because, if $f(i)$ and $f(i+1)$ were zero, $F(i)$ should be identical to $\phi(i-1)$. In practice, however, such degenerate fertility schedules are not encountered; and the restrictions imposed by such theoretical considerations do not warrant the loss of flexibility they would imply in obtaining the best possible fit to the model data.

Table 7 shows the values of the coefficients required for the use of equation (B.4). The table is divided into two parts: the first part presents coefficients for use with fertility rates derived from births in the 12 months before a survey tabulated by age of mother at the time of the survey; and the second presents coefficients for use with fertility rates calculated from births classified by age of mother at the time of delivery.

Step 5: calculation of a fertility schedule for conventional five-year age groups. When age-specific fertility rates have been calculated from births in a 12-month period classified by age of mother at the end of the period, they are specific for unorthodox age groups that are shifted by six months. A fertility schedule for conventional five-year age groups, $f^*(i)$, can be estimated by weighting the rates referring to unorthodox age groups according to equations (B.5) and (B.6), and using the coefficients displayed in table 8. Note that when fertility rates have been calculated from births classified by age of mother at the time of delivery, this step is not required:

TABLE 8. COEFFICIENTS FOR CALCULATION OF WEIGHTING FACTORS TO ESTIMATE AGE-SPECIFIC FERTILITY RATES FOR CONVENTIONAL AGE GROUPS FROM AGE GROUPS SHIFTED BY SIX MONTHS

Age group (i)	Index i (2)	Coefficients x(i) (3)	y(i) (4)	z(i) (5)
15-19	1	0.031	2.287	0.114
20-24	2	0.068	0.999	−0.233
25-29	3	0.094	1.219	−0.977
30-34	4	0.120	1.139	−1.531
35-39	5	0.162	1.739	−3.592
40-44	6	0.270	3.454	−21.497

$$f^*(i) = (1-w(i-1))f(i) + w(i)f(i+1) \qquad \text{(B.5)}$$

where $f(i)$ and $f^*(i)$ are, respectively, the unadjusted and adjusted age-specific fertility rates; and the weighting factor, $w(i)$, is calculated as

$$w(i) = x(i) + y(i)f(i)/\phi(7) + z(i)f(i+1)/\phi(7). \qquad \text{(B.6)}$$

The values of $x(i)$, $y(i)$ and $z(i)$ were obtained by fitting equation (B.6) by least-squares regression to the same model cases used in deriving the coefficients presented in table 7. No weighting factor is needed for $i=7$, as childbearing is assumed to cease after age 50; and $f^*(7)$ is therefore taken to be $(1-w(6))f(7)$. Births reported to women under age 15 can be included among those reported by women aged 15-19.

Step 6: adjustment of period fertility schedule. With the quantities computed in steps 1-4, the ratios $F(i)/F(i)$ are calculated. Ideally, these ratios should be fairly simi-

[6] *Ibid.*

lar for different values of i, although if children ever born are increasingly omitted by older women, the ratios will tend to decrease as age increases (especially over ages 30 or 35). In practice, however, they are often far from being constant, even below age 35; and one can be satisfied if $P(2)/F(2)$ and $P(3)/F(3)$ are reasonably consistent. If this is the case, either one of them can be used as an adjustment factor for the period fertility rates. If they are not very similar, a weighted average of the two can be used (using as weights the number of women in age groups 20-24 and 25-29 as a proportion of all women aged 20-29), as can a simple, unweighted average of them. However, if there is evidence suggesting that the population is experiencing a fertility decline affecting mainly women in the older age groups, the value of $P(2)/F(2)$ is recommended as an adjustment factor because it is less likely to be affected by the decline. In general, $P(1)/F(1)$ should be disregarded because of the intrinsic difficulty in estimating $F(1)$, and the P/F ratios for age groups over 30 cannot be regarded as reliable due to the possible omission of children ever born. Naturally, the more consistent the set of ratios obtained, the more confidence one can have in the adjustment factor selected. Certain patterns of variation of the ratios with age may also reveal the types of problems present. For example, a recent decline in fertility tends to produce a sequence of P/F ratios that increases with age.

Once an adjustment factor has been chosen (one may denote it by K), an adjusted fertility schedule is computed by multiplying the fertility rates for conventional age groups, $f^*(i)$, if the rates were originally for age groups shifted by six months, but $f(i)$ if the rates were originally for conventional age groups, by K, to yield adjusted $f^*(i)$ values:

$$f^*(i) = Kf^*(i), \text{ or } f^*(i) = Kf(i). \qquad \text{(B.7)}$$

Once all the $f^*(i)$ values are available, one may calculate total fertility, TF, which is defined as

$$TF = 5\left[\sum_{i=1}^{7} f^*(i)\right]. \qquad \text{(B.8)}$$

An estimate of the adjusted birth rate can be obtained by multiplying each of the adjusted fertility rates by the number of women in the relevant age group to estimate numbers of births, adding these results for all ages and then dividing their sum by the total population.

The calculation of these and other parameters is described in the following example.

(d) *A detailed example*

Table 9 shows data on the number of children ever born and children born in the year preceding the survey for women who were interviewed during a demographic survey conducted in Bangladesh in 1974.

The steps of the calculation are given below.

Step 1: calculation of reported average parities. Values

TABLE 9. CHILDREN EVER BORN AND BIRTHS IN THE PAST YEAR, BY AGE GROUP OF MOTHER, BANGLADESH, 1974

Age group (1)	Number of women (2)	Children ever born (3)	Births in past year (4)
15-19	3 014 706	1 160 919	320 406
20-24	2 653 155	4 901 382	609 269
25-29	2 607 009	9 085 852	561 494
30-34	2 015 663	9 910 256	367 831
35-39	1 771 640	10 384 001	237 297
40-44	1 479 575	9 164 329	95 357
45-49	1 135 329	6 905 673	38 125

of the reported average parities, $P(i)$, are obtained by dividing the numbers listed in column (3) (children ever born) of table 9 by those appearing in column (2) (number of women). Results are given in table 10; shown below is the way in which $P(3)$ was obtained (it will be recalled that index 3 refers to age group 25-29):

$$P(3) = (9{,}085{,}852/2{,}607{,}009) = 3\,485.$$

In the case of Bangladesh, data were available to apply the El-Badry correction for non-response. The estimated level of non-response was so low, however, as to be insignificant. The figures on number of women shown in column (2) of table 9 therefore include all women, even those for whom parity was not stated.

TABLE 10. AVERAGE PARITIES, PERIOD FERTILITY RATES AND CUMULATED FERTILITY, BY AGE GROUP OF MOTHER, BANGLADESH, 1974

Age group (1)	Index i (2)	Average parity per woman $P(i)$ (3)	Period fertility rate $f(i)$ (4)	Cumulated fertility $\phi(i)$ (5)
15-19	1	0.385	0.1063	0.5315
20-24	2	1.847	0.2296	1.6795
25-29	3	3.485	0.2154	2.7565
30-34	4	4.917	0.1825	3.6690
35-39	5	5.861	0.1339	4.3385
40-44	6	6.194	0.0644	4.6605
45-49	7	6.084	0.0338	4.8285

Step 2: calculation of preliminary fertility schedule. The values of this schedule, denoted by $f(i)$, are computed by dividing the entries in column (4) (births in the past year) of table 9 by those in column (2) (number of women). The value of $f(3)$, for example, is calculated as

$$f(3) = (561{,}494/2{,}607{,}009) = 0.2154.$$

Other values of $f(i)$ are given in table 10.

Step 3: calculation of cumulated fertility schedule. The values of $\phi(i)$, the cumulated fertility schedule, are obtained by adding the values of $f(j)$, beginning with $j=1$ and ending with $j=i$, and then multiplying this sum by five (this number is used because five-year age groups are being considered). Final results are shown in column (5) of table 10. As an example, $\phi(4)$ is computed as

$$\phi(4) = 5(0.1063 + 0.2296 + 0.2154 + 0.1825$$

$$= 5(0.7338) = 3\,6690.$$

Step 4: estimation of average parity equivalents for a period. Period fertility rates were calculated from births in the 12 months preceding the survey, tabulated by age of mother at the time of the survey; therefore, coefficients from part (a) of table 7 should be used to estimate the current average parity equivalents, $F(i)$. The computation of $F(1)$, $F(4)$ and $F(7)$ is illustrated, using the values of $\phi(i)$ and $f(i)$ listed in table 10 (full results are shown in column (4) of table 11):

$$F(1) = \phi(0) + a(1)f(1) + b(1)f(2) + c(1)\phi(7)$$

$$= 0.0 + (2.531)(0.1063) + (-0.188)(0.2296) +$$

$$(0.0024)(4.8285) = 0.237;$$

$$F(4) = \phi(3) + a(4)f(4) + b(4)f(5) + c(4)\phi(7)$$

$$= 2.7565 + (3.442)(0.1825) + (-0.563)(0.1339) +$$

$$(0.0029)(4.8285) = 3.323;$$

$$F(7) = \phi(6) + a(7)f(7) + b(7)f(6) + c(7)\phi(7)$$

$$= 4.6605 + (3.828)(0.0336) + (0.016)(0.0644) +$$

$$(-0.0002)(4.8285) = 4.789.$$

TABLE 11. AVERAGE PARITIES, ESTIMATED PARITY EQUIVALENTS AND *P/F* RATIOS, BANGLADESH, 1974

Age group (1)	Index i (2)	Average parity per woman $\phi(i)$ (3)	Estimated parity equivalent $F(i)$ (4)	P/F ratio $\phi(i)/F(i)$ (5)
15-19	1	0.385	0.237	1.624
20-24	2	1.847	1.209	1.528
25-29	3	3.485	2.338	1.491
30-34	4	4.917	3.323	1.480
35-39	5	5.861	4.094	1.432
40-44	6	6.194	4.503	1.376
45-49	7	6.064	4.789	1.270

Step 5: calculation of a fertility schedule for conventional five-year age groups. Because what is being considered now is a case in which births in the past year were tabulated by age of mother at the time of the survey, the reported period rates, $f(i)$, need to be converted into a fertility schedule, $f^+(i)$, for conventional age groups. Conversion is carried out by using equations (B.5) and (B.6). A detailed example of the calculation of $f^+(1)$, $f^+(4)$ and $f^+(7)$ is given below; other values of f^+ are shown in table 12. The totals of columns (3) and (4) of table 12, labelled $f(i)$ and $f^+(i)$, respectively, do not quite agree because of rounding:

$(i = 1)$ $w(1) = x(1) + y(1)f(1)/\phi(7) + z(1)f(2)/\phi(7)$

$= 0.031 + (2.287)(0.1063)/(4.8285) +$

$(0.114)(0.2296)/(4.8285)$

$= 0.087$

$f^+(1) = (1.0)(0.1063) + (0.087)(0.2296) = 0.1263$

$(i = 4)$ $w(4) = x(4) + y(4)f(4)/\phi(7) + z(4)f(5)/\phi(7)$

$= 0.120 + (1.139)(0.1825)/(4.8285) +$

$(-1.531)(0.1339)/(4.8285)$

$= 0.121$

$f^+(4) = (1 - w(3))f(4) + w(4)f(5)$

$= (0.889)(0.1825) + (0.121)(0.1339)$

$= 0.1784$

$(i = 7)$ $f^+(7) = (1 - w(6))f(7) = (0.834)(0.0336)$

$= 0.0280.$

Step 6: adjustment of period fertility schedule. The first step in selecting an adjustment factor K for the converted fertility rates obtained in the previous step is to calculate the P/F ratios. They are shown in column (5) of table 11.

Though the P/F ratios show a fairly marked decline from age 35 onward, probably in this case because of

TABLE 12. REPORTED PERIOD FERTILITY RATES, FERTILITY RATES FOR CONVENTIONAL AGE GROUPS, ADJUSTED FERTILITY RATES AND ESTIMATED NUMBER OF BIRTHS, BANGLADESH, 1974

Age group (1)	Index i (2)	Reported fertility rate $f(i)$ (3)	Fertility rate for conventional age groups $f^+(i)$ (4)	Adjusted fertility rate $f'(i) = Kf^+(i)$ ($K = 1.500$) (5)	Estimated number of births $w(i) - f(i)P(i)/T$ (6)
15-19	1	0.1063	0.1263	0.1895	571 287
20-24	2	0.2296	0.2323	0.3485	924 625
25-29	3	0.2154	0.2131	0.3197	833 461
30-34	4	0.1825	0.1784	0.2676	539 391
35-39	5	0.1339	0.1282	0.1923	340 694
40-44	6	0.0644	0.0595	0.0893	132 126
45-49	7	0.0336	0.0280	0.0420	47 675
TOTAL		0.9657	0.9858	1.4489	3 389 259
Total fertility		4.83	4.93	7.24	

omission of children ever borne by women over age 35, the ratios for the crucial age range 20-34 are fairly consistent. The adjustment factor has therefore been calculated as the average of the three ratios for women in this age range:

$$K = (1.528 + 1.491 + 1.480)/3 = 1.500.$$

Had the ratios not been so consistent, K would have been calculated as a weighted average of $P(2)/F(2)$ and $P(3)/F(3)$, the weights being the number of women in each age group as a proportion of the women in both age groups. As an illustration, in the example being considered,

$$K' = (1.528)(2,653,155)/(2,653,155 + 2,607,009)$$

$$+ (1.491)(2,607,009)/(2,653,155 + 2,607,009)$$

$$= 1.510.$$

Adjusted age-specific fertility rates for conventional age groups, $f'(i)$, are obtained by multiplying the $f'(i)$ values by the adjustment factor K. Final values for $f'(i)$ are shown in column (5) of table 12.

Total fertility, TF, can be estimated by multiplying the sum of the adjusted age-specific fertility rates $f'(i)$ by five:

$$TF = 5(1.4489) = 7.24.$$

An adjusted birth rate can be obtained by calculating the number of births that would occur to the population being considered if it were subject to the adjusted fertility rates and by dividing the total number of births by the total population. Numbers of births by age are shown in column (6) of table 12. The total number of births is the sum of all these entries; and because the total population considered is 71,315,944, an adjusted value of b is given by

$$b = 3,389,259/71,315,944$$

$$= 0.0475$$

Lastly, the general fertility rate can be calculated by dividing the total number of births by the number of women considered (women aged 15-49)

$$GFR = 3,389,259/14,676,917 = 0.2309.$$

3. *Comparison of current first-birth fertility with the reported proportion of mothers*

(a) *Basis of method and its rationale*

The consistency of information on current and retrospective fertility can also be checked by using data referring only to first births. In the same way that cumulated fertility rates based on all births are a measure closely associated with parity, cumulated first-birth fertility rates can be regarded as a measure of the probability of having had at least one child by the upper age-limit used in the cumulation. The consistency of recent and lifetime fertility information can then be checked by comparing cumulated and interpolated first-birth rates for a recent period with the reported proportions of women in each age group who have had at least one child. Assuming that any discrepancy between these two proportions is due to errors in the reporting of events, rather than to changing fertility, an adjustment factor may be obtained and an adjusted first-birth fertility schedule may be computed. This method of adjustment is, in fact, just a special case of the all-children method described in subsection B.2.

Computationally speaking, these two methods are very similar and there is also some similarity between the assumptions on which they are based. For example, both assume that the type of fertility measured (first child or all children) has remained constant in the recent past and that errors in current fertility are a result of a misperception of the reference period. However, the first-child method is less likely to be affected by changing marital fertility than is the method based on data for all children. Indeed, if a fertility decline is brought about by the use of contraception either to limit family size or to increase the spacing between children, the performance of the first-child method will not be affected; even if all childbearing after the first birth stopped, the first-birth comparison would remain valid. Any change affecting the timing of first births, on the other hand, would bias the results yielded by this method. Therefore, its use is not recommended when there is evidence of rapidly changing age at marriage or of a change in the interval between marriage and the onset of childbearing.

Another advantage of the first-birth method is that it is based on data that are more likely to be accurate. For example, the retrospective information it uses is the proportion of women who have had at least one child. This proportion is only distorted by women of parity one or higher who report themselves as childless, or by women of zero parity reporting themselves to be of parity one or higher. It is not affected by the actual number of children reported by these women and should therefore be more reliable than the equivalent information used by the all-children method (average reported parity). In particular, it should not suffer from the typical tendency of older women to omit some of the children they have had.

Unfortunately, the first-birth method does not solve the main problem: it does not produce an adjustment factor for all births. Logically, an adjustment factor computed from first-birth data applies only to first births. To estimate an adjustment factor applicable to all births from first-birth data one needs to make additional assumptions about the relationship between the errors affecting the reporting of first births and those affecting the reporting of all subsequent births. Although it may be reasonable to suppose that reference-period error is not much affected by birth order, other types of error may vary with birth order; and first-birth fertility rates

may be relatively inflated if too many women report themselves as having only one child. In this respect, it is important to point out that, in the case of the first-birth method, the two pieces of information being used (proportions of mothers and first births in a year) are not strictly independent from the point of view of the information gathered to produce them. Thus, a birth occurring during the year of interest is identified as "first" only if the woman reporting it also reports that her parity is one. Hence, both the answer to the question about children ever born and that to the one about births in the past year are necessary to classify the data used. Therefore, unlike the method based on all births, errors in reporting children ever born may affect the estimated first-birth fertility rates, making the comparison of the observed proportions of mothers with the proportions implied by those rates somewhat less satisfactory.

In spite of these shortcomings, whenever the data required to apply this method have been tabulated, its application is recommended because it can provide useful indications about the overall quality of the data at hand. For example, since in most populations over 90 per cent of all women eventually become mothers, recent first-birth fertility rates that imply a lower proportion of eventual mothers are suspect. Furthermore, since no more than 100 per cent of all women may become mothers, adjustment factors for first-birth fertility rates that imply an eventual overall proportion of mothers higher than one cannot be accepted.

Lastly, the P/F ratios calculated by using data on all births are very often lower than those estimated on the basis of first-birth information. This outcome may be explained by a tendency to report more accurately recent first births than those of higher orders, or by the tendency of women whose first child has died to report a subsequent and recent birth as being the first. Yet, whatever the mechanism, whenever the completeness level of first-birth fertility rates is greater than that of all births, it seems safe to assert that it indicates an upper bound for the completeness of the latter. In other words, the adjustment factor derived from data on first births can, whenever the P_1/F_1 ratios are, on average, lower than those for all births, be regarded as a lower bound for the adjustment factor required by all births.

(b) *Data required*

The data required for this method are described below:

(*a*) The number of first births occurring in a given year, obtained either from a survey or from a registration system, classified by five-year age group of mother;

(*b*) Total number of women of reproductive ages (normally between ages 10 or 15 and 50) classified by five-year age group;

(*c*) The number of women in each age group who have borne at least one child during their lifetime;

(*d*) The reported birth rate or enough information to calculate it (i.e., the total number of births in a given year and the total population in that year).

(c) *Computational procedure*

As mentioned before, the computational procedure is exactly the same as that followed when estimating an adjustment factor for all births (see subsection B.2(c)), except that the data used in this case refer only to first births. For the sake of completeness, the most important steps are summarized below.

Step 1: calculation of reported proportion of mothers in each age group. As in the all-children method, index $i = 1$ refers to age group 15-19, $i = 2$ to age group 20-24 and so on (see table 6). The proportion of mothers in each group, $P_1(i)$, is calculated by dividing the number of women who reported having borne at least one child by the total number of women in each age group. It should be noted that in this case $P_1(i)$ represents the proportion of women in each age group who have had at least one child and is equivalent to average parity in the all-children method. For the treatment of those whose parity is not stated in the calculation of the proportion of mothers in each age group, see subsection A.2 and annex II concerning the El-Badry correction.

Step 2: calculation of period first-birth fertility schedule. One computes this schedule, $f_1(i)$, by dividing the number of first births occurring in a given year to women in age group i by the total number of women in that age group.

Step 3: calculation of cumulated first-birth fertility schedule for a period. This schedule, denoted by $\phi_1(i)$, is five times the sum of the values of $f_1(j)$ from the youngest age group up to and including age group i, that is,

$$\phi_1(i) = 5\sum_{j=0}^{i} f_1(j). \qquad \text{(B.9)}$$

Step 4: estimation of equivalent proportions of women with at least one child from information for a period. These proportions, denoted by $F_1(i)$, are estimated by interpolation within the cumulated fertility schedule $\phi_1(i)$. The interpolation procedure is the same as that described in subsection B.2(c) for the method based on all births, except that first-birth fertility rates are substituted for the usual all-birth fertility rates. The general form of the interpolation equation is

$$F_1(i) = \phi_1(i-1) + a(i)f_1(i) + b(i)f_1(i+1) + c(i)\phi_1(7). \qquad \text{(B.10)}$$

The constants $a(i)$, $b(i)$ and $c(i)$ are presented in table 7; if births in a 12-month period have been classified by age of mother at the end of the period, as is normally the case with census or retrospective survey data, constants from part (a) of the table should be used; if births in a 12-month period have been classified by age of mother at the time of the birth, as is usually the case with registration data, constants from part (b) of the table should be used.

Step 5: calculation of a first-birth fertility schedule for conventional five-year age groups. When births in a year have been tabulated by mother's age at the end of the

year, the reported first-birth fertility rates calculated in step 2 will refer to unconventional age groups roughly six months younger than the usual groups. A first-birth schedule for conventional age groups, $f_1^*(i)$, can be obtained by applying equations (B.5) and (B.6), and the constants given in table 8, to the reported schedule. Normally, however, the interest in first-birth fertility rates is limited to the consistency check described below, so that it is often not necessary to convert the reported schedule into a schedule for conventional age groups.

Step 6: selection of an adjustment factor for period fertility. Possible adjustment factors are obtained by calculating the $P_1(i)/F_1(i)$ ratios. $P_1(1)/F_1(1)$ is usually disregarded because the number of events in age group 15-19 is small, and the interpolation procedure is insufficiently flexible to replicate rapid increases with age in the rates for young women. When the assumptions of constant first-birth fertility in the past and relatively good reporting are correct, all values of P_1/F_1 after the first should be much the same. Furthermore, since the proportion of women with at least one child is not expected to decline with age, there is no reason for preferring early values of P_1/F_1 to later values. It is therefore recommended that the adjustment factor K_1 be calculated as the average of any group of consistent ratios, if such a group exists. This adjustment factor can then be multiplied by the observed $\phi_1(7)$ to obtain an adjusted proportion of women who, according to current rates, will become mothers.

Step 7: adjustment of fertility parameters referring to all births. Using the adjustment factor obtained in the previous step, the birth and general fertility rates derived from reported births, and total fertility, can be multiplied by K_1 to obtain what may be interpreted as lower bounds for their true values if first births appear to be more completely reported than all births (that is, if the P_1/F_1 ratios are lower than the P/F ratios). Strictly speaking, the birth rate and the general fertility rate should be computed by adjusting first the age-specific fertility rates based on all births, then calculating the implied number of births by multiplying by the number of women in each age group, cumulating the results, and then dividing by the relevant denominator (the total mid-year population if a birth rate is being calculated or the female population aged 15-49 if the general fertility rate is of interest), though in practice the accuracy gained by this procedure would be small. An adjusted first-birth fertility schedule, $f_1^*(i)$, can also be obtained if required, by multiplying $f_1^*(i)$ (or $f_1(i)$, if data from a vital registration system are being used) by K_1.

(d) *A detailed example*

Table 13 shows data obtained during a demographic survey conducted in Bangladesh in 1974. Data from the same survey were used to illustrate an application of the all-children method in subsection B.2 (d).

The calculations for this example are described below.

Step 1: calculation of reported proportion of mothers. Column (3) of table 14 shows the values for the proportion of mothers, $P_1(i)$, obtained by dividing the numbers

TABLE 13. NUMBER OF WOMEN, NUMBER OF WOMEN WITH AT LEAST ONE CHILD, AND NUMBER OF FIRST BIRTHS DURING THE 12 MONTHS PRECEDING THE SURVEY, BY AGE GROUP, BANGLADESH, 1974

Age group (1)	Number of women (2)	Number of women who reported at least one child (3)	First births during 12 months before the survey (4)
15-19	3 014 706	854 760	224 716
20-24	2 653 155	2 092 845	153 140
25-29	2 607 009	2 416 367	37 050
30-34	2 015 663	1 926 214	6 306
35-39	1 771 680	1 716 374	2 180
40-44	1 479 535	1 425 415	0
45-49	1 135 129	1 090 567	192

appearing in column (3) (women with at least one child) of table 13 by those listed in column (2) (total number of women). Thus, for example, $P_1(2)$ is obtained as

$$P_1(2) = 2{,}092{,}845/2{,}653{,}155 = 0.789.$$

TABLE 14. PROPORTION OF WOMEN WITH AT LEAST ONE CHILD, PERIOD FIRST-BIRTH FERTILITY RATES AND CUMULATED FIRST-BIRTH FERTILITY SCHEDULE, BY AGE GROUP, BANGLADESH, 1974

Age group (1)	Index i (2)	Proportion of women with at least one child $P_1(i)$ (3)	Observed first birth fertility rate $f_1(i)$ (4)	Cumulated first birth fertility schedule $\phi_1(i) = 5\sum_{j=1}^{i} f_1(j)$ (5)
15-19	1	0.284	0.0745	0.3725
20-24	2	0.789	0.0577	0.6610
25-29	3	0.927	0.0142	0.7320
30-34	4	0.956	0.0031	0.7475
35-39	5	0.969	0.0012	0.7535
40-44	6	0.963	0.0000	0.7535
45-49	7	0.961	0.0002	0.7545

Step 2: calculation of period first-birth fertility schedule. The period fertility schedule, $f_1(i)$, for first births is computed by dividing the number of first births in a year (listed in column (4) of table 13) by the total number of women in each age group (listed in column (2) of table 13). As an example, $f_1(3)$ is computed as

$$f_1(3) = 37{,}050/2{,}607{,}009 = 0.0142.$$

The complete set of $f_1(i)$ values is presented in column (4) of table 14.

Step 3: calculation of cumulated first-birth fertility schedule for a period. This schedule, denoted by $\phi_1(i)$, is calculated by adding the first i values of $f_1(j)$ (listed in column (4) of table 14) and multiplying the result by five (since each fertility rate applies to a five-year age group). The calculation of $\phi_1(2)$ and $\phi_1(4)$ is illustrated below:

$$\phi_1(2) = 5(0.0745 + 0.0577) = 0.6610$$

$$\phi_1(4) = 5(0.0745 + 0.0577 + 0.0142 + 0.0031) = 0.7475.$$

Of course, $\phi_1(4)$, for example, could also be calculated as

$$\phi_1(4) = \phi_1(3) + 5 f_1(4) = 0.7320 + 5(0.0031) = 0.7475$$

All values of $\phi_1(i)$ are listed in column (5) of table 14.

Step 4: estimation of equivalent proportions of women with at least one child from current information. Because the data used here were obtained from a sample survey and the recorded age of mother was, on average, six months older than her age when her first child was born, constants from part (a) of table 7 are substituted into equation (B.10) to calculate the proportions of women with at least one child, $F_1(i)$. The calculations of $F_1(1)$, $F_1(3)$ and $F_1(7)$ are shown below:

$$F_1(1) = \phi_1(0) + a(1)f_1(1) + b(1)f_1(2) + c(1)\phi_1(7)$$

$$= 0.0 + (2.531)(0.0745) + (-0.188)(0.0577) + (0.0024)(0.7545)$$

$$= 0.1795$$

$$F_1(4) = \phi_1(3) + a(4)f_1(4) + b(4)f_1(5) + c(4)\phi_1(7)$$

$$= 0.7320 + (3.442)(0.0031) + (-0.563)(0.0012) + (0.0029)(0.7545)$$

$$= 0.7442$$

$$F_1(7) = \phi_1(6) + a(7)f_1(7) + b(7)f_1(6) + c(7)\phi_1(7)$$

$$= 0.7535 + (3.828)(0.0002) + (0.016)(0.0) + (-0.0002)(0.7545)$$

$$= 0.7541.$$

Note that $\phi_1(0)$ is assumed to be zero because no data are available for women younger than 15. If data for these women were available, the value of $\phi_1(0)$ would have to be calculated. The values of $F_1(i)$ are shown in column (4) of table 15.

TABLE 15. REPORTED AND EQUIVALENT PROPORTIONS OF MOTHERS BY AGE GROUP AND P_1/F_1 RATIOS, BANGLADESH, 1974

Age group (1)	Index i (2)	Reported proportion of mothers P_i (3)	Equivalent proportion of mothers F_i (4)	P_1/F_1 ratio (5)
15-19	1	0.284	0.180	1.578
20-24	2	0.789	0.566	1.394
25-29	3	0.927	0.716	1.295
30-34	4	0.956	0.744	1.285
35-39	5	0.969	0.752	1.289
40-44	6	0.963	0.753	1.279
45-49	7	0.961	0.754	1.275

Step 5: calculation of first-birth fertility schedule for conventional age groups. The procedure used in step 5 of the example on all births (subsection B.2 (d)) could be repeated here in order to convert the reported first-birth fertility schedule for age groups that are shifted by six months to a schedule for conventional age groups, $f_1^+(i)$. However, since the first-birth fertility schedule has no intrinsic value, the process of conversion is not generally justified. This step is therefore omitted.

Step 6: selection of an adjustment factor for period fertility. Ratios of retrospective, P_i, to period, F_i, proportions of mothers are calculated for each age group by dividing the values of P_i in column (3) of table 15 by those of F_i in column (4). Except for those for the first two age groups, the ratios are very consistent; excluding those two, the average of the remaining five ratios is 1.285. Note that two general observations made earlier about the results from the first-births method have been fulfilled: first, the ratios remain rather constant as age increases; and secondly, the ratios are, in general, lower than those obtained using all births. Taking the average ratio computed above as an adjustment factor K_1 for the first-birth fertility rates, and adjusting $\phi_1(7)$ by it, one obtains an estimate of the proportion of women who would ultimately become mothers equal to $(1.285)(0.7545) = 0.970$. Since this proportion cannot theoretically exceed 1.0, and in practice is always somewhat lower than 1.0 because of sterility, the adjusted proportion is as high as can be expected; the estimated adjustment factor of 1.285 for first births could not thus be any larger.

Step 7: adjustment of fertility parameters referring to all births. As mentioned earlier, if first births appear to be better reported than all births, K_1 may also be used as a conservative adjustment factor for parameters obtained from data on all births. The following values can therefore be regarded as lower bounds for the true values:

$$b = (0.0313)(1.285) = 0.0402;$$

$$TF = (4.8285)(1.285) = 6.20;$$

$$GFR = (0.1519)(1.285) = 0.1952.$$

In the case of Bangladesh, these values are all smaller than those obtained in subsection B.2 (d), because the adjustment factor used is considerably lower. The very marked difference between the first-birth and all-birth adjustment factors is to be noted. Since the evidence does not support the existence of a fertility decline in Bangladesh that would artificially inflate the P/F ratios for all births, the fact that the adjustment factor derived from all births is larger than that derived from first births suggests that it should be preferred as an adjustment for overall fertility. It is not possible to establish with certainty what mechanism or set of mechanisms leads to the large difference between the two sets of ratios, but it is likely to be related to the fact that, as pointed out earlier, first-birth fertility rates are not entirely independent of the data on children ever born. The interplay between errors in reported children ever born and births in the past year may well lead to first-birth fertility rates that, as in this case, appear to be more complete than those referring to all births.

Hence, in the case of Bangladesh, the P/F ratios based on all births should be preferred as a basis for the adjustment of overall fertility rates, with the proviso that the adjustment factor derived from them should not be

applied to fertility rates specific by birth order. The first-birth method has been useful in establishing that the data are not as internally consistent as one would wish and that the adjustment factor obtained from all births is acceptable, in that it is not obviously inconsistent with other evidence.

4. *Comparison of period fertility rates with average parities for a hypothetical cohort*

(a) *Basis of method and its rationale*

It has been stressed above that the estimation of an adjustment factor for period fertility on the basis of the comparison of cumulated period fertility rates with lifetime average parities is only valid if fertility has been approximately constant during the 15 years or so preceding the time at which the data were collected. If fertility has been changing, cumulated period fertility rates cannot be expected to equal lifetime fertility; and an adjustment factor calculated on the basis of the comparison of the two will reflect not only possible data errors but the effects of changes through time. Hence, its use for correction purposes will tend to obscure the effects of those changes.

One way of avoiding this problem is to compute average parities that refer to the fertility experienced during a particular period and to compare those parities with cumulated average fertility rates measured during the same period. Suitable parities, referring to a particular period rather than to lifetime experience, can be computed if data on children ever born classified by age of mother are available from two surveys; in such a case, the average parities that a hypothetical cohort subject to intersurvey fertility would exhibit can be constructed on the basis of the intersurvey parity increments for true cohorts.

With an interval of five or 10 years between the surveys, the survivors of a cohort of women at the first survey can be identified at the second, and the change in the average parity of the cohort can be calculated. The resulting sequence of parity increments for different cohorts during the period between the surveys can then be cumulated to calculate average parities for a hypothetical cohort experiencing the level of intersurvey fertility implicit in the observed parity increments. Other uses of average parities for hypothetical cohorts are discussed in section C. Note that in deriving this measure of intersurvey fertility it is assumed that mortality and migration have no effect on actual parity distributions; that is, it is assumed that the average parity of those women who die or migrate between the surveys is not significantly different from the average parity at comparable ages of those women who are alive and present at the end of the period.

The "period" fertility rates from which parity equivalents are to be derived for comparison with the intersurvey parities should ideally refer to the entire intersurvey period. Suitable rates can be obtained if registered births classified by age of mother are available for each calendar year of the period. In this case, all births recorded during the period for each age group can be calculated by addition over calendar years; and average intersurvey fertility rates can be obtained by dividing the births by the number of woman-years lived in each age group, estimated from the female population enumerated at the end-points of the period. A simpler, and generally adequate, procedure is to calculate age-specific fertility rates only for the first and last years of the period, and to approximate average intersurvey rates by the arithmetic mean of these two sets. If registered births are not available, but the two surveys gathered data on births in the past year, age-specific fertility rates for the period may by approximated in the same way by averaging the rates observed at the beginning and at the end of the period. When the births during the 12 months preceding each survey are tabulated by age of mother at the time of the survey, the observed fertility rates will correspond to age groups displaced by six months, and the analysis performed will have to take this fact into account. It is, of course, important that the sets of fertility rates being averaged be consistent with respect to age classification before they are averaged; if they are not consistent at first, because one refers to age groups displaced by six months and the other does not, the former set should be adjusted before proceeding. If age-specific fertility rates for the end-points of the period are not available, a set of rates referring approximately to the mid-point of the period could be used. It should be remembered that only the pattern of the intersurvey age-specific fertility rates is important in applying the Brass method, so that if this pattern was more or less constant over the period, the exact reference date of the rates used does not matter.

Once the intersurvey parities and intersurvey fertility rates have been calculated, the cumulation and interpolation of the latter, and their comparison with the average parities, are carried out exactly as described above in subsection B.2(*c*).

(b) *Data required*

The data required are described below:

(*a*) The number of children ever born classified by five-year age group of mother, taken from two surveys or censuses five or 10 years apart;

(*b*) The number of births during the year preceding each survey classified by five-year age group of mother, or registered births by five-year age group of mother for each intersurvey year (if registered births are used, the female population enumerated by censuses at the end-points of the period considered is also necessary);

(*c*) The number of women in each five-year age group from both surveys or censuses;

(*d*) If the birth rate is to be calculated, the total population recorded by each survey or census.

(c) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities from each survey. The average parities obtained from the first survey are denoted by $P(i, 1)$, and those from the second

survey by $P(i, 2)$. In both cases, they are computed by dividing the reported number of children ever born to women in age group i by the total number of women in age group i. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the possible application of the El-Badry correction.

Step 2: calculation of average parities for a hypothetical intersurvey cohort. The way in which these parities are calculated depends upon the length of the intersurvey interval. If this interval is five years, all the survivors of age group i at the first survey belong to age group $i+1$ at the second survey, and the parity increment between the surveys for the corresponding cohort is equal to $P(i+1, 2)-P(i, 1)$. Such increments can be calculated for each age group, and the hypothetical-cohort parities are then obtained by successively cumulating them. Thus, if the parity increment for the cohort of age group i at the first survey is denoted by $\Delta P(i+1)$, and the parity of age group i for the hypothetical cohort is denoted by $P(i, s)$ (where the s stands for "synthetic"), one has

$$\Delta P(i+1)=P(i+1, 2)-P(i, 1) \text{ for } i=1 \ldots 6 \qquad (B.11)$$

and

$$P(i, s)=\sum_{j=1}^{i} \Delta P(j). \qquad (B.12)$$

The parity increment $\Delta P(i+1)$ for the youngest age group ($i=0$) is taken as being directly equal to $P(1, 2)$. If fertility is changing rapidly, this value of $\Delta P(1)$ will reflect period rates somewhat closer to the second survey than to the mid-point of the interval, slightly overallowing therefore for the change in fertility.

If the intersurvey interval is 10 years, then the survivors of the initial cohort of age group i in the first survey will be the women in age group $(i+2)$ in the second; and the hypothetical cohort parities are obtained by cumulating two parallel sequences of parity increments. Once more, for the youngest age groups, $\Delta P(1)$ is taken as being equal to $P(1, 2)$ and $\Delta P(2)$ to $P(2, 2)$. Other parity increments are calculated as

$$\Delta P(i+2)=P(i+2, 2)-P(i, 1) \text{ for } i=1 \ldots 5. \qquad (B.13)$$

Hypothetical-cohort parities for even-numbered age groups are obtained by summing the parity increments for even-numbered age groups, whereas those for odd-numbered age groups are obtained by summing parity increments for odd-numbered age groups. Thus,

$$P(2, s)=\Delta P(2)=P(2, 2)$$

$$P(4, s)=\Delta P(2)+\Delta P(4)$$

and

$$P(6, s)=\Delta P(2)+\Delta P(4)+\Delta P(6)$$

whereas

$$P(1, s)=\Delta P(1)=P(1, 2)$$

$$P(3, s)=\Delta P(1)+\Delta P(3)$$

$$P(5, s)=\Delta P(1)+\Delta P(3)+\Delta P(5)$$

and

$$P(7, s)=\Delta P(1)+\Delta P(3)+\Delta P(5)+\Delta P(7).$$

Step 3: calculation of intersurvey fertility schedule. The method of calculating this schedule, denoted by $f(i)$, depends upon the data available. One possible procedure is to calculate age-specific fertility rates referring roughly to the first and last years of the intersurvey period by using data on the reported number of births during the year preceding each survey. In such a case, for each survey one would divide the reported births for each five-year age group of mother by the reported number of women in the same age group and then obtain age-specific fertility rates for the intersurvey period by calculating the arithmetic mean of each pair of end-point rates. Because of age-group incompatibilities, it is important to avoid combining a schedule derived from a question on births in the past year with another based on registered births; either consistent sources of current fertility rates must be used or the schedule based on births in the past year must be adjusted for the fact that its age classification is likely to be displaced by six months (see step 5 of subsection B.2 (c)). Only when compatibility has been ensured by such an adjustment may averaging be performed. If data on births classified by age of mother are not available for the end-points of the intersurvey period, the use of an age-specific fertility schedule referring approximately to the middle of the period would be acceptable.

Step 4: calculation of cumulated fertility for the hypothetical intersurvey cohort. The calculation of cumulated fertility, denoted by $\phi(i)$, is exactly the same as step 3 in subsection B.2 (c); and its description is not repeated here.

Step 5: estimation of average parity equivalents for the hypothetical intersurvey cohort. The estimation of these equivalents, denoted by $F(i)$, is performed exactly as described in step 4 of subsection B.2 (c). It is not described again here.

Step 6: calculation of a fertility schedule for the usual five-year age groups. This step, which is omitted when the source of current fertility data is a vital registration system, is exactly the same as step 5 described in subsection B.2 (c). Its description is not repeated at this time.

Step 7: adjustment of the intersurvey fertility schedule. This step is exactly the same as step 6 of subsection B.2 (c), and its description is omitted.

(d) *A detailed example*

Data from Thailand for 1960 and 1970 permit the

TABLE 16. FEMALE POPULATION, CHILDREN EVER BORN AND REGISTERED BIRTHS BY AGE GROUP OF WOMEN, THAILAND, 1960 AND 1970

Age group (1)	Female population		Number of children ever born		Number of registered births	
	1960 (2)	1970 (3)	1960 (4)	1970 (5)	1960 (6)	1970 (7)
15-19	1 236 294	1 825 371	88 610	246 839	49 799	95 532
20-24	1 204 153	1 361 717	1 038 074	1 353 569	238 243	299 135
25-29	1 046 464	1 143 377	2 402 581	2 754 376	230 736	267 369
30-34	869 376	1 077 088	3 290 345	4 112 920	183 152	213 814
35-39	679 940	957 607	3 388 799	4 866 424	119 887	154 439
40-44	563 812	766 332	3 232 209	4 537 467	49 348	63 259
45-49	482 966	597 454	2 855 997	3 656 488	10 356	12 345

application of the hypothetical-cohort technique. The censuses in both those years published data on children ever born classified by age of mother, and registered births classified in the same way are available for 1960 and 1970 (years that closely approximate the exact end-points of the intercensal period). Although the intercensal period is not exactly 10 years in length—the census dates were 15 April 1960 and 1 April 1970—it is close enough to being exactly 10 years for a cohort in 1960 to be identified as a cohort 10 years older in 1970. The basic data required are shown in table 16 and the steps of the calculations are given below.

Step 1: calculation of reported average parities from each survey. The two sets of average parities at the end-points of the intercensal period are calculated by dividing the number of children ever born, given in columns (4) and (5), by the total number of women, shown in columns (2) and (3), for each age group. Results are shown in table 17. As an example, the average parity for age group 2 from the 1960 census, denoted by $P(2, 1)$, is obtained as

$$P(2, 1) = 1{,}038{,}074 / 1{,}204{,}153 = 0.8621,$$

and the average parity for age group 4 from the 1970 census, denoted by $P(4, 2)$, is obtained as

$$P(4, 2) = 4{,}112{,}920 / 1{,}077{,}088 = 3.8186$$

TABLE 17. REPORTED AVERAGE PARITIES, 1960 AND 1970, AND PARITIES FOR THE HYPOTHETICAL INTERCENSAL COHORT, BY AGE GROUP, THAILAND

Age group (1)	Average parity		Cohort parity increment $\Delta P(i)$ (3)	Parity for hypothetical cohort $P(i, s)$ (4)
	1960 $P(i, 1)$ (2)	1970 $P(i, 2)$ (3)		
15-19	0.0717	0.1309	(0.1309)	0.1309
20-24	0.8621	0.9940	(0.9940)	0.9940
25-29	2.2959	2.4090	2.3373	2.4682
30-34	3.7825	3.8186	2.9565	3.9506
35-39	4.9840	5.0819	2.7860	5.2542
40-44	5.7328	5.9210	2.1385	6.0890
45-49	5.9135	6.1201	1.1361	6.3903

Step 2: calculation of average parities for a hypothetical intersurvey cohort. The intercensal interval in the case under consideration is 10 years, so the second way of deriving hypothetical-cohort parities, employing two different sums of parity increments, is used. The hypothetical-cohort parity for age group i is denoted by $P(i, s)$. The first two values are obtained directly from the parities reported at the second census; thus,

$$P(1, s) = \Delta P(1) = P(1, 2) = 0.1309$$

and

$$P(2, s) = \Delta P(2) = P(2, 2) = 0.9940.$$

Subsequent values of $P(i, s)$ are obtained by summing certain cohort parity increments, one sum using only increments derived from odd-numbered age groups and another only those from even-numbered age groups. The parity increments, denoted by $\Delta P(i)$ and shown in table 17, are calculated as

$$\Delta P(i+2) = P(i+2, 2) - P(i, 1).$$

Thus, for example, the parity increment for $i = 5$ is calculated as

$$\Delta P(5) = P(5, 2) - P(3, 1) = 5.0819 - 2.2959 = 2.7860.$$

Average parities for a hypothetical intercensal cohort are then obtained by cumulating the cohort parity increments. Since the intercensal period is 10 years, two sums are required: parities for odd-numbered age groups are obtained by adding odd-numbered parity increments, while those for even-numbered age groups are obtained by summing even-numbered parity increments. The resulting average parities are shown in column 5 of table 17; the following two examples illustrate the computational procedure:

$$P(3, s) = \Delta P(1) + \Delta P(3) = 0.1309 + 2.3373 = 2.4682;$$

$$P(6, s) = \Delta P(2) + \Delta P(4) + \Delta P(6)$$

$$= 0.9940 + 2.9565 + 2.1385$$

$$= 6.0890.$$

Step 3: calculation of the intersurvey fertility schedule. Table 16 shows the number of births registered in 1960

and 1970 classified by age of the mother and also the female population enumerated by the 1960 and 1970 censuses classified according to the same age groups. Age-specific fertility rates for 1960 and 1970 are obtained by dividing the registered births for each age group, given in columns (6) and (7), by the enumerated female population of the same age group, shown in columns (2) and (3). Thus, if $f(i, 1)$ denotes the age-specific fertility rate of age group i in 1960 and $f(i, 2)$ denotes the corresponding rate in 1970,

$$f(2, 1) = 238{,}243 / 1{,}204{,}153 = 0.1979$$

and

$$f(4, 2) = 213{,}814 / 1{,}077{,}088 = 0.1985.$$

Values of $f(i, j)$ for all age groups are shown in columns (2) and (3) of table 18. Average fertility rates for the intercensal period 1960-1970, $f(i)$, are then obtained by summing the rates for 1960 and 1970 for each age group and dividing by two. The final $f(i)$ values are shown in column (4) of table 18. The next examples illustrate the computational procedure:

$$f(2) = (f(2, 1) + f(2, 2))/2$$

$$= (0.1979 + 0.2198)/2 = 0.2088$$

and

$$f(5) = (f(5, 1) + f(5, 2))/2$$

$$= (0.1763 + 0.1613)/2 = 0.1688.$$

TABLE 18. AGE SPECIFIC FERTILITY RATES, 1960 AND 1970, AND AVERAGE AGE FERTILITY RATES FOR THE INTERCENSAL PERIOD, THAILAND

Age group (i)	Age-specific fertility rates: 1960 f(i, 1) (2)	1970 f(i, 2) (3)	1960-1970 f(i) (4)
15-19	0.0403	0.0507	0.0455
20-24	0.1979	0.2198	0.2088
25-29	0.2396	0.2338	0.2367
30-34	0.2105	0.1985	0.2045
35-39	0.1763	0.1613	0.1688
40-44	0.0875	0.0825	0.0850
45-49	0.0214	0.0213	0.0214

It should be mentioned that this estimate of the average intercensal fertility schedule, being just an arithmetic mean of the schedules observed at the two end-points, is only approximate in nature. If the age patterns of the fertility schedules at the end-points are very different, it is likely that the age pattern of fertility may have changed sharply during the intercensal period; and a better estimate of an average intercensal fertility schedule would be obtained by averaging the schedules observed during each year of the period or by calculating directly an intercensal schedule for the period by dividing all the births recorded during the period for each age group of mother by an estimate of person-years lived by women in each age group during the same period. In the case of Thailand, the age-specific fertility rates for 1960 and 1970 shown in table 18 are fairly similar and do not suggest the existence of a sharp change in the age pattern of fertility over the intercensal period, so that the $f(i)$ estimates listed in table 18 are adequate.

Fertility rates for 1960 and 1970 have been calculated by using the births registered during each year and the female population enumerated by each census. Strictly speaking, these rates should have been calculated by using estimates of the female population at the middle of each year, rather than the populations enumerated by censuses taking place in April. Since the mid-year populations would be slightly larger than those enumerated, the fertility rates calculated using them as denominators would be slightly lower. However, if the mid-year populations are estimated by applying the same growth rate to the enumerated number of women in each age group, the resulting fertility rates will just be constant multiples of those obtained in table 18 and their age pattern will not be affected. Since only this pattern is relevant in applying the P/F ratio method, the results yielded by it will not be essentially affected by the lack of adjustment of the female population, although, of course, the estimates of the completeness of birth registration obtained are valid only with respect to the population actually used as denominator. In this case, the use of the enumerated population instead of that corresponding to the mid-year would increase the completeness estimates only by about three quarters of 1 per cent.

Step 4: calculation of cumulated fertility for the hypothetical intersurvey cohort. The intercensal fertility rates, $f(i)$, shown in column (4) of table 18 are multiplied by the width of the age group to which they refer and

TABLE 19. CUMULATED FERTILITY SCHEDULE, PARITY EQUIVALENTS, HYPOTHETICAL-COHORT PARITIES, P/F RATIOS AND ADJUSTED INTERCENSAL FERTILITY RATES, THAILAND, 1960-1970

Age group (i)	Cumulative fertility schedule a(i) (1)	Parity equivalent P(i) (2)	Hypothetical cohort parity P(i, j) (4)	P/F ratio P(i, j)/F(i) (5)	Adjusted intercensal fertility rate f'(i) (6)
15-19	0.2275	0.0632	0.1309	2.071	0.0606
20-24	1.2715	0.7193	0.9940	1.382	0.2781
25-29	2.4550	1.6679	2.4682	1.321	0.3153
30-34	3.4775	2.9743	3.9505	1.328	0.2724
35-39	4.3315	3.9218	5.2542	1.340	0.2248
40-44	4.7405	4.5482	6.0890	1.339	0.1132
45-49	4.8539	4.8206	6.3903	1.326	0.0285

summed successively to produce the values of the cumulated fertility schedule, $\phi(i)$. Because all age groups in this instance are five years in length, the process just described is equivalent to that illustrated below for $i = 2$ and 5, where cumulation of the values of $f(i)$ is carried out first and multiplication by the length of the age groups later:

$$\phi(2) = 5(0.0455 + 0.2088) = 1.2715$$

$$\phi(5) = 5(0.0455 + 0.2088 + 0.2367 + 0.2045 + 0.1688)$$

$$= 5(0.8643) = 4.3215.$$

The complete set of $\phi(i)$ values is shown in column (2) of table 19.

Step 5: estimation of average parity equivalents for the hypothetical intersurvey cohort. Since, in this case, information on births by age of mother was obtained from a registration system, the reported age of mother is likely to be the one she had at the time of the birth. Therefore, the values of average parity equivalents, $F(i)$, are calculated by substituting the coefficients from part (b) of table 7 in equation (B.4), which has the form:

$$F(i) = \phi(i-1) + a(i)f(i) + b(i)f(i+1) + c(i)\phi(7).$$

As examples,

$$F(3) = \phi(2) + (2.760)f(3) + (-0.594)f(4) + (0.0133)\phi(7)$$

$$= 1.2715 + (2.760)(0.2367) + (-0.594)(0.2045) + (0.0133)(4.8535)$$

$$= 1.8679$$

and

$$F(7) = \phi(6) + (3.535)f(7) + (-0.007)f(6) + (-0.0002)(4.8535)$$

$$= 4.7465 + (3.535)(0.0214) + (-0.007)(0.0850) + (-0.0002)(4.8535)$$

$$= 4.8206.$$

All values of $F(i)$ are shown in column (3) of table 19.

Step 6: calculation of a fertility schedule for the usual five-year age groups. This step is omitted because in this case the information on births by age of mother was obtained from a registration system where women are likely to report the age they had at the time of delivery.

Step 7: adjustment of the intercensal fertility schedule. Comparable values of $P(i)$ and $F(i)$ are now available for the period 1960-1970, so that P/F ratios can be calculated and an adjustment factor K can be selected from them. Column (3) of table 19 shows the $F(i)$ values, column (4) shows the $P(i, s)$ values copied from table 17 and column (5) shows the $P(i, s)/F(i)$ ratios. These ratios are reasonably consistent, except for the first, the value of which suggests the existence of less complete registration of births by very young mothers; part of this above-average omission affects the P/F ratio for the second age group through its dependence upon $\phi(1)$. Since most of the P/F ratios are consistent, the way in which an adjustment factor is selected is not of great importance; the average of the ratios for age groups 3–6 is likely to be as satisfactory as any, so

$$K = (1.321 + 1.328 + 1.340 + 1.339)/4.0 = 1.332.$$

Column (6) of table 19 shows the adjusted fertility schedule, $f^*(i)$, for the period 1960-1970. Total fertility may be estimated either by summing the $f^*(i)$ values and multiplying by five or by multiplying $\phi(7)$ by K. In either case, the estimate of TF obtained is 6.46, compared with the unadjusted value of 4.85. K is an adjustment factor for registered births, so its reciprocal, $1/K$, is an estimate of the completeness of birth registration, found to be 75.1 per cent (this completeness is measured with respect to the female population enumerated by the censuses). The intercensal birth rate may be estimated by summing total births registered during the years 1960-1969, multiplying the total by K, and dividing by the person-years lived by the entire population from 1960 to 1970.

5. *Comparison of mean number of births registered by a cohort of women with the reported average parity of the same cohort*

(a) *Basis of method and its rationale*

Subsection B.4 presented a method for comparing average parities with average parity equivalents derived from period fertility rates without the necessity of assuming constant fertility. The most important aspect of this method is that average parities are calculated for a period rather than for a series of cohorts. It requires, however, that data on children ever born be available for two points in time, five or 10 years apart. If only one source of data on children ever born exists, or if the intersurvey period is not five or 10 years in length, an alternative procedure that does not require the assumption of constant fertility may be used. However, this procedure requires the availability of a fairly long series of annual data on registered births classified by age of mother. This method makes use of the cohort nature of reported average parities and compares them with parity equivalents obtained from the recorded fertility rates pertaining to the relevant cohorts.

If one considers women aged 30-34 at some census, a year before the census they were aged 29-33, then 10 years before the census they were aged 20-24, and 20 years before the census they were aged 10-14. Therefore, assuming that childbearing effectively begins at age 15, the children ever born reported by these women at the time of the census reflect the cumulated fertility experience of the women over the preceding 20 years. If mor-

mality and migration are assumed to be unrelated to the fertility experience of women, and fertility rates can be calculated for those 20 years, average parity equivalents for each cohort can be constructed and compared with the reported average parity of women at the time of the census. This method is mainly of use with data on births from a vital registration system, which is normally the only source of information about births over a 20-year period; but if fertility schedules are available from other sources for regular five-year intervals, there is no reason to prevent the use of such schedules. The description here, however, is given in terms of data from a vital registration system.

The difficulty with applying this general idea is that a cohort represented by a conventional five-year age group at the time of the census would not have been a conventional five-year age group in earlier years. Thus, the population in age group 30-34 at the time of a census would have been aged 29-33 a year earlier, 28-32 two years earlier and so on. If births are tabulated by single year of age of mother, this problem is not serious, because single-year fertility rates can be calculated for each year and then summed by cohort with relative ease. The calculations would be lengthy, however, and age-heaping might have a non-trivial effect on the fertility rates, so it is convenient to have an approach that can be applied to rates for conventional five-year age groups. Such a procedure is described here.

(b) *Data required*

The data required for this method are described below:

(*a*) The number of children ever born by five-year age group of mother, taken from a census;

(*b*) Registered births by five-year age group of mother for each of 15 or 20 years preceding the census;

(*c*) The number of women in each age group from the census, and from one or more earlier censuses, to allow the estimation of the female population by five-year age group for each of the 15 or 20 years preceding the final census.

(c) *Computational procedure*

The following steps are required for the computational procedure.

Step 1: calculation of reported average parities. Average parities for each age group from the final census, denoted by $P(i)$, are obtained by dividing the number of children ever born reported by women in each age group by the total number of women in each age group. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the use of the El-Badry correction (described in annex II).

Step 2: estimation of mid-year female population by age group for each year preceding the census. The exact procedure to be followed in estimating the series of mid-year female populations by age group depends upon the dates of the census enumerations available, so an attempt is made here to describe the procedure in general terms. It is assumed that census enumerations cover, or almost cover, the 20 years or so for which registered fertility rates are to be cumulated. The reference date of each census should then be calculated in terms of years, the decimal part being obtained by dividing the number of days from 1 January to the date of the census by 365, the number of days in a year. The exponential growth rate, $r(i)$, of each age group i is then obtained by dividing the difference between the natural logarithms of the female population of age group i at the second and first censuses by the length of the intercensal period in years, as shown in equation (B.14):

$$r(i) = (\ln N(i, 2) - \ln N(i, 1))/(t_2 - t_1) \quad \text{(B.14)}$$

where $N(i, j)$ is the female population of age group i at census j; t_1 is the date of the first census expressed in decimals; and t_2 is the date of the second census expressed in the same fashion. The required denominators for each year can then be calculated for each mid-year between t_1 and t_2 by expanding exponentially the initial population for the period using equation (B.15):

$$N(i, \tau) = N(i, 1) \exp(r(i)(\tau + 0.5 - t_1)) \quad \text{(B.15)}$$

where $N(i, \tau)$ is the female population of age group i required as denominator for calendar year τ.

The objective of this method is to measure the completeness of birth registration, with a view to adjusting births registered during a recent period for omission, and thus to estimate the recent levels of fertility. The effects of other errors, such as changes in the completeness of census enumeration through time, should therefore be allowed for before cumulating age-specific fertility rates for comparison with average parities. Hence, when there is evidence suggesting that changes in the completeness of enumeration have taken place, it is desirable to adjust the censuses *before* calculating the population denominators. However, it is not necessary to adjust each census for absolute underenumeration; it is only necessary to ensure that the completeness of enumeration of the different censuses shall be the same.

Step 3: calculation of age-specific fertility rates from births registered during the years preceding the census. Age-specific fertility rates are to be calculated for calendar years, so it is convenient to cumulate the rates to the end of each year. The census providing average parities is unlikely to have as reference date exactly the end of a year, but fortunately average parities for a specified age group change slowly even when fertility is changing rapidly. The parities from the census can therefore be regarded as referring to the year-end nearest to the census date, and registered rates can be cumulated up to the relevant year-end. Thus, if the census date is on or before 30 June, registered fertility rates would be cumulated to the end of the preceding calendar year, whereas if the census date is after 30 June, registered fertility rates would be cumulated to the end of the calendar year during which the census took place.

Age-specific fertility rates are required for a total of 20 calendar years. The rate for age group i and calendar year τ, $f(i, \tau)$, is calculated as

$$f(i, \tau) = B(i, \tau)/N(i, \tau) \qquad \text{(B.16)}$$

where $B(i, \tau)$ is the number of births registered in calendar year τ as having occurred to women of age group i.

If registered births by age of mother are not available for a few of the 20 calendar years required, the application of the method will be only slightly affected if rates for the odd blank year are estimated from neighbouring rates. For example, if registered births are not available for one year in the series, the fertility rates for that year can be estimated as the average of the rates in the preceding and following years. Or if fertility rates are only available for the last 16 of the 20 years required, the rates for the earliest available year can be adopted for the four preceding years without much danger of introducing sizeable errors. However, such extrapolation is more dangerous if more recent years are involved because imputation in this case affects more age groups, is likely to cover more of the years of peak childbearing and is less likely to reflect adequately any changes in fertility that might have actually taken place.

Step 4: cumulation of registered fertility for different female birth cohorts to estimate parity equivalents. As mentioned in step 3, reported average parities are assumed to refer exactly to the end of a calendar year, whereas age-specific fertility rates calculated from registered births refer to whole calendar years. Hence, each age-specific fertility rate encompasses the childbearing experience of two female birth cohorts if the latter are defined with respect to age at the end of a calendar year. It is therefore necessary to split the observed age-specific fertility rates into two parts, each contributed by a cohort for which reported average parity is available, in order to estimate parity equivalents, $F(i)$. For example, one may assume that average parities by five-year age group are available for the end of year t. Women aged 25-29 at the end of year t were aged 24-28 at the beginning of year t, so during year t their cumulated fertility would have been increased by most of the age-specific fertility rate for age group 25-29 in year t (but not by all of it, because some of those aged 29 years who give birth in year t would be age 30 by the end of the year) and by a small amount of the rate for age group 20-24 (to allow for births to those aged 24 years who were 25 by the end of the year). Assuming that, within each age group, the distribution of women by age is rectangular and their fertility by age is constant, 90 per cent of $f(3, t)$, the rate for women 25-29 in year t, and 10 per cent of $f(2, t)$ contribute to the cumulated fertility of women aged 25-29 years at the end of year t. The same women would have begun year $t-1$ aged 23-27 and ended it aged 24-28, so their cumulated fertility would have a rather greater contribution from age group 20-24 and a rather smaller contribution from age group 25-29 than in the previous case; making the same assumptions as above, their cumulated cohort fertility would increase by 70 per cent of $f(3, t-1)$ and by 30 per cent of $f(2, t-1)$ during year $t-1$. It should be noted in passing that the proportions of these age-specific fertility rates that are not contributed to the cohort in question are contributed to another cohort; for example, 30 per cent of $f(3, t-1)$ is added to the cumulated fertility of the cohort aged 30-34 at the end of year t, and 70 per cent of $f(2, t-1)$ is added to that of the cohort aged 20-24. This example shows that the essence of the calculation of cumulated cohort fertility lies in splitting by cohort the fertility rates for each age group and calendar year into two parts and then summing the portions relevant to each cohort.

Unfortunately, the simple assumption made above concerning the constancy of fertility within each age group is not satisfactory for splitting the age-specific fertility rates for younger women; and since the recent fertility experience of these women is that of greatest interest, an alternative estimation procedure is necessary. Hill[7] devised such a procedure. It is based on a set of separation factors derived from the Brass[8] fertility polynomial. The set of factors used in any particular instance depends upon the general shape of the age-specific fertility schedule proposed by Brass and on its specific age-location in the case at hand. The general equation defining cumulated fertility in terms of separation factors and observed annual age-specific fertility rates is

$$F(i, t) = \sum_{j=1}^{i} \left[\sum_{m=1}^{5} \left(s(m, j) f(j, t-k) + (1 - s(m, j-1)) f(j-1, t-k) \right) \right] \qquad \text{(B.17)}$$

where $F(i, t)$ = cumulated fertility of women of age group i at the end of year t;

$s(m, j)$ = separation factor for location m of age group j;

$f(j, t-k)$ = age-specific fertility rate for age group j in year $t-k$, where $k = 5(j-1) + m$.

Values of $s(m, j)$ are listed in table 20, and they should be selected for each calendar year t according to the value of $f(1, t)/f(2, t)$.

Equation (B.17) states that cumulated cohort fertility at the end of year t is the sum of all the portions of the registered fertility rates contributed by women of the cohort. The number of years the sum will cover depends upon the age of the cohort at the end of year t. The oldest members of age group 15-19 at time t would have

[7] Kenneth H. Hill, "Methods for estimating fertility trends using WFS and other data", *World Fertility Survey: Conference, Record of Proceedings*, London, 7-11 July 1980 (Voorburg, The Hague, International Statistical Institute, 1981), vol. 3, pp. 455-506.

[8] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, N.C., Carolina Population Center, Laboratories for Population Studies, 1975).

Table 20. Separation factors for splitting annual age-specific fertility rates by cohort

Age group of fertility rate *Age* (1)	Age of cohort on 31 December (2)	Index *i* (3)	Separation factors *f*(*i*, *j*) for *P*(1)/*P*(2) = 0.7 (4)	0.6 (5)	0.5 (6)	0.4 (7)	0.3 (8)	0.2 (9)	0.1 (10)
15-19	11-15	1	0.062	0.046	0.022	0.007	-	-	-
	12-16	2	0.196	0.180	0.127	0.085	0.033	-	-
	13-17	3	0.406	0.365	0.304	0.222	0.124	0.037	-
	14-18	4	0.626	0.593	0.544	0.474	0.371	0.239	0.040
	15-19	5	0.870	0.857	0.836	0.807	0.764	0.691	0.557
20-24	16-20	1	0.094	0.090	0.086	0.082	0.077	0.069	0.059
	17-21	2	0.288	0.279	0.270	0.261	0.249	0.233	0.209
	18-22	3	0.488	0.478	0.468	0.458	0.444	0.426	0.398
	19-23	4	0.693	0.684	0.677	0.668	0.657	0.642	0.619
	20-24	5	0.898	0.894	0.891	0.888	0.883	0.877	0.868
25-29	21-25	1	0.108	0.105	0.103	0.101	0.099	0.096	0.093
	22-26	2	0.320	0.314	0.309	0.304	0.299	0.293	0.285
	23-27	3	0.525	0.519	0.513	0.507	0.501	0.494	0.486
	24-28	4	0.723	0.717	0.713	0.708	0.703	0.698	0.690
	25-29	5	0.910	0.908	0.906	0.904	0.902	0.900	0.897
30-34	26-30	1	0.121	0.118	0.116	0.114	0.112	0.109	0.107
	27-31	2	0.351	0.344	0.338	0.334	0.329	0.324	0.318
	28-32	3	0.562	0.553	0.547	0.542	0.536	0.530	0.523
	29-33	4	0.752	0.746	0.741	0.736	0.731	0.727	0.721
	30-34	5	0.923	0.920	0.918	0.916	0.914	0.912	0.910
35-39	31-35	1	0.145	0.139	0.134	0.130	0.127	0.124	0.120
	32-36	2	0.404	0.390	0.380	0.371	0.364	0.356	0.349
	33-37	3	0.622	0.606	0.595	0.585	0.576	0.568	0.559
	34-38	4	0.801	0.788	0.779	0.771	0.764	0.758	0.750
	35-39	5	0.942	0.937	0.933	0.930	0.928	0.925	0.922
40-44	36-40	1	0.227	0.198	0.181	0.169	0.159	0.151	0.143
	37-41	2	0.577	0.517	0.482	0.456	0.435	0.417	0.399
	38-42	3	0.805	0.745	0.707	0.680	0.657	0.636	0.617
	39-43	4	0.933	0.893	0.866	0.845	0.828	0.812	0.797
	40-44	5	0.989	0.977	0.967	0.960	0.953	0.947	0.941

reached 15 at the beginning of year $t-4$, so that if no childbearing is assumed to occur before age 15, the sum need only be calculated for the years from $t-4$ to t. For age group 30-34, on the other hand, childbearing occurs during the years from $t-19$ to t, so the sum begins in year $t-19$ with a small portion of the age-specific fertility rate for women aged 15-19 and continues through 20 years, to finish with a large portion of the rate in year t for women aged 30-34, plus a small portion of the rate in year t for women aged 25-29.

In conclusion, it should be mentioned that although the form of equation (B.17) may give the impression that its application is very complicated, the basic idea underlying it is fairly simple, and if working-sheets are laid out following the procedure described in the detailed example the necessary calculations are tedious but straightforward.

Step 5: estimation of completeness of birth registration. The cumulated cohort fertility from registered births, $F(i)$, calculated in the previous step has been constructed so as to be comparable to reported cohort parity, $P(i)$, at the final census. Therefore, the ratio $F(i)/P(i)$ provides a measure of the average completeness of registration of the births that occurred to cohort i. If the completeness of registration had remained approximately constant over a period of 15 years or so, the F/P ratios should have more or less the same values for all cohorts, and an average of the ratios for age groups 20-24, 25-29 and 30-34 can be adopted as an estimate of the completeness of birth registration over the period. Hence, its reciprocal can be used as an adjustment factor for any or all of the age-specific fertility schedules calculated in step 3. If the completeness of birth registration has improved over time, the F/P ratios for the younger cohorts will be higher than for older cohorts. In such a case, the most recent fertility schedule (referring to the year of the final census) may be adjusted by $P(2)/F(2)$, the ratio reflecting the most recent level of completeness; $P(1)/F(1)$ should not be used in general as an adjustment factor because of the intrinsic difficulty in approximating $F(1)$ accurately. When the F/P ratios indicate that completeness has been improving through time, no obvious basis exists for adjusting the fertility schedules referring to earlier years.

(d) *A detailed example*

The data required for the application of the method described in this section are available for Thailand prior to 1970. Table 21 shows registered births classified by five-year age group of mother for the period 1950-1969 (only the births for women under 35 are shown, since the cumulation of cohort fertility is only carried out up to the cohort of women aged 30-34 at the end of the period). Although not shown, some of the births registered could not be classified according to age of mother because the latter information was missing. Therefore, there is a question as to what to do with these births. If their proportion over all registered births is not large and it does not change much over time, the

difference introduced by distributing them according to births of known age of mother will be minor. Hence, the simplest procedure is to exclude them, in which case the estimates of registration completeness obtained refer only to births of known age of mother. If the births with unspecified age of mother are a substantial proportion of all registered births, or if their proportion changes over time, their redistribution and addition to the births with mothers of known age is likely to affect the results; because there is usually no sound basis to determine how their redistribution with respect to age of mother should be performed, the safest procedure is to exclude them from the analysis. In table 21, they have been excluded. Table 22 shows the female population enumerated by the censuses of 1947, 1960 and 1970, and the number of children ever born classified by age of mother from the 1970 census.

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities. The average parities by age group of mother, $P(i)$, are calculated for 1970 from the data displayed in table 22 by dividing the reported number of children ever born in each age group (column (5)) by the total number of women in each age group in 1970 (column (4)). Results for each age group are shown in column (6) of table 22; as an example,

$$P(4) = (4{,}112{,}920/1{,}077{,}088) = 3.819.$$

Step 2: estimation of mid-year female population by age group for each year preceding the census. Table 22 shows the reference dates of the population censuses of 1947, 1960 and 1970. These dates can be converted into units of years by calculating the number of days from the beginning of the year to the census date and dividing it by 365 (or 366 in a leap year). The reference date of the 1947 census was 23 May 1947. There are 31 days in January, 28 in February (29 in a leap year), 31 in March, 30 in April; and in this case, 23 of the 31 days in May are needed. The decimal portion of the census date is thus (31 + 28 + 31 + 30 + 23)/365, or 143/365, = 0.392. Hence, the full decimal date is 1947.392. Because 1960 was a leap year, the decimal portion of the date is (31 + 29 + 31 + 25)/366 = 0.317, and the full decimal date is 1960.317. The 1970 date is 1970.249.

At this point, any adjustments necessary to improve the consistency of the completeness of enumeration of the censuses should be applied. Application of a method described in chapter IX suggests that the 1970 census was about 5 per cent less complete than the 1960 census, so the raw 1970 census figures appearing in column (4) of table 22 have been adjusted by a factor of 1.05 and are shown in column (7) of the same table. The 1947 and 1960 censuses appear to have achieved approximately the same level of completeness. Thus, columns (2), (3) and (7) of table 22 are regarded as showing an approximately consistent set of population figures.

Age-specific population growth rates are then calculated for both intercensal periods using equation (B.14) and the exact census dates calculated above. Thus, for the period 1947-1960 and for the female population aged 15-19 ($i = 1$):

$$r(1) = |\ln(1{,}236{,}294) - \ln(979{,}613)|/|1960.317 - 1947.392|$$

$$= |14.02763 - 13.79491|/12.925$$

$$= 0.0180.$$

The growth rates for all age groups and for both intercensal periods are shown in table 23; it will be noticed

TABLE 21. REGISTERED BIRTHS, BY FIVE-YEAR AGE GROUP OF MOTHER, THAILAND, 1950-1969

Year (1)	Age group 15-19 (2)	20-24 (3)	25-29 (4)	30-34 (5)
1950	32 352	139 307	131 729	101 258
1951	31 422	140 562	137 998	107 652
1952	31 083	150 681	141 402	104 569
1953	35 097	172 015	160 075	110 508
1954	38 528	189 795	182 055	125 260
1955	40 323	196 868	185 435	127 204
1956	44 611	216 365	207 804	143 083
1957	44 160	210 923	208 855	147 567
1958	43 607	209 405	212 248	153 627
1959	45 445	224 553	235 221	170 281
1960	49 799	238 243	250 736	183 152
1961	51 048	239 183	253 366	184 200
1962	53 686	246 039	263 988	196 153
1963	57 335	253 232	273 504	205 771
1964	64 153	270 198	299 972	231 271
1965	75 314	265 482	289 160	227 407
1966	81 238	260 053	274 279	215 830
1967	84 833	268 539	271 177	222 601
1968	92 338	295 524	288 179	236 962
1969	91 788	285 713	267 990	215 613

TABLE 22. TOTAL POPULATION AND FEMALE POPULATION AGED 15-34, BY AGE GROUP, FOR THE CENSUS YEARS 1947, 1960 AND 1970; CHILDREN EVER BORN IN 1970 AND ADJUSTED POPULATION IN 1970, THAILAND

Age group (1)	Enumerated female population: 1947 23 May (2)	1960 23 April (3)	1970 1 April (4)	Children ever born, 1970 (5)	Average parity, 1970 (6)	Adjusted female population, 1970 (7)
15-19	979 613	1 236 294	1 885 371	246 839	0.131	1 979 640
20-24	792 366	1 204 153	1 361 717	1 353 569	0.994	1 429 803
25-29	638 334	1 046 464	1 143 377	2 754 376	2.409	1 200 546
30-34	570 842	869 876	1 077 088	4 112 920	3.819	1 130 942
Total population	8 681 257	13 109 767	17 273 512			18 137 188

that they are rather variable, suggesting the existence of differential age-misreporting or fluctuations by age in enumeration completeness. However, beyond noting the possibility of errors, not much else can be done.

Mid-year population denominators can now be calculated for each year, from 1950 to 1969, by applying equation (B.15). A few examples will make the procedure clear. The female population aged 15-19 in mid-1950 (that is, at 1950.5 in decimal terms) is needed.

TABLE 23. AGE-SPECIFIC GROWTH RATES FOR THE FEMALE POPULATION, 1947-1960 AND 1960-1970, AFTER ADJUSTMENT: THAILAND

Age group (1)	Factor i (2)	Growth rate of female population: 1947-1960 (3)	1960-1970 (4)
15-19	1	0.01801	0.04740
20-24	2	0.03238	0.01729
25-29	3	0.03824	0.01383
30-34	4	0.03259	0.02643

The growth rate for age group 15-19 between 1947 and 1960 is 0.01801, and the period from the 1947 census to mid-1950 is 1950.5 − 1947.392 years. Hence, the estimated female population in 1950.5, $N(1, 1950)$, is obtained from the 1947 census population, $N(1, 1947)$, as

$$N(1, 1950) = N(1, 1947)\exp((0.01801)(1950.5 - 1947.392))$$
$$= (979{,}613)\exp((0.01801)(3.108))$$
$$= (979{,}613)(1.05757)$$
$$= 1{,}036{,}009.$$

For 1959 and age group 25-29,

$$N(3, 1959) = N(3, 1947)\exp((0.03824)(1959.5 - 1947.392))$$
$$= (638{,}334)(1.588849)$$
$$= 1{,}014{,}216.$$

For 1960, however, the 1960-1970 growth rate would be used:

$$N(3, 1960) = N(3, 1960)\exp((0.01383)(1960.5 - 1960.317))$$
$$= (1{,}046{,}464)(1.002534)$$
$$= 1{,}049{,}116.$$

The estimated mid-year populations are shown in table 24. Note that, for the sake of clarity, the calculations have been performed with a higher number of significant digits than is really required. For efficiency of calculation, it would be worth working with only four significant digits, rounding the female populations in each case to thousands.

Step 3: calculation of age-specific fertility rates from registered births. Age-specific fertility rates for each year are calculated by dividing the number of births registered for each age group of women (table 21) by the estimated mid-year female population of the age group (table 24). Thus, the age-specific fertility rate for age group 15-19 in 1950 is calculated as

$$f(1, 1950) = B(1, 1950)/N(1, 1950)$$
$$= 32{,}352/1{,}036{,}009$$
$$= 0.0312.$$

TABLE 24. ESTIMATED MID-YEAR FEMALE POPULATION, BY AGE GROUP: THAILAND, 1950-1969

Year (1)	Age group: 15-19 (2)	20-24 (3)	25-29 (4)	30-34 (5)
1950	1 036 009	876 238	718 892	611 692
1951	1 054 838	905 095	746 915	632 619
1952	1 074 008	934 882	776 030	654 237
1953	1 093 526	965 649	806 280	676 573
1954	1 113 399	997 428	837 710	699 648
1955	1 133 633	1 030 254	870 364	723 488
1956	1 154 235	1 064 159	904 291	748 117
1957	1 175 211	1 099 181	939 541	773 562
1958	1 196 568	1 135 355	976 165	799 851
1959	1 218 313	1 172 719	1 014 216	827 010
1960	1 247 065	1 207 969	1 049 116	874 094
1961	1 307 599	1 229 036	1 063 726	897 504
1962	1 371 036	1 251 471	1 078 539	921 541
1963	1 437 625	1 272 280	1 093 559	946 222
1964	1 507 409	1 294 469	1 108 788	971 564
1965	1 580 581	1 317 045	1 124 229	997 585
1966	1 657 304	1 340 015	1 139 885	1 024 303
1967	1 737 752	1 363 385	1 155 760	1 051 736
1968	1 822 106	1 387 163	1 171 855	1 079 904
1969	1 910 552	1 411 355	1 188 174	1 108 826

Similarly, the rate for those aged 25-29 years in 1960 is calculated as

$$f(3, 1960) = B(3, 1960)/N(3, 1960)$$

$$= 250,736/1,049,116$$

$$= 0.2390.$$

All values of $f(i, j)$ are shown in table 25. For the period 1960-1969, age-specific fertility rates for women aged 35-39, 40-44 and 45-49 are also shown. These rates are not needed to apply the method described here, but they are needed in applying that described in subsection B.6.

Step 4: cumulation of registered fertility for different female birth cohorts to estimate parity equivalents. Some of the age-specific fertility rates shown in table 25 are not required in cumulating the fertility for the cohorts of interest since they reflect entirely the childbearing of cohorts older than 30-34 in 1970; to be specific, the rates above the dotted lines are not needed and have been calculated in part for the sake of completeness and in part to be used in subsection B.6. All the rates below the lines, however, need to be split between two cohorts in order to estimate parity equivalents, $F(i)$; and the portions corresponding to each cohort need to be cumulated separately. A convenient way of carrying out this process is to work with five-year periods. Table 26 shows the recommended way of laying out the calculations. The 20-year period considered is divided into four subperiods, each five years in length; within each period, each year is identified by an index value m ranging from 1 to 5, indicating the location of the cohorts with respect to the age groupings of the age-specific fertility rates. Thus, for 1969, m is equal to 5; and the cohorts are, on average, six months younger than the ages indicated by the age groups of the fertility rates; the cohort aged 30-34 at the end of 1969 began the year aged 29-33, for instance. For 1965, m is equal to 1; and the cohorts are, on average, 4.5 years younger than the ages indicated by the fertility rate groupings. The index m is used to select the separation factors necessary to split the observed fertility rates (see table 20). The other indices employed in table 26 are: k, a measure of years before the census; i, the cohort index; and j, which indicates the age range of the age-specific fertility rates being split.

For each group of five calendar years, the procedure is the same, though the number of rates that are split declines by one each time one moves five years into the past. The first step is to calculate, for each year, the value of $f(1)/f(2)$, the ratio of the age-specific fertility rate for women aged 15-19 to that for women aged 20-24. This ratio is an indicator of the age pattern of early childbearing, and its value is necessary for interpolating between the columns of table 20. Then, the $f(1, j)$ fertility rates for each of the five years preceding the census are split and the portion of each rate belonging to the census cohort aged 15-19, and its complement, belonging to the census cohort aged 20-24, are identified. The separation factors for these rates are obtained from table 20 and depend upon the index m, the age group j and the value of $f(1)/f(2)$. For 1969, m is 5 and $f(1)/f(2)$ is 0.2372. The required separation factor is therefore a value between 0.764 for an $f(1)/f(2)$ of 0.3, and 0.691

TABLE 25. AGE-SPECIFIC FERTILITY RATES CALCULATED FROM REGISTERED BIRTHS AND INTERPOLATED MID-YEAR FEMALE POPULATION, THAILAND, 1950-1969

Year (1)	Age group: 15-19 (2)	20-24 (3)	25-29 (4)	30-34 (5)	35-39 (6)	40-44 (7)	45-49 (8)
1950	0.0312	0.1590	0.1832	0.1603	-	-	-
1951	0.0298	0.1553	0.1848	0.1650	-	-	-
1952	0.0289	0.1612	0.1822	0.1551	-	-	-
1953	0.0321	0.1781	0.1985	0.1586	-	-	-
1954	0.0346	0.1907	0.2173	0.1741	-	-	-
1955	0.0356	0.1911	0.2131	0.1711	-	-	-
1956	0.0386	0.2033	0.2298	0.1863	-	-	-
1957	0.0376	0.1919	0.2223	0.1560	-		
1958	0.0364	0.1846	0.2174	0.1834	-	-	-
1959	0.0373	0.1915	0.2319	0.2010	-	-	-
1960	0.0399	0.1972	0.2390	0.2095	0.1751	0.0870	0.0213
1961	0.0390	0.1946	0.2384	0.2025	0.1626	0.0796	0.0197
1962	0.0392	0.1968	0.2466	0.2129	0.1753	0.0855	0.0216
1963	0.0399	0.1990	0.2538	0.2175	0.1803	0.0870	0.0219
1964	0.0426	0.2087	0.2705	0.2380	0.1965	0.0935	0.0239
1965	0.0436	0.2016	0.2577	0.2280	0.1884	0.0926	0.0235
1966	0.0490	0.1941	0.2406	0.2107	0.1736	0.0874	0.0228
1967	0.0488	0.1970	0.2400	0.2117	0.1717	0.0869	0.0247
1968	0.0501	0.2130	0.2461	0.2176	0.1795	0.0906	0.0234
1969	0.0480	0.2024	0.2255	0.1945	0.1591	0.0824	0.0225

TABLE 26 CALCULATION OF PARITY EQUIVALENTS FOR DIFFERENT FEMALE BIRTH COHORTS, THAILAND 1950-1969

Age groups			Time					
			1965	1966	1967	1968	1969	
			-4	-3	-2	-1	0	
			1	2	3	4	5	
i	j	f(i)/f(j)=	0.2361	0.2344	0.2417	0.2380	0.2332	Sum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					(a) 1965-1969			
1	1	$f(1)$	0.0476	0.0490	0.0488	0.0507	0.0480	-
		$s(m,1)$	0.0000	0.0070	0.0730	0.2920	0.3180	-
		Cohort 1	0.0000	0.0003	0.0036	0.0148	0.0345	0.0532
		Cohort 2	0.0476	0.0487	0.0452	0.0359	0.0135	0.1909
2	2	$f(2)$	0.2016	0.1941	0.1970	0.2130	0.2024	-
		$s(m,2)$	0.0720	0.2410	0.4350	0.6480	0.8790	-
		Cohort 2	0.0145	0.0468	0.0857	0.1380	0.1779	0.4629
		Cohort 3	0.1871	0.1413	0.1113	0.0750	0.0245	0.5432
3	3	$f(3)$	0.2577	0.2406	0.2400	0.2461	0.2235	-
		$s(m,3)$	0.0970	0.2960	0.4970	0.7000	0.9010	-
		Cohort 3	0.0250	0.0712	0.1193	0.1723	0.2032	0.5910
		Cohort 4	0.2327	0.1694	0.1207	0.0738	0.0223	0.6189
4	4	$f(4)$	0.2280	0.2107	0.2117	0.2176	0.1945	-
		$s(m,4)$	0.1160	0.3270	0.5350	0.7290	0.9130	-
		Cohort 4	0.0231	0.0689	0.1128	0.1586	0.1776	0.5410
		Cohort 5	0.2029	0.1418	0.0989	0.0590	0.0169	0.5195
5	5	$f(5)$	0.1884	0.1736	0.1717	0.1795	0.1591	-
		$s(m,5)$	0.1160	0.3270	0.5330	0.7290	0.9130	-
		Cohort 5	0.0207	0.0568	0.0915	0.1309	0.1453	0.4452
		Cohort 6	0.1677	0.1168	0.0802	0.0486	0.0138	0.4271
6	6	$f(6)$	0.0926	0.0874	0.0869	0.0906	0.0824	-
		$s(m,6)$	0.1250	0.3600	0.5730	0.7600	0.9260	-
		Cohort 6	0.0116	0.0315	0.0497	0.0689	0.0763	0.2380
		Cohort 7	0.0810	0.0559	0.0372	0.0237	0.0061	0.2020
7	7	$f(7)$	0.0235	0.0238	0.0247	0.0234	0.0225	-
		$s(m,7)$	0.1540	0.4260	0.6460	0.8180	0.9490	-
		Cohort 7	0.0036	0.0101	0.0160	0.0191	0.0214	0.0702
		Cohort 8	(0.0199)	(0.0137)	(0.0087)	(0.0043)	(0.0011)	(0.0477)

Age groups			Time					
			1960	1961	1962	1963	1964	
			-9	-8	-7	-6	-5	
			1	2	3	4	5	
i	j	f(i)/f(j)=	0.2003	0.2004	0.1992	0.2000	0.2001	Sum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					(b) 1960-1964			
2	1	$f(1)$	0.0399	0.0390	0.0392	0.0399	0.0426	-
		$s(m,1)$	0.0000	0.0000	0.0270	0.2400	0.6940	-
		Cohort 2	0.0000	0.0000	0.0011	0.0096	0.0296	0.0403
		Cohort 3	0.0399	0.0390	0.0381	0.0303	0.0130	0.1603
3	2	$f(2)$	0.1972	0.1946	0.1968	0.1990	0.2085	-
		$s(m,2)$	0.0690	0.2330	0.4260	0.6420	0.8770	-
		Cohort 3	0.0136	0.0453	0.0838	0.1278	0.1830	0.4535
		Cohort 4	0.1836	0.1493	0.1130	0.0712	0.0251	0.5428
4	3	$f(3)$	0.2390	0.2384	0.2466	0.2538	0.2705	-
		$s(m,3)$	0.0960	0.2930	0.4940	0.6980	0.9000	-
		Cohort 4	0.0229	0.0699	0.1218	0.1772	0.2435	0.6353
		Cohort 5	0.2161	0.1685	0.1248	0.0766	0.0271	0.6131
5	4	$f(4)$	0.2095	0.2025	0.2129	0.2175	0.2380	-
		$s(m,4)$	0.1090	0.3240	0.5300	0.7270	0.9120	-
		Cohort 5	0.0228	0.0656	0.1128	0.1581	0.2171	0.5764
		Cohort 6	0.1867	0.1369	0.1001	0.0594	0.0209	0.5040
6	5	$f(5)$	0.1751	0.1626	0.1753	0.1803	0.1965	-
		$s(m,5)$	0.1240	0.3560	0.5680	0.7580	0.9250	-
		Cohort 6	0.0217	0.0579	0.0996	0.1367	0.1818	0.4977
		Cohort 7	0.1534	0.1047	0.0757	0.0436	0.0147	0.3921
7	6	$f(6)$	0.0870	0.0796	0.0855	0.0876	0.0935	-
		$s(m,6)$	0.1510	0.4170	0.6360	0.8120	0.9470	-
		Cohort 7	0.0131	0.0332	0.0544	0.0711	0.0885	0.2603
		(Cohort 8)	(0.0739)	(0.0464)	(0.0281)	(0.0165)	(0.0050)	(0.1729)

TABLE 26 *(continued)*

Age group			Time					
			1951	1956	1957	1958	1959	
			−14	−15	−12	−11	−10	
			1	2	3	4	5	Sum
i (1)	j (2)	$f(1)/f(2)=$ (3)	= 1963 (4)	= 1969 (5)	0.1958 (6)	0.1974 (7)	0.1948 (8)	(9)
				(c) *1955-1959*				
3	1	$f(1)$	0.0356	0.0386	0.0376	0.0364	0.0373	.
		$s(m, 1)$	0.0000	0.0000	0.0260	0.2340	0.6840	.
		Cohort 3	0.0000	0.0000	0.0010	0.0085	0.0255	0.0350
		Cohort 4	0.0356	0.0386	0.0366	0.0279	0.0118	0.1505
4	2	$f(2)$	0.1917	0.2033	0.1919	0.1844	0.1915	.
		$s(m, 2)$	0.0680	0.2310	0.4250	0.6410	0.8770	.
		Cohort 4	0.0130	0.0470	0.0816	0.1187	0.1679	0.4277
		(Cohort 5)	0.1781	0.1563	0.1103	0.0667	0.0236	0.5345
			Time					
Age group			1950	1951	1952	1953	1954	
			−19	−18	−17	−16	−15	
			1	2	3	4	5	Sum
i (1)	j (2)	$f(1)/f(2)=$ (3)	0.1962 (4)	0.1919 (5)	0.1939 (6)	0.1980 (7)	0.1918 (8)	(9)
				(d) *1950-1954*				
4	1	$f(1)$	0.0312	0.0298	0.0289	0.0321	0.0346	.
		$s(m, 1)$	0.0000	0.0000	0.0210	0.2000	0.6670	.
		Cohort 4	0.0000	0.0000	0.0006	0.0064	0.0231	0.0301
		(Cohort 5)	0.0312	0.0298	0.0283	0.0257	0.0115	0.1265

Note. i = cohort index; j = age range of the age-specific fertility rates being split; k = measure of years before the census; m = index value indicating the location of cohorts with respect to the age groupings of the age-specific fertility rates; $f(1)/f(2)$ = ratio of the age-specific fertility rate for women aged 15-19 to that for women aged 20-24.

for an $f(1)/f(2)$ of 0.2. The required value can be found by linear interpolation, as shown below:

$$s(5, 1) = (0.764)(0.372) + (0.691)(1.0 - 0.372) = 0.738.$$

Note that the interpolation factor, 0.372, is simply the value of $f(1)/f(2)$, 0.2372, less the lower bound of the interval, 0.2, and divided by the width of the interval, 0.1. It is thus simple to calculate by moving the decimal point of $f(1)/f(2)$ one place to the right and taking only the decimal part of the resulting number. The portion of the fertility rate belonging to the youngest female census cohort considered is then found by multiplying this rate by the separation factor; for 1969, the resulting portion of $f(1, 1969)$ to be cumulated into $F(1)$ is (0.738)(0.0480), or 0.0345. The remainder, 0.0135, is the portion of $f(1,1969)$ contributed by the second census cohort (aged 20-24), and it is recorded for later cumulation. In the case of the first cohort, all its childbearing has occurred during the five years immediately preceding the census, so its cumulated fertility at the end of 1969, $F(1)$, is just the sum of all the portions belonging to the cohort for the years from 1965 through 1969:

$$F(1) = 0.0 + 0.0003 + 0.0036 + 0.0148 + 0.0345 = 0.0532.$$

This sum appears in column (9) of table 26, part (a). The complementary portions of each rate are also cumulated; and their sum, 0.1909, is recorded in column (9) for later use.

The same process is then repeated using the fertility rates for the next age group, 20-24. Separation factors are again obtained from table 20, on the basis of the values of m and of the observed $f(1)/f(2)$. Thus, for 1966, m is 2 and $f(1)/f(2)$ is 0.2524, so the interpolation factor is 0.524; and $s(m, 2)$ is obtained as

$$s(m, 2) = (0.249)(0.524) + (0.233)(1.0 - 0.524)$$

$$= 0.241.$$

The portion of $f(2, 1966)$ contributed by the cohort aged 20-24 at the time of the census (cohort 2) is thus $f(2, 1966)(0.241)$, or 0.0468; its complement, 0.1473, has been contributed by the third cohort (aged 25-29 in 1970). The total contribution of cohort 2 to the fertility rates for age group 20-24 during the period 1965-1969 is then found by adding the estimated contributions for each year of the period, giving 0.4629; the complement of this quantity, 0.5452, is the contribution of cohort 3. Both values are recorded in column (9).

It is now possible to calculate the fertility accumulated by the cohort aged 20-24 in 1970 during the five years preceding the census. It is the sum of the cohort's contribution to the age-specific fertility rates for those aged 15-19 (0.1909) and its contribution to the rates for those aged 20-24 (0.4629). This sum, 0.6538, is not yet an estimate of the total cumulated fertility for the cohort, since it does not include the cohort's childbearing experience during the period from 6 to 10 years before the census; the missing estimates are calculated in part (b) of table 26.

Similar calculations are carried out for the age-specific fertility rates for women aged 25-29 and 30-34 registered during the period 1965-1969, each rate being divided up by cohort contribution and the portions attributable to each cohort being cumulated over the five years. The portion of the rates for those aged 30-34

TABLE 27. CONTRIBUTIONS TO COHORT FERTILITY, BY AGE GROUP AND FIVE-YEAR PERIOD: THAILAND, 1950-1969

Age group (1)	Number of years ago (2)	Reference period (3)	Age group of cohort at end of 1969			
			15-19 (4)	20-24 (5)	25-29 (6)	30-34 (7)
15-19	15-19	1950-1954	-	-	-	0.0301
	10-14	1955-1959	-	-	0.0350	0.1505
	5-9	1960-1964	-	0.0403	0.1603	-
	0-4	1965-1969	0.0532	0.1909	-	-
20-24	10-14	1955-1959	-	-	-	0.4277
	5-9	1960-1964	-	-	0.4535	0.5428
	0-4	1965-1969	-	0.4629	0.5457	-
25-29	5-9	1960-1964	-	-	-	0.6353
	0-4	1965-1969	-	-	0.5910	0.6189
30-34	0-4	1965-1969	-	-	-	0.5430
Parity equivalent, $F(i)$			0.0532	0.6941	1.7850	2.9483

that is contributed by women aged 35-39 at the time of the census is not needed in this application, but it has been calculated for the sake of completeness. The calculations in parts (a) and (b) of table 26 have in fact been extended to cover all the childbearing experience of cohorts aged 35-49 in 1970, mainly because these estimates are used later in subsection B.6.

Once the calculations for the five years immediately preceding the 1970 census have been completed, the same procedure is applied for each of the years of the period 1960-1964. The results are shown in part (b) of table 26. Note how the fertility rates for the oldest age group are excluded. Strictly speaking, only those for the first three age groups need to be included, since the oldest cohort considered, those aged 30-34 in 1970, was aged under 30 in 1965. Once the rates for the period from 6 to 10 years (1960-1964) before the census have been split and cumulated, the same procedure is applied to the rates for the period from 11 to 15 years before (it need only be applied to the first two age-specific fertility rates) as shown in part (c) of table 26 and to those for the period from 16 to 20 years before (for only the first age-specific rate) as shown in part (d) of the same table.

The final parity equivalents, $F(i)$, can now be obtained by summing the portions contributed by each cohort for each five-year period. To avoid errors, such portions can be copied from column (9) of table 26 (labelled "sum") and arranged in columns by cohort, age group of the fertility rates from which the contribution came and period to which the rates refer, as shown in table 27. Cumulation of the entries in each column leads to the desired $F(i)$ values.

It is worth taking note that table 27 may reveal other interesting features of the registration data. Consider, for example, the set of contributions to cohort fertility of the period fertility rates for age group 15-19; the diagonals show the change in such contributions through time. Reading from right to left, it is clear that these contributions have been increasing, indicating either rising fertility among this age group or improving registration of their births. Given the assumptions on which this estimation method is based, the former situation will not affect a final estimate of completeness of birth registration, but the latter will, and the estimates of completeness obtained in such circumstances will be averages of recent levels.

Step 3: estimation of completeness of birth registration. For each cohort, the completeness of birth registration is estimated as the ratio of the parity equivalent, $F(i)$, calculated from registered births to reported average parity, $P(i)$, as obtained from the census. Table 28 shows the results.

The estimates of completeness of registration of births for women aged 20-24, 25-29 and 30-34 are moderately consistent, suggesting an average level of completeness of around 74 per cent. The estimate for women aged 15-19 is very low, however, and the other estimates rise

TABLE 28. ESTIMATES OF COMPLETENESS OF BIRTH REGISTRATION: THAILAND, 1969

Age group of [illegible] (1)	Reported average parity $P(i)$ (2)	Parity equivalent $F(i)$ (3)	Completeness of birth registration $R(i)=F(i)/P(i)$ (4)
15-19	0.131	0.0532	0.406
20-24	0.994	0.6941	0.698
25-29	2.409	1.7850	0.741
30-34	3.819	2.9483	0.772

with age of woman. It seems likely that registration of births is substantially less complete among very young women and that this differential also reduces the completeness estimate derived from the reports of women aged 20-24. The estimates of completeness based on the reports of women aged 25-29 and 30-34 are affected relatively little by the excess omission at early ages, so that in this case a better estimate of average completeness of birth registration would be an average of 0.741 and 0.772, the final estimate therefore being 0.757. Hence, an estimate of fertility for 1969 could be obtained by inflating the registered age-specific fertility rates for that year by a factor of 1.0/0.757, or 1.321. It should be noted, however, that the adjusted fertility schedule might not be a good indicator of the age pat-

tern of childbearing, because of the relatively higher omission of births by young women.

The results presented in table 28 do not suggest that birth registration completeness has been changing, so the adjustment factor of 1.321 can also be applied to the observed age-specific fertility rates for years preceding 1969. However, because the contributions of fertility rates registered before 1960 to cumulated cohort fertility are small, the estimated adjustment factor cannot be validly applied to the fertility rates registered before 1960. Although not observed in the case of Thailand, evidence of a trend towards more complete registration, such as a tendency for the estimates of completeness to decline with age, should warn against adjusting registered births for particular years.

6. *Comparison of the cohort fertility registered between two censuses with cohort parity increments*

(a) *Basis of method and its rationale*

When information on average parity from two censuses (or surveys) is available in such a way that cohort parity increments can be calculated (see subsection B.4) and age-specific fertility rates can be calculated from registered births for the intersurvey period, a more specific version of the method described in the previous section can be applied. The change in parity of a cohort as it ages from one census or survey to the next is a measure of the childbearing experience of the cohort during the interval. An equivalent measure can be calculated from registered births by splitting the fertility rates registered during the interval by cohort and cumulating the contributions of each cohort. The ratio of cumulated cohort fertility during the interval estimated from period rates, such as those calculated from registered births, to the cohort change in average parity from the beginning to the end of the interval provides a measure of completeness of birth registration specific both to a cohort and to a time period. The main assumptions made in the previous section in order to adjust period fertility on the basis of cumulated cohort fertility from the beginning of childbearing, namely, that registration completeness be constant both by age of mother and by period, are no longer required. On the other hand, parity increments are very sensitive to changes in the completeness of reporting of children ever born, as a result of which the estimates of registration completeness obtained by the method described in this section are also quite sensitive to such changes, which are generally most marked for older women.

(b) *Data required*

The data required for this method are described below:

(*a*) Children ever born classified by five-year age group of mother for two points in time, five or 10 years apart;

(*b*) Registered births classified by five-year age group of mother for each calendar year of the period between the two surveys;

(*c*) The total number of women in each age group at the beginning and end of the period, or enough information to estimate the mid-year female population by five-year age group for each year for which birth registration data are available.

(c) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities for the first and second surveys. Average parities obtained from the first survey are denoted by $P(i, 1)$ and those obtained from the second by $P(i, 2)$. In both cases, they are computed by dividing the reported number of children ever born to women in age group i by the total number of women in age group i. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the possible application of the El-Badry correction (described in annex II).

Step 2: calculation of cohort parity increments. For each cohort of women of age group i at the second census or survey, the average parity of the same cohort at the first survey can be identified. Then, the cohort parity increment, $\Delta P(i)$, can be calculated as

$$\Delta P(i) = P(i, 2) - P(i - n, 1) \qquad (B.18)$$

where n is the number of five-year periods between the two surveys.

Step 3: estimation of mid-year female population by age group for each year of the intersurvey period. The procedure to be followed is exactly the same as that described in step 2 of subsection B.5(c). Its description is not repeated here.

Step 4: calculation of age-specific fertility rates from registered births for each year of the intersurvey period. Again, the procedure is exactly the same as that described in step 3 of subsection B.5 (c). Its description is omitted here.

Step 5: calculation of intersurvey increments in cohort fertility from registered births. In essence, the calculation of intersurvey increments in cohort fertility, denoted by $\Delta F(i)$, is very similar to the calculation of lifetime cohort parity equivalents from period rates. Each age-specific fertility rate for a given year of the intersurvey period (taken to be exactly five or 10 years in length) is split into two portions, according to the cohort structure of the age group to which the rate refers during the given calendar year; one portion is the period contribution to the fertility increment of one cohort and the other is the contribution to the fertility increment of the next cohort. The main differences between this procedure and that described in step 4 of subsection B.5 (c) is that here all calculations are limited to the intersurvey period, and all cohorts of reproductive age at the time of the second survey are considered. The splitting of age-specific fertility rates is carried out using the separation

factors in table 20; the general definition of the cohort fertility increments is

$$\Delta F(i) = \sum_{j=1}^{n} \sum_{m=1}^{5} [s(m,k)f(k,h) + (1-s(m,k-1))f(k-1,h)] \qquad (B.19)$$

where $\Delta F(i)$ = parity increment for cohort i at the second survey;

n = number of five-year periods in the intersurvey interval;

$s(m,k)$ = separation factor from table 20 required for age group k, where $k = (i-n+j)$;

$f(k,h)$ = age-specific fertility rate for age group k in year h, where $h = 5(j-n-1)+m$ years before the second survey (assumed to have taken place at the end of a year).

The detailed example given below illustrates the way in which this equation is used in practice.

Step 6: estimation of completeness of birth registration. Estimates of the completeness of birth registration for the offspring of cohort i during the intersurvey period are obtained by dividing the fertility increment, $\Delta F(i)$, estimated for the cohort by the parity increment, $\Delta P(i)$, of the same cohort. If the estimates for different cohorts are consistent, the intersurvey births classified by age group can be inflated by an average of the $\Delta P(i)/\Delta F(i)$ ratios and adjusted age-specific fertility rates can be obtained by dividing the inflated number of births by an estimate of the female person-years lived in each age group over the intercensal period.

(d) *A detailed example*

The detailed example is again for Thailand, since data on children ever born are available both for 1960 and for 1970. Average parities for each census have already been presented in table 17 and annual age-specific fertility rates for the intercensal period can be found in table 25. However, some of the raw data necessary for the calculation of the latter rates are not presented. The steps followed in applying the method to this example are given below.

Step 1: calculation of reported average parities for the first and second surveys. The method of calculating average parities has been described in step 1 of subsection B.4 (c) and is not repeated here; the $P(i,1)$ values for 1960 and 1970 are shown in columns (2) and (3) of table 29. Note that the average parities are taken as referring to the year-end nearest to the census dates, in this case to the end of 1959 and the end of 1969, respectively (the exact census dates can be found in table 22).

Step 2: calculation of cohort parity increments. As the intersurvey interval in this case is 10 years, the n in equation (B.18) is 2. Thus, the parity increment for a cohort in age group i at the second survey is calculated by subtracting from the average parity of women of age group i at the second survey, $P(i,2)$, the average parity of the same cohort 10 years earlier, $P(i-2,1)$. For example, for the cohort aged 25-29 in 1970,

$$\Delta P(3) = P(3,1970) - P(1,1960)$$

$$= 2.409 - 0.072 = 2.337.$$

The parity increments are shown in column (4) of table 29.

TABLE 29. REPORTED AVERAGE PARITIES 1960 AND 1970, AND COHORT PARITY INCREMENTS DURING THE INTERCENSAL PERIOD, THAILAND

Age group (1)	Average parity 1960 (2)	Average parity 1970 (3)	Cohort parity increment 1960-1970 (4)
15-19	0.072	0.131	0.131
20-24	0.862	0.994	0.994
25-29	2.296	2.409	2.337
30-34	3.783	3.819	3.957
35-39	4.984	5.082	2.786
40-44	5.733	5.921	2.138
45-49	5.914	6.120	1.936

Step 3: estimation of mid-year female population by age group for each year of the intersurvey period. Most of the required estimates have already been made in step 2 of subsection B.5 (d), the results being shown in table 24. Since the procedure to estimate the remaining female populations is exactly the same as that used there, it is not illustrated again.

Step 4: calculation of age-specific fertility rates from registered births for each year of the intersurvey period. Most of the required fertility rates have already been calculated in step 3 of subsection B.5 (d) and all the required rates are shown in table 25. Although the raw data needed to calculate some of these rates have not been presented, the calculation procedure is identical to that illustrated in subsection B.5 (d) and is not repeated here.

Step 5: calculation of intersurvey increments in cohort fertility from registered births. In this example, the starting-point of the cumulation is the beginning of 1960 and the final point is the end of 1969. Fertility rates for the intervening decade have already been calculated and split by cohorts identified at the end of 1969 in table 26, parts (a) and (b). The cohort fertility increments, denoted by $\Delta F(i)$, can therefore be obtained simply by cumulating the portions of the observed age-specific fertility rates attributable to each cohort. Table 30 shows these portions arranged by cohort and five-year period (they were copied from column (9) of table 26, parts (a) and (b), labelled "Sum"). Table 30 also shows the sum, $\Delta F(i)$, of the portions corresponding to each cohort. The process of splitting fertility rates by cohort is exactly the same as that used in subsection B.5 (c) (step 4) to construct table 26, so it is not described again in detail. However, in order to enable the reader to grasp the nature of the cumulation carried out in this case, one

TABLE 30. CONTRIBUTIONS OF PERIOD FERTILITY TO INTERSURVEY COHORT FERTILITY BY FIVE-YEAR PERIOD AND ESTIMATED INTERSURVEY INCREMENTS IN COHORT FERTILITY, THAILAND, 1960-1969

Cohort's age group at end of 1969 (1)	Index i (2)	Contribution to cohort fertility from age group i: 1960-1964 f_{i-2} (3)	1960-1964 f_{i-1} (4)	1965-1969 f_{i-3} (5)	1965-1969 f_i (6)	Estimated cohort fertility increment $\Delta F(i)$ (7)
15-19	1	-	-	-	0.0532	0.0532
20-24	2	-	0.0403	0.1909	0.4629	0.6941
25-29	3	0.1603	0.4535	0.5452	0.5910	1.7500
30-34	4	0.5428	0.6353	0.6189	0.5430	2.3400
35-39	5	0.6131	0.5764	0.5195	0.4452	2.1542
40-44	6	0.5040	0.4977	0.4272	0.2380	1.6667
45-49	7	0.3421	0.2600	0.2020	0.0702	0.9246

may consider the case of the cohort aged 30-34 at the end of 1969. At the beginning of 1960, this cohort was aged 20-24, and therefore all the annual fertility rates for age group 20-24 during the period 1960-1964 contributed a portion to the intercensal fertility of this cohort (the exact amount contributed by each rate is found in part (b) of table 26, in the panel corresponding to $f(2)$ and the line labelled "Cohort 4". When the annual contributions of the $f(2)$ rates are summed over the period 1960-1964, their total contribution to the intersurvey fertility of the cohort aged 30-34 at the end of 1969 is 0.5428. In a similar way, the total contribution of the $f(3)$ rates (those for age group 25-29) from 1960 to 1964 is 0.6353 of a child, and that of the same rates from 1965 to 1969 is 0.6189 of a child. Lastly, the total contribution of the annual $f(4)$ rates (for age group 30-34) during the period 1965-1969 is 0.5430 of a child. Hence, the total increment in intersurvey or intercensal fertility for the cohort aged 30-34 at the end of 1969 is 2.3400 births per woman.

Step 6: estimation of the completeness of birth registration. Cohort-specific estimates of the completeness of birth registration between 1960 and 1969 can now be obtained by calculating the ratios of $\Delta F(i)$ to $\Delta P(i)$. Results are shown in table 31. Note that the estimates for the cohorts aged 15-19 and 20-24 are the same as those obtained in subsection B.5(d) (table 28) where they were based on the lifetime, rather than the intersurvey, estimates of cohort fertility. This outcome was to be expected because all the childbearing of these two cohorts is assumed to have taken place during the period 1960-1969, so that their lifetime and intersurvey fertility estimates are identical.

TABLE 31. ESTIMATES OF COMPLETENESS OF BIRTH REGISTRATION, THAILAND, 1960-1969

Cohort's age at end of 1969 (1)	Index i (2)	Intersurvey cohort fertility increment $\Delta F(i)$ (3)	Intersurvey cohort parity increment $\Delta P(i)$ (4)	Estimated completeness of birth registration $\Delta P(i)/\Delta F(i)$ (5)
15-19	1	0.063	0.131	0.405
20-24	2	0.694	0.994	0.698
25-29	3	1.750	2.337	0.749
30-34	4	2.340	2.957	0.791
35-39	5	2.154	2.786	0.773
40-44	6	1.667	2.138	0.780
45-49	7	0.925	1.136	0.814

Two features of the results require some comment, namely, the low estimates of completeness for women under 25 and the relative consistency of the estimates for the central age range (25-44).

In interpreting the results of this method, it is important to bear in mind its assumptions. It has been assumed that the reporting of children ever born is complete at both censuses. However, if there is a tendency among older women to omit children ever born, cohort parity increments will be reduced and the estimates of registration completeness will be increased. A relatively minor deterioration of the completeness of parity reporting by all cohorts from one census to the next will produce estimates of completeness that increase with age. In a similar way, a relative amelioration of the completeness of parity reporting will lead to completeness estimates that decrease with age.

The second assumption made is that parity increments are unaffected by migration and mortality. If low-parity women are more likely to migrate than those of higher parities, areas of in-migration are likely to display reduced parity increments, and vice versa. If low-parity women are more likely to die than high-parity women, parity increments will be inflated. In the case of Thailand, neither of these two effects is likely to be important, since both international migration and mortality are low.

Another assumption of importance is that the denominators used are accurate. Age-reporting errors that are not the same when a birth is registered as when the population is enumerated will distort the pattern of period age-specific fertility rates. The rather wide variations exhibited by the age-specific growth rates given in table 23 suggest that there are problems with age-reporting, but their effects on the final estimates of completeness are very hard to predict. Denominators may also be distorted by changes in the completeness of enumeration from one census to the next; in the case of Thailand, the 1970 overall census count is about 5 per cent less complete than that yielded by the 1960 census; although the former count was adjusted for this change, differential completeness of enumeration by age group

might still affect the results. Changes in enumeration completeness might also affect average parities; if women with children are more likely to be enumerated than women without, average parities will be inflated by omission.

However, none of these considerations explains adequately the low estimates of completeness obtained for the cohorts aged 15-19 and, to a lesser extent, those aged 20-24. Of course, the procedure used to split the period fertility rates is not perfect and it is most likely to be inaccurate at 15-19, but possible methodological inaccuracy cannot explain the large differential observed. Furthermore, since average parities were calculated without making any adjustment for non-response, they are more likely to be too small than too large. Hence, on the basis of this evidence alone, it would appear that birth registration is really less complete for young mothers than for the older group. Yet, those familiar with the data from Thailand and with the typical errors affecting age-reporting in East Asian countries may suggest another explanation for the outcome observed. It is customary in East Asian cultures to reckon age as of conception rather than as of birth. Therefore, when a question on age is posed (as during the 1960 census) the reports collected tend to reflect age as of the next birthday rather than age at the last birthday, as is normally expected. To avoid such misreporting problems, date of birth should be requested instead of age (as was done during the 1970 census). Because age is also the item recorded in the vital registration system, the direct intercensal comparison of average parity as reported in the 1960 and 1970 censuses with that reconstructed from vital registration is likely to yield biased results because of the change in the age-recording scheme. Furthermore, younger cohorts are those most likely to suffer from such biases, since age reported as of next birthday would make them lose systematically some of their high-fertility members. In this context, at least part of the low completeness estimates observed for the cohorts aged 15–19 and 20-24 at the time of the 1970 census may be due to non-comparable age-reporting schemes in the different data-collection systems producing the data used as input. It must be noted, however, that even when such deficiencies in the basic data are taken into account and some adjustment is performed to eliminate the biases they imply, the low completeness estimates associated with younger mothers persist, albeit at a somewhat lower level than that shown in this example.

C Estimation of age-specific fertility from the increment of cohort parities between two surveys

1. *Basis of method and its rationale*

Data on children ever born tabulated by standard five-year age group of women for a single census or survey convey much information about the past fertility experience of the women. Unfortunately, however, if fertility has been changing, it is not possible to use the average parities of women in different age groups to obtain estimates of the age patterns of either cohort or period fertility.

Yet, if information on children ever born is available from two surveys approximately five or 10 years apart, the change in the average number of children ever borne by a particular cohort of women reflects their intercensal fertility; and it becomes possible to estimate an intercensal age-specific fertility schedule. Arretx[9] developed a method for using such information with a 10-year interval between the surveys; Coale and Trussell recently developed an elegant method based on the concept of the hypothetical cohort for using such information with an interval of five or 10 years between the surveys. The latter method is relatively simple and a variant of it is described here, illustrated by cases with such intervals.

The general warning given in subsection A 2 about the use of information on children ever born in estimating fertility should be kept in mind in this instance. There is a distinct tendency, even in countries with otherwise reasonably good data, for older women to omit some of their children, perhaps those who have died or those who have left home. As a result, average parities often fail to increase at a plausible rate, or may even decrease after age 35 or 40. The calculation of age-specific fertility rates from parities that suffer from such a degree of omission will result in underestimates of the fertility of older women; and if the error is relatively minor, its effects may not be obvious. Thus, fertility estimates based on average parities of older women must be interpreted with caution, particularly if they indicate low fertility in relation to that estimated from the reports of younger women. Average parities for a hypothetical cohort are in fact very sensitive to changes in parity reporting from one survey to the other, and the calculation of such parities provides a useful consistency check of the raw data.

The method based on the increment of cohort parities between two surveys estimates the average age-specific fertility rates in effect during the intersurvey period by constructing the average parities of a hypothetical, intersurvey cohort; a cumulated fertility schedule is then derived from these parities by interpolation, and age-specific fertility rates are obtained from cumulated fertility by successive subtraction.

The method is intended for situations in which it is possible to calculate average parities by age group of women for two points in time approximately five or 10 years apart. If the interval between the surveys is five years, the women in any five-year age group at the second survey represent the survivors of the women in the next younger five-year age group at the first survey. The difference in the average parity of the cohort between the first and the second surveys reflects its childbearing experience between the two surveys, if it is assumed that the women who died or migrated between the two surveys had, on average, lifetime fertility that

[9] Carmen Arretx, "Fertility estimates derived from information on children ever born using data from censuses", *International Population Conference, Liège, 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 2, pp. 247-261.

was not systematically different from that of the native women who remained. By cumulating the intersurvey parity increments, it is possible to estimate average parities for a hypothetical cohort experiencing throughout its hypothetical lifetime the age-specific fertility rates in effect during the intersurvey period. If the length of this period is 10 years, a five-year age group at the second survey represents the survivors of the five-year age group who were two groups younger at the first survey; and it is still possible to calculate the cohort parity increment for each cohort in order to construct the average parities of a hypothetical intersurvey cohort. The method may be applied when the data come entirely or partially from nationally representative sample surveys, for although cohorts of particular individuals will not be identical on each occasion, their average parities will be representative of those of the sampled female population.

It is worth noting that although the strength of the intercensal parity evolution method is its robustness to changing fertility, the technique presented here can also be used to estimate age-specific fertility rates using parity data from only one census or survey when fertility has not been changing during the reproductive life spans of the women concerned. It may also be mentioned that the two data sets need not refer to two points exactly five or 10 years apart. Unless fertility is changing very rapidly, a four-year interval or an 11-year interval will provide reasonable estimates. In such a case, one is no longer following a cohort from survey to survey, but this factor is not very important because the average parity of an age group will not change rapidly from one year to the next.

A final general observation is that if the required information is available, the El-Badry correction procedure to estimate the level of non-response, described in annex II, should be applied when calculating average parities by age group. For a further discussion of the treatment of women whose parity is not stated, see subsection A.2.

2. *Fertility estimation from the increment of cohort parities between two surveys*

(a) *Data required*

The data required for this method are described below:

(*a*) Children ever born classified by five-year age group of mother for two points in time approximately five or 10 years apart;

(*b*) Number of women aged 15–49, classified by five-year age group for the same two points in time.

(b) *Computational procedure*

The computational procedure is slightly different if the intersurvey period is five years than if it is 10 years in length. However, the only difference occurs in step 2, where the average parities of the hypothetical cohort are derived. Therefore, two versions of step 2 are described here: step 2A to be used with a five-year interval; and step 2B to be used with a 10-year interval. All other steps are described once.

Step 1: calculation of average parities for both surveys. The observed average parities from the first survey are denoted by $P(i, 1)$ and those from the second survey by $P(i, 2)$, where $i = 1$ indicates age group 15-19; $i = 2$ the 20-24 age group; and so on. In both cases, the average parities are computed by dividing the reported number of children ever born to women in age group i by the total number of women in age group i. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the possible application of the El-Badry correction (annex II).

Step 2A: calculation of average parities for a hypothetical cohort: five-year intersurvey period. The survivors of cohort i at the first survey belong to age group $i + 1$ at the second survey; and the parity increment for the cohort, denoted by $\Delta P(i + 1)$, is equal to the average parity of the cohort at the second survey minus that at the first survey. That is,

$$\Delta P(i+1) = P(i+1, 2) - P(i, 1). \qquad \text{(C.1)}$$

Such parity increments are calculated for values of i from 1 to 6; the value of $\Delta P(1)$, corresponding to $i = 0$, may also be calculated from equation (C.1), with $P(0, 1)$, the average parity at the first survey of women aged 10-14, being taken as equal to zero; this is equivalent, of course, to taking $\Delta P(1)$ as being directly equal to $P(1, 2)$.

Once the cohort parity increments have been obtained from equation (C.1), the average parities for the hypothetical cohort are obtained by successive summation of the increments. Thus, on the basis of intersurvey fertility, the parity of women in age group 1, denoted by $P(1, s)$, is equal to $\Delta P(1)$, or, as it may also be written, $P(1, 2)$; the parity of women in age group 2 is equal to the intersurvey parity for age group 1 plus the intersurvey parity increment for age group 2 at the second survey; this increment is $\Delta P(2)$, so

$$P(2, s) = \Delta P(1) + \Delta P(2).$$

In general terms, one may write:

$$P(i, s) = \sum_{j=1}^{i} \Delta P(j). \qquad \text{(C.2)}$$

Step 2B: calculation of average parities for a hypothetical cohort: 10-year intersurvey period. In this case, the survivors of the initial cohort of age group i at the time of the first survey will be the women in age group $i + 2$ at the time of the second survey, and the hypothetical-cohort parities are obtained by the cumulation of two parallel sequences of cohort parity increments. The parity increments are obtained by subtracting from the average parity of women of age group $i + 2$ at the

second survey the average parity of the women of age group i at the first survey. Thus,

$$\Delta P(i+2) = P(i+2, 2) - P(i, 1)$$

$$\text{for } i = 1, \ldots, 5. \tag{C.3}$$

$\Delta P(1)$ and $\Delta P(2)$ are simply put equal to $P(1, 2)$ and $P(2, 2)$, respectively (this procedure will distort the results slightly when fertility is changing very rapidly). Hypothetical-cohort parities for even-numbered age groups are obtained by summing the parity increments for even-numbered age groups, whereas those for odd-numbered age groups are obtained by summing parity increments for odd-numbered age groups. Thus, for even numbers,

$$P(2, s) = \Delta P(2) \tag{C.4}$$

$$P(4, s) = \Delta P(2) + \Delta P(4) \tag{C.5}$$

and

$$P(6, s) = \Delta P(2) + \Delta P(4) + \Delta P(6) \tag{C.6}$$

whereas

$$P(1, s) = \Delta P(1) \tag{C.7}$$

$$P(3, s) = \Delta P(1) + \Delta P(3) \tag{C.8}$$

$$P(5, s) = \Delta P(1) + \Delta P(3) + \Delta P(5) \tag{C.9}$$

and

$$P(7, s) = \Delta P(1) + \Delta P(3) + \Delta P(5) + \Delta P(7) \tag{C.10}$$

where $P(i, s)$ is the hypothetical average parity for age group i.

Step 3: interpolation between the hypothetical-cohort parities to estimate cumulated fertility. Average parities for the hypothetical cohort calculated in the previous step can be used to estimate cumulated fertility up to the exact ages of interest. The process of estimation followed is directly linked to that used in subsection B.2 (c) to estimate average parities from observed cumulated fertility. Indeed, it can be said that this process is just the inverse of that described in subsection B.2 (c), since the problem now is to go from average parities to cumulated fertility, while the *P/F* estimation method yields estimates of average parities from cumulated fertility.

One interpolation procedure described in subsection B.2 (c) fitted a second-order polynomial to sections of the cumulated fertility schedule and average parities were computed by integrating this polynomial over the desired age range. Hence, average parities can be approximated by integrals of polynomials whose coefficients define the shape of the cumulated fertility schedule, thus providing a means of estimating a value of this schedule at any exact age within the fitting range. Following this principle, an expression allowing the estimation of cumulated fertility from observed parities was obtained by fitting a third-order polynomial to successive average parities. It has the form:

$$\phi(i, s) = 0.9283P(i, s) + 0.4547P(i+1, s)$$

$$-0.0585P(i+2, s) - 0.3245\phi(i-1, s)$$

$$\text{for } i = 1, \ldots, 5. \tag{C.11}$$

where $\phi(i, s)$ is fertility cumulated up to the upper limit of age group i.

When the performance of this estimating equation was tested by using the Coale-Trussell model fertility schedules, it was found that it did not perform very satisfactorily at the upper extreme of the fertile period. Therefore, an empirically derived correction was introduced to estimate $\phi(6, s)$, resulting in the following equation:

$$\phi(6, s) = 0.0209P(4, s) - 0.5574P(5, s) + 1.0478P(6, s)$$

$$+0.2869P(7, s) + 0.2018\phi(4, s). \tag{C.12}$$

Unfortunately, no satisfactory correction was found for estimating $\phi(7, s)$ based on polynomial fits. Yet, since fertility rates are usually very low at ages 45 and older, it is recommended that $\phi(7, s)$ be estimated directly from the observed $P(7, s)$ by using the following equation:

$$\phi(7, s) = 1.007P(7, s). \tag{C.13}$$

Step 4: calculation of intersurvey age-specific fertility rates. Intersurvey age-specific fertility rates, $f(i, s)$, are calculated by subtracting the cumulated fertility to the lower boundary of age group i from that to the upper boundary of the same group and dividing the difference by five. Thus, in general:

$$f(i, s) = (\phi(i, s) - \phi(i-1, s))/5. \tag{C.14}$$

Note that cumulated fertility to age 15, which is denoted by $\phi(0, s)$, is generally assumed to be zero.

Step 5: calculation of total fertility and the birth rate. The estimated total fertility for the intersurvey period is equal to $\phi(7, s)$. To obtain an estimate of the birth rate, the mid-period female population classified by five-year age group and the mid-period total population are required. Both can be obtained by averaging, when censuses are available for the beginning and the end of the period. Then, the births occurring at the middle of the intersurvey period can be found by multiplying the estimated mid-period female population by the relevant intersurvey age-specific fertility rates and summing over all age groups. The birth rate is then obtained by dividing the total number of births by the mid-period population. However, when adequately spaced censuses are not available, the calculation of an acceptable mid-period population may not be possible. In such a case, birth-rate estimates may be obtained independently for the first and second surveys using the intersurvey age-

TABLE 32. FEMALE POPULATION AND CHILDREN EVER BORN, BY AGE GROUP, THAILAND, 1960, 1970 AND 1975

Age group (1)	Index (2)	1960 Female population (3)	1960 Children ever born (4)	1970 Female population (5)	1970 Children ever born (6)	1975 Female population (7)	1975 Children ever born (8)
15-19	1	1 236 294	88 610	1 885 371	246 839	13 329	1 698
20-24	2	1 304 153	1 038 074	1 361 717	1 353 569	10 304	9 108
25-29	3	1 046 464	2 402 581	1 143 277	2 754 376	8 008	17 027
30-34	4	869 836	3 290 345	1 073 088	4 112 920	6 561	22 866
35-39	5	679 940	3 088 799	957 607	4 866 424	6 400	29 821
40-44	6	563 812	3 232 209	766 332	4 537 467	5 546	31 595
45-49	7	482 966	2 855 997	597 454	3 656 488	4 521	28 537

specific fertility rates obtained in step 4 and their average may be used as an estimate of the intersurvey birth rate.

(c) *Detailed examples*

Information on children ever born classified by age of mother is available from the 1960 and 1970 censuses of Thailand and from a large sample survey conducted in 1975. The 10-year cohort parity increment procedure may be applied between 1960 and 1970, and the five-year method between 1970 and 1975. The basic data are shown in table 32.

First to be considered are the calculations in the case of the 10-year interval in order to retain historical order.

Step 1: calculation of average parities for both surveys. Average parities are obtained by dividing the number of children ever born, classified by age group of women (shown in columns (4) and (6) of table 32 for 1960 and 1970, respectively), by the total number of women in each age group (shown in columns (3) and (5) of table 32 for 1960 and 1970, respectively). Results are shown in table 33; but, as an example, the average parity of women aged 25-29 in 1960 is obtained as

$$P(3, 1) = 2{,}402{,}581 / 1{,}046{,}464$$

$$= 2.2959.$$

Step 2B: calculation of average parities for a hypothetical cohort: 10-year intersurvey. Using equation (C.3), cohort parity increments are calculated by subtracting from each recorded parity in 1970 the parity of the corresponding cohort, 10 years younger, in 1960. Thus, in the case of age group 30-34 in 1970,

$$\Delta P(4) = P(4, 2) - P(2, 1)$$

$$= 3.8186 - 0.8621$$

$$= 2.9565.$$

Values of $\Delta P(i)$ for all age groups are shown in column (3) of table 33. Note that for age groups 1 and 2, the parity increment is taken as being equal to the observed parities for age groups 1 and 2 at the time of the second census.

Once the cohort parity increments have been calculated, the average parities for hypothetical cohorts are obtained by summation, as shown in equations (C.4)-(C.10). The average parities for even-numbered age groups are obtained by summing even-numbered cohort parity increments; thus, to calculate $P(4, s)$, using equation (C.5),

$$P(4, s) = \Delta P(2) + \Delta P(4)$$

$$= 0.9940 + 2.9565$$

$$= 3.9505.$$

For odd-numbered age groups, average parities for a hypothetical cohort are obtained by summing odd-numbered cohort parity increments; thus, to calculate $P(5, s)$, using equation (C.9),

$$P(5, s) = \Delta P(1) + \Delta P(3) + \Delta P(5)$$

$$= 0.1309 + 2.3373 + 2.7860$$

$$= 5.2542.$$

TABLE 33. AVERAGE PARITIES FOR 1960 AND 1970, COHORT PARITY INCREMENTS AND AVERAGE PARITIES FOR THE INTERSURVEY HYPOTHETICAL COHORT, THAILAND

Age group (1)	Index (2)	Average parity 1960 (3)	Average parity 1970 (4)	Cohort parity increment (5)	Hypothetical-cohort parity (6)
15-19	1	0.0717	0.1309	0.1309	0.1309
20-24	2	0.8621	0.9940	0.9940	0.9940
25-29	3	2.2959	2.4090	2.3373	2.4682
30-34	4	3.7825	3.8186	2.9565	3.9505
35-39	5	4.9840	5.0819	2.7860	5.2542
40-44	6	5.7328	5.9210	2.1345	6.0890
45-49	7	5.9135	6.1201	1.1361	6.3903

Column (6) of table 33 shows the hypothetical-cohort average parities for the period 1960-1970.

Step 3: interpolation between the hypothetical-cohort parities to estimate cumulated fertility. Using equations (C.11)-(C.13), cumulated fertility is estimated from the average parities obtained in step 2. It is assumed that $\phi(0, s) = 0.0$, so that for $i = 1$:

$$\phi(1, s) = 0.9283(0.1309) + 0.4547(0.9940) -$$

$$0.0585(2.4682) = 0.4291.$$

For other values of i, previously estimated ϕ values are used as input in equation (C.11). For example:

$$\phi(3, s) = 0.9283(2.4682) + 0.4547(3.9505) -$$

$$0.0585(5.2542) - 0.3245(1.6747) = 3.2367.$$

When $i = 6$, equation (C.12) is used, as shown below:

$$\phi(6, s) = 0.0209(3.9505) - 0.5574(5.2542) +$$

$$1.0478(6.0890) + 0.2869(6.3903) + 0.2018(4.6498)$$

$$= 6.3056.$$

Lastly, for $i = 7$,

$$\phi(7, s) = 1.007(6.3903) = 6.4350$$

Column (4) of table 34 displays all the $\phi(i, s)$ values.

Step 4: calculation of intersurvey age-specific fertility rates. Following equation (C.14), age-specific fertility rates are obtained from the cumulated fertility values, $\phi(i, s)$, by finding the difference between successive values of $\phi(i, s)$ and dividing the difference by five to obtain an annual rate; $\phi(1, s)$ is cumulated fertility by age 20, so

$$f(1, s) = \phi(1, s)/5,$$

but thereafter,

$$f(i, s) = (\phi(i, s) - \phi(i - 1, s))/5.$$

To give two examples:

$$f(1, s) = 0.4291/5 = 0.0858$$

and

$$f(6, s) = (\phi(6, s) - \phi(5, s))/5 = (6.3056 - 5.7635)/5$$

$$= 0.1084.$$

Column (5) of table 34 shows the complete set of $f(i, s)$ values.

Step 5: calculation of total fertility and the birth rate. Total fertility estimated for the intersurvey period is immediately available, being equal to $\phi(7, s)$, or in this case 6.44. An approximate estimate of the birth rate can be obtained by estimating the mid-period female population of each age group by adding the 1960 and 1970 populations and dividing by two, then estimating an average number of annual births by multiplying the female population by the age-specific fertility rate shown in table 34, summing the births for all age groups of women, and dividing the total by the average of the 1960 and 1970 total populations. This method can be used because censuses are the sources of the basic data. As an example, the average female population for age group 25-29, $N(3, s)$, is found as

$$N(3, s) = (N(3, 1) + N(3, 2))/2.0$$

$$= (1,046,464 + 1,143,377)/2.0$$

$$= 1,094,921.$$

The annual number of births to women in this age group is then found by multiplying by the estimated $f(3, s)$. So, if the number of births to age group i is denoted by $B(i, s)$,

$$B(3, s) = N(3, s) f(3, s) = (1,094,921)(0.3124) = 342,053.$$

Repeating these steps for other values of i and summing over all i values produces a total number of births of 1,355,416; given that the total population in 1960 was 26,257,916 and in 1970 it was 34,397,374, the average mid-period population is 30,327,645, and the birth rate b can be estimated as

$$b = 1,355,416/30,327,645 = 0.0447.$$

As a commentary on the results obtained, it may be

TABLE 34. AVERAGE PARITIES FOR THE HYPOTHETICAL COHORT, ESTIMATED SCHEDULE OF CUMULATED FERTILITY AND AGE-SPECIFIC FERTILITY RATES: THAILAND, 1960-1970

Age group (1)	Index i (2)	Hypothetical-cohort parity $P(i, s)$ (3)	Cumulated fertility $\phi(i, s)$ (4)	Intersurvey age-specific fertility rate $f(i, s)$ (5)	Age-specific fertility rate from registered births (6)
15-19	1	0.1309	0.4291	0.0858	0.0606
20-24	2	0.9940	1.6747	0.2491	0.2781
25-29	3	2.4682	3.2367	0.1124	0.3153
30-34	4	3.9505	4.6498	0.2826	0.2724
35-39	5	5.2542	5.7635	0.2227	0.2248
40-44	6	6.0890	6.3056	0.1084	0.1132
45-49	7	6.3903	6.4350	0.0259	0.0285

noted that the hypothetical-cohort average parities are higher than either of the observed values, confirming the fact that the hypothetical-cohort parities are very sensitive to changes in the level of fertility. However, they are also very vulnerable to error, especially to errors that affect one of the observed sets of average parities more than the other. Yet, when the estimated age-specific fertility rates derived from the hypothetical-cohort parities are compared with those obtained from births registered in Thailand during the intercensal period and adjusted for level as described in subsection B.4 (d) (the figures shown in column (6) of table 34 were copied from table 19), the similarity is reassuring, although differences for the first two and the last age groups are fairly marked.

In general, whenever the additional data required exist, the procedure outlined in subsection B.4 (c) for comparing cumulated intersurvey fertility rates with hypothetical-cohort average parities is to be preferred to the method described here, since the former method is less sensitive to the omission of children ever born from the reports of older women.

Next to be discussed is the case of the five-year interval. As steps 1, 3 and 4 are identical to those just described, merely the results obtained are given; step 2A, however, is described in detail.

Step 1: calculation of average parities for both surveys. Columns (3) and (4) of table 35 show the average parities for 1970 and 1975, respectively.

Step 2A: calculation of average parities for a hypothetical cohort: five-year intersurvey period. Following equation (C.1), cohort parity increments are calculated by subtracting from each recorded parity in 1975 the parity of the corresponding cohort, five years younger, in 1970. Thus, in the case of age group 30-34 in 1975,

$$\Delta P(4) = P(4, 2) - P(3, 1) = 3.4851 - 2.4090 = 1.0761.$$

TABLE 35. AVERAGE PARITIES FOR 1970 AND 1975, COHORT PARITY INCREMENTS AND HYPOTHETICAL-COHORT AVERAGE PARITIES, THAILAND

Age group (1)	Index i (2)	Average parity: 1970 (3)	Average parity: 1975 (4)	Cohort parity increment (5)	Hypothetical cohort parity (6)
15-19	1	0.1309	0.1274	0.1274	0.1274
20-24	2	0.9940	0.8839	0.7530	0.8804
25-29	3	2.4090	2.1262	1.1322	2.0126
30-34	4	3.8186	3.4851	1.0761	3.0887
35-39	5	5.0819	4.6595	0.8409	3.9296
40-44	6	5.9210	5.6969	0.6150	4.5446
45-49	7	6.1201	6.3121	0.3931	4.9357

Values of $\Delta P(i)$ for all cases are shown in column (5) of table 35. It should be noted that $\Delta P(1)$ is put equal to $P(1, 2)$.

Average parities for hypothetical cohorts are obtained by cumulating successive cohort parity increments. According to equation (C.2),

$$P(i, s) = \sum_{j=1}^{i} \Delta P(j).$$

Thus, $P(4, s)$ is obtained by summing the values of $\Delta P(j)$ from 1 to 4:

$$P(4, s) = \Delta P(1) + \Delta P(2) + \Delta P(3) + \Delta P(4)$$

$$= 0.1274 + 0.7530 + 1.1322 + 1.0761$$

$$= 3.0887.$$

Other values are obtained in a similar fashion, and the complete set of average parities for the hypothetical cohort is shown in column (6) of table 35. Note that in the case of a five-year intersurvey interval, there is only one chain of summation.

Step 3: interpolation between the hypothetical-cohort parities to estimate cumulated fertility. As in the case of a 10-year interval, cumulated fertility is estimated by using equations (C.11)-(C.13). When $i = 1$, $\phi(0, s)$ is assumed to equal zero. In other cases, the application is straightforward. The complete set of $\phi(i, s)$ estimates is shown in column (4) of table 36.

TABLE 36. HYPOTHETICAL-COHORT AVERAGE PARITIES, ESTIMATED SCHEDULE OF CUMULATED FERTILITY AND AGE-SPECIFIC FERTILITY RATES, THAILAND, 1970-1975

Age group (1)	Index i (2)	Hypothetical cohort parity $P(i, s)$ (3)	Intersurvey cumulated fertility $\phi(i, s)$ (4)	Age-specific fertility rate $f(i, s)$ (5)
15-19	1	0.1274	0.4008	0.0802
20-24	2	0.8804	1.4217	0.2042
25-29	3	2.0126	2.5815	0.2320
30-34	4	3.0887	3.3505	0.1938
35-39	5	3.9296	4.2734	0.1446
40-44	6	4.5446	4.7686	0.0990
45-49	7	4.9357	4.9702	0.0403

Step 4: calculation of intersurvey age-specific fertility rates. Age-specific fertility rates are obtained by subtracting fertility cumulated to the lower boundary of each age group from that cumulated to the upper boundary and dividing the result by five, as shown in equation (C.14). For example,

$$f(2, s) = (\phi(2, s) - \phi(1, s))/5$$

$$= (1.4217 - 0.4008)/5$$

$$= 0.2042.$$

Column (5) of table 36 shows the full set of intersurvey fertility rates $f(i, s)$.

Step 5: calculation of total fertility and the birth rate. Total fertility is equal to the value of $phi(7, s)$, that is, 4.97. It is more difficult in this case to estimate the birth rate, since the 1975 population comes from a sample survey, and the mid-period population cannot be obtained by averaging the 1970 and 1975 populations. However, an approximate estimate of the birth rate during the period between the surveys can be obtained by calculating the birth rates for 1970 and for 1975 implied by the intersurvey fertility rates and averaging them. The births in 1970 (not an estimate of the true number of births in 1970, but rather the number of births that would have occured in 1970 given the intersurvey fertility schedule) are obtained by summing the products of the enumerated female population appearing in column (5) of table 32 and the estimated age-specific fertility rates given in column (5) of table 36; then the birth rate is found by dividing this sum by the total 1970 population. The total births are 1,141,687, and the total enumerated population is 34,397,374, so the estimated birth rate is 0.0332. The births for 1975 are estimated by summing the products of the 1975 female population (column (7) of table 32) and the estimated fertility rates (column (5) of table 36) to give 7,959 births; since the total survey population in 1975 was 230,060, the estimated birth rate is 0.0346. Therefore, an estimate of the intersurvey birth rate is obtained by averaging these two, giving a final value of 0.0339.

To conclude, it may be mentioned that whereas between 1960 and 1970 every average parity for the hypothetical cohort was higher than the recorded parities for the beginning-point and end-point, between 1970 and 1975 the hypothetical-cohort average parities were lower than the parities recorded for the beginning-point or end point. The suitability of this method for situations in which fertility is changing is thus clearly demonstrated; it must be remembered, however, that the results will be seriously distorted if children ever born tend to be omitted from the reports provided by their mothers or if the extent of such omission changes from one survey to the next.

D. Estimation of fertility from information on children ever born classified by duration of marriage

1. *Basis of methods and their rationale*

Most of the methods proposed for estimating fertility schedules from data on children ever born are not robust to the presence of substantial age-misreporting. If in a given population there is less distortion in the reporting of marriage duration than in the reporting of age, a method that estimates fertility schedules by using data on children ever born classified by duration of marriage may yield better fertility estimates than a method based on data classified by age. An additional advantage of a duration-based method derives from the fact that fertility schedules by duration of marriage are more uniform than schedules by age, making the method more robust to recent changes in marriage patterns.

Coale, Hill and Trussell[10] propose one such method of estimation, which is called here the "P/P' ratio method". Its applicability is, however, somewhat limited by the fact that it can only be used in populations in which there is and has been very little voluntary control of fertility and in which only a small proportion of all births occurs outside marriage. On the other hand, this method does not require an essentially unchanging age at marriage.

The underlying rationale for the procedure can be stated simply. The age pattern of natural fertility, defined as marital fertility in the absence of voluntary fertility control,[11] is very similar in different populations, although their levels of fertility may differ. Moreover, natural fertility does not vary much from the beginning of reproductive maturity (around age 20) until the early thirties, when a fairly steep decline begins. Therefore, populations that marry early and do not practise voluntary fertility control have rather similar patterns (though not necessarily similar levels) of fertility by duration of marriage. Hence, except for a scale factor, the sequence of average parities by duration of marriage among different populations subject to natural fertility should be similar. It should be possible, therefore, to compare the reported sequence of average parities (for durations of marriage under 5 years, of 5-9 years and of 10-14 years) with a standard embodying the natural fertility pattern and, thus, to determine the level of natural fertility in the population in question. Marital fertility could then be estimated as equal to standard natural fertility multiplied by the estimated level in the population in question, and an estimate of the overall fertility schedule could be obtained as the product of the proportion married and the estimated marital fertility for each age group. In practice, in selecting a standard natural fertility schedule, some allowance is made for the population's distribution by age of entry into marriage and the effect this distribution has through known, though small, variations of natural fertility with age.

When using this method, a problem arises as to which women to consider. The standard schedule of average parity by duration of marriage, $P''(i)$, refers only to women who are still in their first union. Thus, when estimating the level at which natural fertility is experienced, only reports of women still in their first union should be used. Of course, the estimate of fertility obtained from this type of data would not refer to all currently married women. Besides, fertility data are not commonly tabulated according to these specifications (for women still in their first union separately from the rest). Therefore, it is often necessary to use an alternative type of data. The average parities for currently married women by time elapsed since first union,

[10] Ansley J. Coale, Allan G. Hill and T. James Trussell, "A new method of estimating standard fertility measures from incomplete data", *Population Index*, vol. 41, No. 2 (April 1975), pp. 182-212.

[11] Louis Henry, "Some data on natural fertility", *Eugenics Quarterly*, vol. X(III, No. 2 (June 1961), pp. 81-91.

regardless of the number of unions, provides such an alternative. Use of these data makes it possible to estimate the level of natural fertility experienced by all currently married women, but the effect of "dead time" between unions will affect the ratios of the observed average parities by duration groups, $P(i)$, to the corresponding parities derived from the standard, $P^*(i)$. These ratios are denoted by $R(i)$, where i determines the duration of marriage group, 1 denoting duration group 0-4, 2 duration group 5-9 and so on. The "dead time" effect will increase with marriage duration, since the probability of marital dissolution, at least through widowhood, increases with the time elapsed since first union, while the rapidity of remarriage may decrease. If marriages are fairly stable, however, this effect (a tendency for the $R(i)$ ratios to decline with i) will be small; and the use of data for all currently married women is probably the best option if they are available. The third possibility is to use average parities for ever-married women. In this case, the problem of "dead time" between or after unions will be more serious, and the tendency for the $R(i)$ ratios to decline with i will be more marked. The use of a level (or adjustment factor) of natural fertility based on the first three duration groups to estimate the fertility of ever-married women of all age groups will overestimate the fertility of the older age groups, where the proportion currently married is lower. This third possibility should not be used unless the data are only tabulated for ever-married women. In cases of this type, however, although parity information may only be available for ever-married women, the data on marital status may show currently married women classified by five-year age group. Then the expected biases in the final estimates of age-specific fertility rates may be minimized by a combination of two strategies: selecting an adjustment factor for natural fertility on the basis of the first three duration groups; and using the observed proportions of currently married women to weight the adjusted natural fertility schedule.

As in the case of the method that used data on children ever born classified by age, the different adjustment factors obtained by evaluating the ratios of the observed over the expected parities corresponding to a given duration group should be approximately equal for the different duration groups. Large differences between the ratios corresponding to different duration groups reveal either that the assumptions are not satisfied (for example, the population is not subject to natural fertility) or that the data may be affected by substantial misreporting.

Because the P/P^* method is applicable only to populations in which there is little practice of voluntary birth control and in which childbearing by women not currently married is infrequent, it should not be used (at least not without major *ad hoc* adjustments) for parts of Africa and Latin America where several forms of cohabitation other than recognized marriage are prevalent and where a significant proportion of all births are illegitimate.

Furthermore, this method is not applicable without modification to populations, such as that of India, in which many marriages occur before the age of menarche and where there is no close relation between the age at marriage and the age at which exposure to the risk of conception begins. In some Muslim populations to which this method is potentially applicable (with natural fertility and very little illegitimacy), the date of marriage for many couples precedes the beginning of cohabitation. As a result, the observed average parities by duration of marriage are lower than would be expected, especially during the first five-year duration interval, in which the proportion of childless women is higher than would be expected in a population not practising contraception. A procedure for adjusting estimates of fertility to allow for the effects of delay in cohabitation after marriage is described by Coale, Hill and Trussell.[12]

The converse problem is that in countries where consensual unions form a sizeable proportion of all unions, the duration of marriage reported may be that of the current union or the time elapsed since the beginning of the first stable union, rather than that elapsed since the first union. In such cases, the average parity for each duration group, and for the first group in particular, will be too high. A procedure for estimating the average extent of any non-correspondence between reported duration of marriage and apparent duration of cohabitation from the proportions of women with at least one child in duration groups 1 and 2 is also described by Coale, Hill and Trussell.[13]

None of the adjustment procedures that may be applied to the basic data before using the P/P^* method are described in this *Manual*. The main reason for this omission is that most of today's populations are unlikely to be subject to natural fertility within marriage and to lack, at the same time, acceptable information on the pattern of marital fertility they experience.

If data allowing the estimation of the pattern of marital fertility are available in the form of births occurring during a given year classified by the mother's duration of marriage, another method, analogous to that described in subsection B.2, can be used to compare period fertility rates with reported average parities and to adjust the level of the former rates by that of the latter. Once again the cumulated period fertility rates will refer to the end-point of each duration group and will therefore not be strictly comparable to average parities, but parity equivalents, $G(i)$, may be obtained by interpolating between consecutive cumulated fertility values. Indeed, the interpolation process will be much more satisfactory than in the case of data classified by age, since the cumulated duration-specific fertility schedule is likely to be more regular than the cumulated age-specific fertility schedule. Furthermore, this method will be less affected by changing age at marriage than will the method based on data classified by age; and it does not require that marital fertility follow the pattern

[12] A. J. Coale, A. G. Hill and T. J. Trussell, *loc. cit.*

[13] *Ibid.*

of natural fertility. This method is called the "*P/G* method".

However, the *P/G* method is not immune to the effects of errors frequently present in the basic data. Two types of errors are common:

(*a*) Women with no children are incorrectly recorded as being among those whose parity is not stated. The presence of this type of error is revealed by a sharp decline in the proportion of women of unknown parity with increasing age or duration of marriage, paralleling the genuine decline of the proportion of childless women. If the cases classified as "parity unknown" are omitted from the calculations, average parity is overestimated, particularly at duration 0-4, whereas if all such women are assigned a parity of zero, average parity is underestimated. El-Badry[14] proposed a method of adjustment, described in annex II;

(*b*) Misstatement of the time elapsed since first union (duration of marriage). This error biases the values of average parity in two duration intervals (both the true interval and that to which the woman is mistakenly assigned), because the transfer of women from one interval to another is usually selective with respect to their parity. For example, the women who falsely report a duration of first marriage as 5-9 years rather than 0-4 very probably belong near the upper boundary of the lower interval and are of generally higher parity than the average for their true interval but of lower parity than the average for the interval in which they are reported. An upward transfer—an overstatement of duration—thus introduces a downward bias in the recorded average parity for both duration groups. Similarly, a downward transfer is likely to inflate the recorded average parity for both, and some mixture of these two types of transfer will tend to reduce the differential in average parity between the two duration groups.

Errors in stated duration of marriage are analogous to errors of reference period in the reporting of events during a fixed interval, such as the past year, and to misstatements of age or of a child's date of birth. The extent and direction of the bias probably depend upon the wording of the question by which duration is ascertained, as well as upon the education of the enumerator and respondent, and upon the general cultural context. It is not clear which way of collecting data on duration—a question on date of first marriage, time elapsed since first marriage or age at first marriage—is best, but the first method seems likely to be least affected by rounding errors.

Lastly, when data on current fertility and parity information are obtained from different sources it is important to verify that duration of marriage is measured in the same way in both. If, for example, the first source records duration of current marriage while the second measures time elapsed since first marriage, the basic data would be incompatible and should not be used as input for the *P/G* method.

2. *Estimation of level of natural fertility from reported parity by duration of marriage*

(a) *Data required*

The data listed below are required for this method:

(*a*) A value of a_0, the earliest age at which a significant number of marriages take place;

(*b*) The female population classified by five-year age group and marital status (single, married, widowed and divorced);

(*c*) The number of children ever born classified by the duration of first marriage and current marital status of their mothers;

(*d*) The total population.

(b) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities by duration of marriage. In the calculation of the reported average parities, $P(i)$, the index i refers to the different duration groups. Thus, $i = 1$ indicates duration group 0-4; $i = 2$, duration group 5-9; and $i = 3$, duration group 10-14. $P(i)$ is defined as the ratio of the number of children ever born to currently married women in duration group i to the number of currently married women belonging to the same group. If data for currently married women are not available, data for ever-married women may be used. However, as stated above, use of the latter data may introduce some undesirable biases (see subsection D.1). Furthermore, if the proportion of currently married (or ever-married) women whose parity was not stated is non-trivial, the question arises as to the way in which these cases should be treated. For guidelines about the treatment of women whose parity is not stated, refer to subsection A.2 and annex II (El-Badry's correction).

Step 2: calculation of the female singulate mean age at marriage. The female singulate mean age at marriage, *SMAM*, is calculated from the proportions single, classified by age group, as described in detail in annex I.

Step 3: calculation of expected average parities by duration of marriage. To calculate the expected average parities, $P^*(i)$, one needs an estimate of the female singulate mean age at marriage, obtained in step 2, and an estimate of the youngest age at which a significant number of women marry, denoted by a_0. As a rule, a_0 is not known exactly; but it can be guessed with fair accuracy from what is known about the population being studied. Its values generally fall within the range from 12 to 15 years. Therefore, if early marriages are known to take place in the given population, a_0 can be chosen to have a value of 12 or 13. For a population in which early marriages are not common, a value of 14 or even 15 may be appropriate for a_0.

Expected average parities by duration of marriage are obtained from table 37, beginning with a round value of

[14] M. A. El-Badry, "Failure of enumerators to make entries of zero: errors in recording childless cases in population censuses", *Journal of the American Statistical Association*, vol. 56, No. 296 (December 1961), pp. 909-924.

TABLE 37. EXPECTED AVERAGE PARITIES FOR SELECTED VALUES OF THE YOUNGEST AGE AT WHICH A SIGNIFICANT NUMBER OF WOMEN MARRY AND OF THE FEMALE SINGULATE MEAN AGE AT MARRIAGE WHEN MARITAL FERTILITY IS EXPERIENCED AT THE LEVEL INDICATED IN TABLE 38

Youngest age at marriage a_0 (1)	Index i (2)	Duration group (3)	Average parities, $P^*(i)$, for singulate mean age at marriage equal to: 18 (4)	19 (5)	20 (6)	21 (7)	22 (8)	23 (9)	24 (10)	25 (11)	26 (12)	27 (13)
12	1	0-4	1.072	1.090	1.097	1.097	1.091	1.080	1.066	1.050	1.032	1.012
	2	5-9	3.338	3.331	3.316	3.279	3.230	3.171	3.106	3.036	2.963	2.888
	3	10-14	5.500	5.445	5.364	5.261	5.141	5.009	4.868	4.722	4.575	4.427
13	1	0-4	1.097	1.113	1.119	1.117	1.109	1.097	1.081	1.063	1.041	1.018
	2	5-9	3.385	3.381	3.357	3.318	3.266	3.205	3.136	3.062	2.984	2.904
	3	10-14	5.555	5.501	5.419	5.315	5.192	5.054	4.907	4.754	4.598	4.442
14	1	0-4	1.120	1.135	1.138	1.134	1.125	1.111	1.093	1.074	1.052	1.029
	2	5-9	3.424	3.417	3.392	3.351	3.298	3.238	3.163	3.085	3.004	2.920
	3	10-14	5.601	5.548	5.467	5.362	5.237	5.097	4.944	4.784	4.621	4.456
15	1	0-4	1.140	1.152	1.154	1.148	1.137	1.122	1.104	1.083	1.060	1.036
	2	5-9	3.455	3.446	3.419	3.377	3.324	3.260	3.187	3.107	3.022	2.935
	3	10-14	5.638	5.585	5.505	5.402	5.278	5.137	4.981	4.816	4.644	4.472

a_0, and then estimating $P^*(1)$, $P^*(2)$ and $P^*(3)$ by interpolation between columns, using the observed value of the female singulate mean age at marriage.

Step 4: selection of an adjustment factor. To select the adjustment factor, K, the values of $R(i)$ are calculated as the ratios of the reported parities to the expected parities for each duration group i. Thus,

$$R(i) = P(i)/P^*(i). \qquad \text{(D.1)}$$

The $R(i)$ values thus indicate the level of the reported average parities in relation to the level of the expected parities based on a natural fertility schedule. Three values of $R(i)$ are computed in each case, one for each of the duration categories under consideration. Ideally, all three values should be similar; and if they are, an average of the three may be selected as a final adjustment factor, K, to allow for the level of natural fertility in the population being studied. When the values of $R(i)$ are not similar or show a clear increasing and decreasing trend with i, the method in its simplest form should not be used. In such a case, it may be necessary to correct the original data by using some of the procedures mentioned in subsection D.1.

Step 5: calculation of adjusted age-specific marital fertility schedule. Once an adjustment factor, K, has been successfully identified, an adjusted age-specific marital fertility schedule, $g(j)$, for the population in question is calculated by multiplying the model natural fertility schedule appearing in table 38 by the adjustment factor K.

TABLE 38. AGE-SPECIFIC MARITAL FERTILITY RATES WHEN NATURAL FERTILITY IS EXPERIENCED AT THE STANDARD LEVEL

Age group (j)	Index i	Marital fertility rate $g(j)$
10-14	0	0.1350
15-19	1	0.4312
20-24	2	0.4694
25-29	3	0.4418
30-34	4	0.3988
35-39	5	0.3226
40-44	6	0.1668
45-49	7	0.0252
Total marital fertility		11.8540

Step 6: calculation of adjusted age-specific fertility schedule. The adjusted age-specific marital fertility schedule estimated in step 5 may be transformed into an adjusted age-specific fertility schedule, $f(j)$, referring to the entire female population (and not just to those currently married), under the assumption that all childbearing occurs within marriage, by multiplying each marital fertility rate, $g(j)$, by the proportion of currently married women in the age group being considered. The sum of the age-specific fertility rates, $f(j)$, thus obtained, multiplied by five, provides an estimate of total fertility.

Step 7: calculation of adjusted birth rate. An adjusted birth rate is obtained by adding the products of the estimated age-specific marital fertility schedule obtained in step 5 and the number of currently married women in different age groups and dividing this total (which is an estimate of the total number of births) by the total population.

(c) *A detailed example*

Tables 39 and 40 show data obtained from the 1976 census of Egypt. Because the incidence of illegitimate births is very low in the Egyptian population and because there is little evidence substantiating the existence of a fertility decline within marriage, the method described below may be used to estimate age-specific fertility rates.

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities by dura-

TABLE 39. CHILDREN EVER BORN AND EVER-MARRIED WOMEN, BY TIME ELAPSED SINCE FIRST MARRIAGE, EGYPT, 1976

Duration group (i)	Index i (2)	Ever-married women (3)	Children ever born (4)
0-4	1	1 249 636	964 962
5-9	2	1 121 552	2 615 104
10-14	3	1 017 492	2 900 481

TABLE 40. NEVER-MARRIED WOMEN AND TOTAL NUMBER OF WOMEN, BY AGE GROUP, EGYPT, 1976

Age group (1)	Index i (2)	Never-married women (3)	Total number of women (4)	Proportion of single women (5)	Proportion of currently married women (6)
15-19	1	1 540 388	1 849 952	0.8327	0.1673
20-24	2	605 528	1 556 231	0.3891	0.6109
25-29	3	190 214	1 359 752	0.1399	0.8268
30-34	4	76 995	1 088 690	0.0707	0.8774
35-39	5	48 737	1 027 457	0.0474	0.8756
40-44	6	46 062	944 083	0.0488	0.8096
45-49	7	28 491	736 495	0.0387	0.7681
50-54	8	33 259	739 863	0.0450	0.6217

tion of marriage. The values of the reported average parities, $P(i)$, are obtained by dividing each of the entries in column (4) of table 39 (number of children ever born) by the corresponding entries in column (3) of that table (number of women ever married in each duration group). Column (3) of table 41 shows the final set of parities. As an example, $P(2)$ is calculated here:

$$P(2) = 2,615,104/1,121,552 = 2.3317.$$

TABLE 41. REPORTED AND EXPECTED AVERAGE PARITIES, BY DURATION OF MARRIAGE, EGYPT 1976

Duration group (1)	Index i (2)	Reported average parity $P(i)$ (3)	Expected average parity $P^*(i)$ (4)	Ratio of reported to expected parity $R(i) = P(i)/P^*(i)$ (5)
0-4	1	0.772	1.127	0.69
5-9	2	2.332	3.332	0.70
10-14	3	3.624	5.270	0.69

It should be noted that in this case data for ever-married women are used because the data for children ever born by duration of marriage were not tabulated by current marital status.

Step 2: calculation of female singulate mean age at marriage. The singulate mean age at marriage for females is calculated from the proportion of females single in each age group, as is explained in annex I. The proportions single are shown in column (5) of table 40. They are calculated by dividing the entries in column (3) by those listed in column (4) of that table. As an example, the proportion single in age group 30-34, denoted by $U(4)$, is calculated below:

$$U(4) = 76,995/1,088,690 = 0.0707.$$

Following the procedure described in detail in annex I, with $RN = U(7)$, the value of the singulate mean age at marriage for females is calculated to be 21.74 years.

Step 3: calculation of expected average parities by duration of marriage. Female marriage is known to take place somewhat late in Egypt, so a_0 was assumed to be 14. The expected average parities, $P^*(i)$, are obtained by interpolating between the values shown in table 37, given the assumed value of a_0 and the estimated value of the singulate mean age at marriage. Table 42 illustrates the way in which this interpolation is carried out. For a description of the linear interpolation procedure, see annex IV. Since the values for $a_0 = 14$ are only given for

TABLE 42. CALCULATION OF EXPECTED AVERAGE PARITIES BY DURATION GROUP, EGYPT, 1976

Duration group (1)	Index i (2)	Expected average parity, $P^*(i)$, when: Singulate mean age at marriage is 21 ($\theta = 0.26$) (3)	Singulate mean age at marriage is 22 ($1 - \theta = 0.74$) (4)	Singulate mean age at marriage is 21.74 (5)
0-4	1	1.134	1.125	1.127
5-9	2	3.351	3.298	3.332
10-14	3	5.362	5.237	5.270

the singulate mean ages at marriage that are equal to integer numbers of years, the two singulate mean ages that bracket the observed value identify the $P^*(i)$ values between which interpolation is to be performed. In this case, the ages are 21 and 22. Hence, the interpolation factor θ is calculated as follows:

$$\theta = (22 - 21.74)/(22 - 21) = 0.26.$$

This interpolation factor is then used to weight adjacent expected parities, so that

$$P^*(i) = \theta P^*_{21}(i) + (1 - \theta) P^*_{22}(i).$$

The final set of expected $P^*(i)$ values is shown in column (5) of table 42.

Step 4: selection of an adjustment factor. An adjustment factor, K, is selected from the values of $R(i) = P(i)/P^*(i)$, the ratios between the reported and the estimated parities at each duration interval. The values of $R(i)$ are shown in column (5) of table 41. They are clearly very consistent. Their trend does not invalidate the hypothesis made above about the similarity between the actual pattern of marital fertility in Egypt and that of natural fertility. Although recent evidence has shown that in the large urban centers of Egypt (Cairo and Alexandria) women are limiting their fertility by the use of contraceptives, their relative weight in the entire population and the recency of such practices do not seem to have affected the overall picture to a significant degree. In addition, the consistency of the observed $R(i)$ ratios does not suggest that the effect of delayed cohabitation is of importance. However, because the combined biases due to delayed cohabitation and to the adoption of fertility control may also produce the pattern seen in the $R(i)$ ratios, their importance cannot be adequately assessed on the basis of

these results. Other information is needed to make a definitive statement about fertility in the country as a whole. But, continuing with this example, the choice of the mean of the observed $R(i)$ values as adjustment factor K is appropriate. Thus,

$$K = (0.69 + 0.70 + 0.69)/3.0 = 0.69.$$

Step 5: calculation of adjusted age-specific marital fertility schedule. In this step, an adjusted age-specific marital fertility schedule, $g(j)$, is calculated by multiplying each entry of the schedule given in table 38 by the adjustment factor K calculated in the previous step. The resulting set of $g(j)$ values is shown in column (3) of table 43. As an example, $g(5)$ is

$$g(5) = (0.69)(0.3226) = 0.2226.$$

Step 6: calculation of adjusted age-specific fertility schedule. The marital age-specific fertility schedule shown in column (3) of table 43 can be transformed into an age-specific fertility schedule referring to all women, $f(j)$, by multiplying each of the estimated marital fertility rates by the proportion of currently married women in the corresponding age group. These proportions,

TABLE 43. ADJUSTED MARITAL FERTILITY AND ESTIMATED AGE SPECIFIC FERTILITY RATES AND NUMBER OF BIRTHS, EGYPT, 1976

Age group (i)	Index j (2)	Adjusted marital fertility g(j) (3)	Estimated age-specific fertility f(j) (4)	Estimated number of births B(j) (5)
15-19	1	0.2837	0.0459	84 913
20-24	2	0.3239	0.1911	247 396
25-29	3	0.3048	0.2520	342 658
30-34	4	0.2752	0.2415	262 919
35-39	5	0.2226	0.1949	200 251
40-44	6	0.1151	0.0932	87 989
45-49	7	0.0174	0.0134	9 869
TOTAL		1.9427	1.0320	1 285 995

shown in column (6) of table 40, were obtained from a tabulation showing the female population classified by marital status and by age. In this step, the proportions currently married are used instead of the proportions of ever-married women in order to minimize the biases due to the fact that ever-married women have not, in general, been continuously in the married state. Thus, the estimated age-specific fertility rates appearing in column (4) of table 43 are obtained as is illustrated below in the case of $f(3)$:

$$f(3) = (0.3048)(0.8268) = 0.2520.$$

Once age-specific fertility rates are available, total fertility is estimated as five times their sum, that is,

$$TF = 5.0(1.0320) = 5.16.$$

Step 7: calculation of adjusted birth rate. The number of births expected if currently married women in each age group were to experience the estimated marital fertility rates, $g(j)$, shown in table 43 and if there were no illegitimate fertility can be obtained either by multiplying those rates by the number of currently married women or, alternatively, by multiplying the age-specific fertility rates derived in the previous step, $f(j)$, by the total number of women in each age group. The resulting estimated numbers of births by age group are shown in column (5) of table 43. As an example,

$$B(5) = (0.1949)(1,027,457) = 200,251.$$

By adding the number of estimated births over all ages and dividing the result by the total population, in this case 36,626,204 persons, an estimate of the birth rate is obtained. Thus,

$$b = 1,285,995/36,626,204 = 0.0351.$$

3. *Comparison of period duration-specific fertility rates with average parities by duration group*

(a) *Data required*

The data required for this procedure are described below:

(*a*) The number of children ever born, classified by time elapsed since first marriage of mother (five-year duration groups);

(*b*) The ever-married female population, classified by five-year duration of marriage group (duration of marriage should be measured as the time elapsed since first marriage);

(*c*) The births in a year, classified by the duration of the mother's first marriage (this information may come either from a survey question on births in the past year or date of most recent birth, or from a vital registration system).

(b) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities by duration of marriage. Average parities by duration of marriage, $P(i)$, are calculated by dividing the number of children ever born reported by women in duration of marriage group i by the number of women in that duration group. It is important to make sure that a consistent treatment of marital status is employed: if, for example, the children ever born are reported by ever-married women, then the denominator of the average parities should also be ever-married women. If children ever born refer only to women still in their first marriage, the denominator used should be women still in their first marriage, and so on. Furthermore, if the proportion of women in the selected marital category who did not state their parity is non-trivial, a question arises as to how to treat such cases. For guide-lines about the treatment of women whose parity is not stated, refer to subsection A 2 and to annex II on El-Badry's correction.

Step 2: calculation of duration-specific period fertility rates. These rates, denoted by $g(i)$, are calculated for each duration group i by dividing the births reported to have occurred during a year to women of duration group i (the births being obtained either from a question on date of most recent birth in a survey or from a vital registration system) by the number of ever-married women in duration group i. Note once again that consistency should be maintained with regard to the women covered; and if ever-married women were used in step 1, they should be used again in this step (in such a case, the births in a year should also refer to all ever-married women).

Step 3: calculation of cumulated fertility schedule by duration. The duration-specific fertility rates, $g(i)$, are cumulated and multiplied by five in order to estimate fertility cumulated to the upper limit of age group i, $\Gamma(i)$. Thus

$$\Gamma(i) = 5\left[\sum_{j=1}^{i} g(j)\right]. \qquad \text{(D.2)}$$

Step 4: estimation of average parity equivalents. Average parity equivalents, $G(i)$, are estimated by interpolation using the current fertility rates, $g(i)$, and the cumulated fertility schedule, $\Gamma(i)$. As in the case where data are classified by age, the estimation procedure uses different equations, depending upon the way in which duration of marriage has been measured. When the source of the data is a vital registration system, recorded marital duration is likely to be that at the time of the birth, and the estimation equation to be used is

$$G(i) = \Gamma(i-1) + 2.917g(i) - 0.417g(i+1). \quad \text{(D.3)}$$

Average parities are usually available only for four or five duration groups, that is, up to duration group 15-19 or 20-24. In order to exploit all the information available, the last or upper value of $G(i)$ is estimated by using the following equation:

$$G(i) = \Gamma(i-1) + 0.417g(i-1) + 2.083g(i). \quad \text{(D.4)}$$

When data on current fertility by duration of marriage come from a survey question on births in the past year, the recorded duration of marriage will almost certainly be that at the time of the survey and not the duration at the time of the birth; therefore, fertility rates calculated from these data will refer to duration groups shifted, on average, six months towards lower durations. Hence, a different equation to estimate parity equivalents is needed. The parallel with the case in which data are classified by age is obvious. However, the two cases are not quite the same, since in the present instance it is assumed that childbearing does not begin until marriage, and it is also assumed that if cohabitation begins with first marriage the burst of childbearing nine months after and in the following few months compensates for the period immediately following marriage during which no births occur. The first duration period is thus only 4.5 years; and at the beginning of it, cumulated fertility equals zero. Hence, for the first duration group, 0-4, $G(i)$ is estimated by

$$G(1) = 3.137g(1) - 0.324g(2). \qquad \text{(D.5)}$$

For subsequent duration groups, the general interpolation equations proposed by Coale and Trussell in the case of data classified by age are also used in the duration case.

$$G(i) = \Gamma(i-1) + 3.392g(i) - 0.392g(i+1) \quad \text{(D.6)}$$

for $i = 2, 3, \ldots, j-1$, and

$$G(j) = \Gamma(j-1) + 0.392g(j-1) + 2.608g(j) \quad \text{(D.7)}$$

where the index j indicates the last duration group considered. Because fertility changes more smoothly with marriage duration after the first two years or so than it does with age, there is no need to resort to more sensitive, model-based interpolation procedures. The Coale-Trussell procedure based on the fitting of a second-order polynomial to cumulated fertility is adequate.

Step 5: comparison of reported average parity and average parity equivalents for duration-of-marriage groups. When marital fertility has remained reasonably constant, a comparison of reported average parities with the parity equivalents estimated on the basis of period fertility should provide a basis for adjusting current births. Such a comparison is carried out by calculating the ratios $P(i)/G(i)$. If these ratios are reasonably consistent by duration group, at least for the first three or four values of i, the average of the first three ratios can be used as an adjustment factor for the reported number of current births, whether the latter are classified by age or by duration of marriage. The adjustment of births classified by age of mother is likely to be more useful, but it is valid only if the error in reporting births is constant across duration, as well as across age groups. If such an assumption seems tenable in a given case, births by age of mother may be adjusted, age-specific fertility rates can be calculated, and other measures of fertility may be derived. (See subsection B.2 (c) and take note that, if data on births come from a survey, the age-specific fertility rates will have to be adjusted for the fact that they probably refer to age groups shifted by six months.)

In cases where it is not possible to assume that errors in data classified by age are similar to those in data classified by duration, or where data classified by age are not available, the number of reported births classified by duration can be adjusted and an adjusted marital fertility schedule, as well as an adjusted birth rate, can be derived. The latter rate will, of course, be a valid estimate only if childbearing occurs mainly within marriage in the population in question.

(c) *A detailed example*

Data on children ever born and date of most recent

TABLE 44. EVER-MARRIED WOMEN, CHILDREN EVER BORN AND BIRTHS IN THE YEAR PRECEDING THE SURVEY, BY TIME ELAPSED SINCE FIRST MARRIAGE, 1976

Duration group (i)	Index i (1)	Number of ever-married women (2)	Children ever born (3)	Births in preceding year (4)	Reported average parity P(i) (5)	Duration-specific fertility rate g(i) (6)
0-4	1	725	858	324	1.1834	0.4469
5-9	2	696	2 396	317	3.4425	0.4555
10-14	3	396	2 214	215	5.3926	0.3607
15-19	4	574	3 961	167	6.9007	0.2909
20-24	5	471	3 863	88	8.2017	0.1868
25-29	6	333	3 028	28	9.0931	0.0841

birth classified by time elapsed since first marriage are available from the fertility survey of a Muslim population in 1976. Table 44 shows the reported number of ever-married women in each duration group, the total number of children ever born to them and the births they reported as having occurred during the year preceding the survey.

The steps of the computation procedure are given below.

Step 1: calculation of reported average parities by duration of marriage. Children ever born, given in column (4) of table 44, are divided by the corresponding number of ever-married women, shown in column (3). The values for reported average parities by duration of marriage, $P(i)$, thus obtained are shown in column (6). As an example, $P(3)$ is calculated below:

$$P(3) = 3{,}214/596 = 5.3926.$$

Step 2: calculation of duration-specific period fertility rates. These rates, denoted by $g(i)$, are calculated by dividing the births that occurred in the year preceding the survey, shown in column (5) of table 44, by the corresponding number of ever-married women, shown in column (3). The resulting marital fertility rates, $g(i)$, are shown in column (7). As an example,

$$g(4) = 167/574 = 0.2909.$$

Step 3: calculation of cumulated fertility schedule by duration. To obtain each value of the cumulated fertility schedule, $\Gamma(i)$, the duration-specific fertility rates, $g(i)$, are cumulated, up to and including group i, and then multiplied by five. Thus, for duration group 4 (15-19 years since first marriage), the calculations are

$$\begin{aligned}\Gamma(4) &= 5.0(g(1)+g(2)+g(3)+g(4)) \\ &= 5.0(0.4469+0.4555+0.3607+0.2909) \\ &= 7.7700.\end{aligned}$$

All values of $\Gamma(i)$ are shown in column (5) of table 45.

TABLE 45. CUMULATED FERTILITY SCHEDULE, PARITY EQUIVALENTS AND RATIOS OF AVERAGE PARITIES TO PARITY EQUIVALENTS, BY TIME ELAPSED SINCE FIRST MARRIAGE, 1976

Duration group (i)	Index i (1)	Average parity P(i) (2)	Period fertility g(i) (3)	Cumulated fertility Γ(i) (4)	Parity equivalents G(i) (5)	Ratio of average parity to parity equivalent P(i)/G(i) (6)
0-4	1	1.1834	0.4469	2.2345	1.2543	0.944
5-9	2	3.4425	0.4555	4.5120	3.6382	0.946
10-14	3	5.3926	0.3607	6.3155	5.6215	0.959
15-19	4	6.9007	0.2909	7.7700	7.2290	0.955
20-24	5	8.2017	0.1868	8.7040	8.3707	0.980
25-29	6	9.0931	0.0841	9.1245	8.9966	1.011

Step 4: estimation of average parity equivalents. The data on births were obtained from fertility histories, and although it is possible to derive from this information data classified by marriage duration at the time of the birth, the data displayed in table 44 refer to marriage duration at the time of the interview. Therefore, the true duration groups to which the marital fertility rates, $g(i)$, refer are six months younger than shown. Values of the current average parity equivalents, $G(i)$, are therefore obtained using equations (D.5), (D.6) and (D.7). For $i = 1$:

$$\begin{aligned}G(1) &= 3.137g(1) - 0.324g(2) \\ &= 3.137(0.4469) - 0.324(0.4555) \\ &= 1.2543.\end{aligned}$$

For i equal to 2, 3, 4 and 5, equation (D.6) is used, so for duration group 3, for example,

$$\begin{aligned}G(3) &= \Gamma(2) + 3.392g(3) - 0.392g(4) \\ &= 4.5120 + (3.392)(0.3607) - (0.392)(0.2909) \\ &= 5.6215.\end{aligned}$$

For the last value of i, namely 6, equation (D.7) is used:

$$\begin{aligned}G(6) &= \Gamma(5) + 0.392g(5) + 2.608g(6) \\ &= 8.7040 + (0.392)(0.1868) + (2.608)(0.0841) \\ &= 8.9966.\end{aligned}$$

Column (6) of table 45 shows all six values of $G(i)$.

Step 5: comparison of reported average parity and average parity equivalents for duration-of-marriage groups. The ratios $P(i)/G(i)$ are calculated for each duration group, the $P(i)$ values being obtained from column (3) of table 45 and the $G(i)$ values coming from column (6). The complete set of P/G ratios is shown in column (7). The ratio values are fairly consistent, although they display some tendency to increase as duration increases. This tendency may be indicative of the fact that marital fertility has decreased somewhat, although it is not possible to discard entirely the possibility of it being caused by errors in the data (particularly in the parity reports for higher duration groups). According to the P/G ratios for the first four duration groups, period fertility rates would overestimate marital fertility by about 5 per cent. Multiplying them by the adjustment factor $K = 0.95$ would yield current fertility estimates consistent with the reported parities. Total marital fertility adjusted in this way amounts to 8.67 children per ever-married woman.

It is of interest to mention that although the Muslim population in question is one in which the practice of delaying cohabitation after marriage may be widespread, the trend of the P/G ratios calculated on the basis of the data available does not appear to be affected by such practice. In fact, the P/G ratios are robust to the existence of any practice that delays or postpones childbearing immediately after marriage, as long as it affects in the same way both retrospective fertility (parities) and period fertility (births in the past year). Biases are to be expected only if there have been changes in the prevalence of such practices in recent years or if there is a non-negligible incidence of pre-marital fertility in the population being studied. In the latter case, pre-marital births will increase the parities of the lower duration groups; but because only ever-married women are considered, they are not likely to be adequately reflected by period fertility rates. Thus, if the basic data were accurate in all other respects, a non-negligible incidence of pre-marital fertility would result in a series of P/G ratios that decline as duration of marriage increases.

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Chapter III

ESTIMATION OF CHILD MORTALITY FROM INFORMATION ON CHILDREN EVER BORN AND CHILDREN SURVIVING

A. BACKGROUND OF METHODS

1. *Use of data on child survivorship*

It is well known that the proportions of children ever born who have died are indicators of child mortality and can yield robust estimates of childhood mortality. The births to a group of women follow some distribution over time, and the time since birth is the length of exposure to the risk of dying of each person. The proportion dead among the children ever borne by a group of women will therefore depend upon the distribution of the children by length of exposure to the risk of dying (that is, upon the distribution in time of the births) and upon the mortality risks themselves. By allowing for the effects of the distribution of the births in time, such a proportion of dead children can be converted into a conventional mortality measure expressing their average experience. Specifically, the proportions of children dead classified by the mother's five-year age group or duration of marriage can provide estimates of the probabilities of dying between birth and various childhood ages. In certain cultures, women appear to be more likely to state duration of marriage correctly than to give correct information about their age, so the estimation procedure based on data classified by duration of marriage may be preferred. However, the use of data classified by duration is not recommended in countries where consensual unions are frequent and relatively unstable.

Brass[1] was the first to develop a procedure for converting proportions dead of children ever born reported by women in age groups 15-19, 20-24, etc. into estimates of the probability of dying before attaining certain exact childhood ages. Following the notation in the literature and using the symbol $D(i)$ to denote the proportion dead among children ever born to women in successive five-year age groups (where $i = 1$ signifies age group 15-19; $i = 2$ denotes 20-24; etc.), Brass developed a procedure to convert $D(i)$ values into estimates of $q(x)$, where $q(x) = 1.0 - l(x)$, the probability of dying between birth and exact age x. The basic form of the estimation equation proposed by Brass is

$$q(x) = k(i)\, D(i) \qquad \text{(A.1)}$$

where the multiplier $k(i)$ is meant to adjust for non-mortality factors determining the value of $D(i)$.

Brass found that the relation between the proportion of children dead, $D(i)$, and a life-table mortality measure, $q(x)$, is primarily influenced by the age pattern of fertility, because it is this pattern that determines the distribution of the children of a group of women by length of exposure to the risk of dying. He developed a set of multipliers to convert observed values of $D(i)$ into estimates of $q(x)$, the multipliers being selected according to the value of $P(1)/P(2)$—a good indicator of fertility conditions at younger ages—where $P(i)$ is the average parity or average number of children ever born reported by women in age group i. Brass estimated the $k(i)$ multipliers by using a third-degree polynomial of fixed shape but variable age location to represent fertility,[2] the logit system generated by the general standard (see chapter I, subsection B.4) to provide the mortality element, and a growth rate of 2 per cent per annum to generate a stable age distribution for females.

An important assumption made in the development of this method is that the risk of dying of a child is a function only of the age of the child and not of other factors, such as mother's age or the child's birth order. In practice, it appears that children of young mothers experience mortality risks well above average. For this reason, the estimate of the infant mortality rate $q(1)$ (the probability of dying before age 1) that can be derived from reports of women aged 15-19 frequently suggests heavier child mortality than estimates derived from reports of older women. Therefore, mortality estimates based on the reports of women aged 15-19 are generally disregarded, in part for this reason and in part because the numbers of children born and dead are usually small.

Trying to increase the flexibility of Brass' original method, Sullivan[3] computed another set of multipliers by using least-squares regression to fit equation (A.1) to data generated from observed fertility schedules and the Coale-Demeny life tables.[4] Trussell[5] estimated a third set of multipliers by the same means but using data generated from the model fertility schedules developed by

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[1] William Brass, "Uses of census or survey data for the estimation of vital rates" (E/CN.14/CAS.4/V57), paper prepared for the African Seminar on Vital Statistics, Addis Ababa, 14-19 December 1964.

[2] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, North Carolina, Carolina Population Center, Laboratories for Population Statistics, 1975).

[3] Jeremiah M. Sullivan, "Models for the estimation of the probability of dying between birth and exact ages of early childhood", *Population Studies*, vol. XXVI, No. 1 (March 1972), pp. 79-97.

[4] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966).

[5] T. James Trussell, "A re-estimation of the multiplying factors for the Brass technique for determining childhood survivorship rates", *Population Studies*, vol. XXIX, No. 1 (March 1975), pp. 97-108.

Coale and Trussell.[6] The general theory on which these methods are based is essentially the same, but they arrive at somewhat different multipliers because the data bases used in each case are different. Since the Sullivan variant has no obvious advantages over that proposed by Trussell, whereas the latter is based on a wider range of cases, the Trussell procedure is described here. It must be mentioned, however, that the multipliers presented are a more recent and more satisfactory version of those originally proposed by Trussell in 1975.

It is important to take note that this method of estimation is based on the assumption that fertility and childhood mortality have remained constant in the recent past. If, for example, fertility has been changing, the ratios of average parities obtained from a cross-sectional survey will not replicate accurately the experience of any cohort of women and will not provide a good index of the distribution in time of the births to the women of each age group.

The problems caused by declining fertility can be avoided when data for true cohorts are available (from censuses or surveys taken five or 10 years apart). In this case, an estimation method specifically designed for cohorts experiencing fertility change should be used.[7]

Preston and Palloni[8] propose an alternative approach to estimate the time location of births, which circumvents all the problems associated with changing fertility. This approach is closely related to the "own-children" procedure for estimating fertility from an age distribution (see chapter VIII, section C). If it is possible to link, within households, the records of mothers and their surviving children, it becomes possible to tabulate surviving children according both to their own age and to that of their mothers. Given an age pattern of mortality, say, from one of the Coale-Demeny regional model life tables, the combination of the proportion of children dead and the age distribution of the surviving children of women from some particular age group uniquely determines a level of mortality. The age distribution of surviving children is used to define the age distribution of children ever born without recourse to fertility models. In an actual application, the choice between the age distribution of surviving children and the ratio of consecutive parities to estimate the real distribution of births over time, depends upon data availability and upon a rough assessment of the likelihood each approach has of yielding the best possible estimate. In cases where age-reporting is good and most children live with their mothers, the approach suggested by Preston and Palloni may be the better, particularly if fertility is changing. In cases where age-reporting or completeness of enumeration is poor, or where a sizeable proportion of children do not live with their mothers or cannot be properly linked to them because of poor information, the parity-ratio approach is very likely to be better. A detailed description of the Preston-Palloni method is not included here, in part because in most cases where mortality needs to be estimated indirectly, age distributions are at best only moderately reliable, and in part because the data required for its application are not as widely available as the proportions of children dead. However, the user who has access to the former data for cases where biases due to fertility change may be a problem is encouraged to consider the application of this method.

Probably a more widespread problem is posed by declining mortality. The procedures outlined above all assume that a constant pattern and level of mortality have prevailed in the recent past of the population under study. In most countries, however, mortality has been declining.

Feeney[9] was the first to examine the effects of changing mortality on the performance of the child-mortality estimation procedure. Using infant mortality as an index of mortality level in a one-parameter logit life-table system, he calculated the proportions of children dead that would be observed if infant mortality were changing linearly through time. On the basis of these simulated cases, he showed that for plausible annual rates of change in infant mortality, the $q(1)$ values estimated from data on children ever born and surviving for different age groups of mother could be matched with the $q(1)$ values prevalent during a set of years before the survey; and that this set of years was, for all practical purposes, invariant with respect to the rate of mortality change. Using this empirical finding, Feeney developed an estimation procedure to establish the set of years to which infant mortality rates estimated from data on children ever born and children surviving refer. This procedure was developed from data generated by using a one-parameter logit life-table system derived from the general standard (see chapter I, subsection B.4) and the Brass fertility polynomial.[10] The use of $q(1)$, infant mortality, as an indicator of mortality level and as the estimated parameter makes the underlying age-pattern of mortality important to the results, since similar overall levels of mortality (life tables with the same expectation of life at birth, for example) can be associated with markedly different infant mortality rates. As a result, the Feeney method is likely to yield biased $q(1)$ estimates when the mortality pattern in early childhood of the population under study does not resemble that embodied by the general standard. For this reason, Feeney's original method is not described in detail.

It is fairly straightforward to apply Feeney's approach to data generated with other mortality models. Coale

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[6] Ansley J. Coale and T. James Trussell, "Model fertility schedules: variations in the age structure of childbearing in human populations", *Population Index*, vol. 40, No. 2 (April 1974), pp. 185-258.

[7] The method is presented in section E and it uses the estimation equations and coefficients given in tables 70-71.

[8] Samuel H. Preston and Alberto Palloni, "Fine-tuning Brass-type mortality estimates with data on ages of surviving children", *Population Bulletin of the United Nations, No. 10-1977* (United Nations publication, Sales No. E.78.XIII.6), pp. 72-87.

[9] Griffith Feeney, "Estimating infant mortality rates from child survivorship data by age of mother", *Asian and Pacific Census Newsletter*, vol. 3, No. 2 (November 1976), pp. 12-16, and Griffith Feeney, "Estimating infant mortality trends from child survivorship data", *Population Studies*, vol. XXXIV, No. 1 (March 1980), pp. 109-128.

[10] W. Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data*.

and Trussell[11] carried out this exercise by assuming that period mortality changes can be modelled as movements through successively higher (or lower) levels of a set of model life tables, so that cohort life tables may be obtained by chaining together the mortality rates experienced by true cohorts living through the different periods. In this case, it can also be shown empirically that the child mortality estimate of the Brass type obtained from data for women in age group i, for example, is equal to the corresponding value prevalent during some particular period $t(x)$ years before the survey, and that this period is, for most practical purposes, invariant with respect to the speed of mortality change, so long as the rate of change is roughly constant over time. Because these time-location estimates have been derived in a manner that is consistent with that used in deriving the Trussell multiplying factors employed in estimating child mortality in this chapter, this timing procedure is described here.

An alternative solution to the problem of declining mortality is possible if data on children ever born and surviving are available from two surveys taken five or 10 years apart. It arises, once again, from the use of a hypothetical cohort representing the intersurvey experience; and it provides mortality estimates that refer to the intersurvey period. This estimation approach is not sensitive to the exact shape of mortality changes, but changes in the completeness of reporting of dead children from one survey to the next or population changes that are selective for the number of dead children may seriously affect the results.

To conclude these preliminary remarks on the methods presented in this chapter, it should be pointed out that for several of them two variants are presented: one variant to be applied when data on children ever born and surviving are classified by age of mother; and another when they are classified by duration of first marriage. The variants based on data classified by duration of marriage are, strictly speaking, based on the assumption that women, once married, stay married until age 50 (the assumed upper limit of the potential reproductive life of a woman). Therefore, the duration-based methods should strictly be applied only to data from currently married women still in their first union. However, in practice, no serious biases will arise when they are applied to data pertaining to all ever-married women, as long as their marriage duration is calculated as the time elapsed since first marriage.

As a last word of caution, it must be said that the performance of the duration variants of these methods can be rather poor when "duration" is not accurately measured. Problems in the measurement of this variable have been described in chapter II, subsection A.2, and are only briefly cited now. Duration of marriage is defined as the time elapsed since first union, regardless of whether that union is legal. Data on duration of marriage will be less than ideal when only legal unions are considered; when the time elapsed is not measured from the beginning of the first union, but rather from that of the current union; or when, as in some Muslim cultures, the entrance into a legal marriage predates the initiation of cohabitation. In populations where these problems are likely to arise, the duration variant should not be used.

2. *Organization of this chapter*

All the estimation procedures presented in this chapter have one characteristic in common: they use data on children ever born and surviving. However, the methods can be separated into categories according to the exact type of data they require (whether classified by age or by duration of marriage, for example), or according to the practical constraints that their assumptions impose (whether fertility is assumed to be constant or not). Sections B-E are devoted to the different categories. To aid the user in selecting that most suited for a particular application, brief descriptions of each section follow (see also table 46):

Section B. Estimation of child mortality using data classified by age. In this section, the most recent version of the original Brass estimation procedure is presented (Trussell's method). Estimates of $q(2)$, $q(3)$, $q(5)$, $q(10)$, $q(15)$ and $q(20)$, as well as of the periods to which they refer in cases where a smooth change in mortality can be assumed, are obtained from data on children ever born and surviving classified by age of mother. Fertility patterns are assumed to remain constant;

Section C. Estimation of child mortality using data classified by duration of marriage. In this section, a variant of the original Brass method that may be applied to data classified by duration of first marriage is presented. Estimates of $q(2)$, $q(3)$, $q(5)$, $q(10)$, $q(15)$, $q(20)$ and $q(25)$, as well as of the periods to which they refer in cases where a smooth change in mortality can be assumed, are obtained from data on children ever born and surviving classified by the mother's marriage duration. Marital fertility patterns are assumed to remain constant;

Section D. Estimation of intersurvey child mortality using data for a hypothetical intersurvey cohort. In this section, data from two censuses or surveys five years apart are used to estimate average intersurvey child mortality. The use of hypothetical cohorts circumvents the necessity of assuming that fertility and mortality have remained constant. Therefore, if the data at the two points in time considered are similar in quality and moderately reliable, intersurvey estimates are to be preferred over those derived by other means;

Section E. Estimation of child mortality when the fertility experience of true cohorts is known. In this section, data from two censuses or surveys five or 10 years apart are used to determine the parity ratios for true cohorts; and these ratios, in turn, are employed in estimating child mortality from data collected by the second census or survey. Both an age and a duration variant are described in subsection E.2. The time location of the child mortality estimates obtained is also estimated and constant fertility is not assumed.

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[11] Ansley J. Coale and James Trussell, "Estimating the time to which Brass estimates apply", *Annex I* to Samuel H. Preston and Alberto Palloni, "Fine-tuning Brass-type mortality estimates with data on ages of surviving children", *Population Bulletin of the United Nations, No. 10/1977*, pp. 87-89

TABLE 46. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER III

Section	Type of input data	Estimated parameters
B. Estimation of child mortality rates using data classified by age (one survey)	Children ever born classified by five-year age group of mother Children surviving (or dead) classified by five-year age group of mother Women classified by five-year age group	$q(2)$, $q(3)$, $q(5)$, $q(10)$, $q(15)$ and $q(20)$ Reference period for each $q(x)$ estimate
C. Estimation of child mortality using data classified by duration of marriage (one survey)	Children ever born classified by five-year duration of marriage group of mother Children surviving (or dead) classified by five-year duration of marriage group of mother Ever-married women classified by five-year duration of marriage group	$q(2)$, $q(3)$, $q(5)$, $q(10)$, $q(15)$, $q(20)$ and $q(25)$ Reference period for each $q(x)$ estimate
D. Estimation of intersurvey child mortality using data for a hypothetical intersurvey cohort (data by age from two surveys five years apart)	Children ever born classified by five-year age group of mother from two surveys or censuses five years apart Children surviving (or dead) classified by five-year age group of mother from two surveys or censuses five years apart Women classified by five-year age group from two surveys or censuses five years apart	Intersurvey estimates of $q(2)$, $q(3)$, $q(5)$, $q(10)$, $q(15)$ and $q(20)$
E. Estimation of child mortality when the fertility experience of true cohorts is known	Children ever born classified by five-year age (duration) group of mother from surveys or censuses five or 10 years apart Children surviving (or dead) classified by five-year age (duration) group of mother from the second survey or census Women (ever-married women) classified by five-year age (duration) group	$q(2)$, $q(3)$, $q(5)$ and $q(10)$, and their time reference periods when data are classified by age and surveys are five years apart $q(3)$, $q(5)$ and $q(10)$ with reference periods when data are classified by age and surveys are 10 years apart $q(3)$, $q(5)$ and $q(10)$ with reference periods when data are classified by duration and surveys are five years apart $q(5)$ and $q(10)$ with reference periods when data are classified by duration and surveys are 10 years apart

B. ESTIMATION OF CHILD MORTALITY RATES USING DATA CLASSIFIED BY AGE

1. *Data required*

The data required for this method are listed below:

(*a*) The number of children ever born, classified by sex (see note) and by five-year age group of mother;

(*b*) The number of children surviving (or the number dead), classified by sex (see note) and by five-year age group of mother;

(*c*) The total number of women (irrespective of marital status), classified by five-year age group. Note that all women, not merely ever-married women, must be considered.

Note should be taken also that classification by sex for children ever born and surviving is desirable, not essential. If it is available, child mortality for each sex can be estimated separately; whereas if it is not available, estimates for each sex can only be obtained by assuming that the sex differentials in the population being studied are the same as those embodied by model life tables whose mortality level is consistent with the estimated child mortality of both sexes, or by making some other assumption about the relationship between male and female child mortality.

When data on children ever born are classified by sex, their consistency may be ascertained by computing the sex ratios (defined as the average number of male children per female child) of children ever born by age of mother. Ideally, these sex ratios should not vary systematically with age and their values should be between

1.02 and 1.07. Studies made in countries where birth registration is fairly complete have shown that the sex ratio at birth is remarkably constant and that its usual value is around 1.05 males per female. In populations originating in Africa south of the Sahara, this value appears to be closer to 1.03. In either case, however, its constancy and the fact that women are supposed to declare all the children they have ever borne alive, whether these children survived or not, allows a simple consistency check. In populations other than those originating in sub-Saharan Africa, sex ratios higher than 1.07 or lower than 1.02 suggest differential omission of females or males, respectively, or misreporting of the sex of the reported children.

2. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of average parity per woman. Parity $P(1)$ refers to age group 15-19, $P(2)$ to 20-24 and $P(3)$ to 25-29. In general,

$$P(i) = CEB(i)/FP(i) \qquad (B.1)$$

where $CEB(i)$ denotes the number of children ever borne by women in age group i; and $FP(i)$ is the total number of women in age group i, irrespective of their marital status. Recall that, following the usual convention, variable i refers to the different five-year age groups considered. Thus, the value $i = 1$ represents age group 15-19, $i = 2$ group 20-24 and so on. The treatment of women whose parity is not stated is discussed in chapter II, subsection A.2, and annex II. In general, if the El-Badry technique for estimating true non-response cannot be applied, women of unstated parity should be included in the female population denominator when calculating average parity, since childless women are often misclassified as cases of non-response.

Step 2: calculation of proportion of children dead for each age group of mother. The proportion of children dead, $D(i)$, is defined as the ratio of reported children dead to reported children ever born, that is,

$$D(i) = CD(i)/CEB(i) \qquad (B.2)$$

where $CEB(i)$ is defined as in step 1; and $CD(i)$ is the number of children dead reported by women in age group i.

Step 3: calculation of multipliers. Table 47 presents the estimation equations and the necessary coefficients to estimate the multipliers, $k(i)$, according to the Trussell variant of the original Brass method. A different set of

TABLE 47. COEFFICIENTS FOR ESTIMATION OF CHILD MORTALITY MULTIPLIERS, TRUSSELL VARIANT, WHEN DATA ARE CLASSIFIED BY AGE OF MOTHER

Mortality model (1)	Age group (2)	Index i (3)	Mortality ratio $q(x)/D(i)$ (4)	Coefficients $a(i)$ (5)	$b(i)$ (6)	$c(i)$ (7)
North	15-19	1	$q(1)/D(1)$	1.1119	−2.9287	0.8507
	20-24	2	$q(2)/D(2)$	1.2390	−0.6865	−0.2745
	25-29	3	$q(3)/D(3)$	1.1884	0.0421	−0.5456
	30-34	4	$q(5)/D(4)$	1.2046	0.3037	−0.5656
	35-39	5	$q(10)/D(5)$	1.2586	0.4236	−0.5898
	40-44	6	$q(15)/D(6)$	1.2240	0.4222	−0.5456
	45-49	7	$q(20)/D(7)$	1.1772	0.3486	−0.4624
South	15-19	1	$q(1)/D(1)$	1.0819	−3.0005	0.8689
	20-24	2	$q(2)/D(2)$	1.2846	0.6181	−0.3024
	25-29	3	$q(3)/D(3)$	1.2223	0.0851	−0.4704
	30-34	4	$q(5)/D(4)$	1.1905	0.2631	−0.4487
	35-39	5	$q(10)/D(5)$	1.1911	0.3152	−0.4291
	40-44	6	$q(15)/D(6)$	1.1564	0.3017	−0.3958
	45-49	7	$q(20)/D(7)$	1.1307	0.2596	−0.3538
East	15-19	1	$q(1)/D(1)$	1.1461	−2.2536	0.6259
	20-24	2	$q(2)/D(2)$	1.2231	−0.4301	−0.2245
	25-29	3	$q(3)/D(3)$	1.1593	0.0581	−0.3479
	30-34	4	$q(5)/D(4)$	1.1404	0.1991	−0.3487
	35-39	5	$q(10)/D(5)$	1.1540	0.2511	−0.3506
	40-44	6	$q(15)/D(6)$	1.1336	0.2556	−0.3428
	45-49	7	$q(20)/D(7)$	1.1201	0.2362	−0.3268
West	15-19	1	$q(1)/D(1)$	1.1415	−2.7020	0.7662
	20-24	2	$q(2)/D(2)$	1.2563	−0.5381	−0.2633
	25-29	3	$q(3)/D(3)$	1.1854	0.0633	−0.4177
	30-34	4	$q(5)/D(4)$	1.1720	0.2341	−0.4272
	35-39	5	$q(10)/D(5)$	1.1865	0.3080	−0.4452
	40-44	6	$q(15)/D(6)$	1.1746	0.3314	−0.4537
	45-49	7	$q(20)/D(7)$	1.1639	0.3190	−0.4435

Estimation equations:

$$k(i) = a(i) + b(i)(P(2)/P(3)) + c(i)(P(2)/P(3))$$
$$q(x) = k(i)\,D(i)$$

[a] Ratio of probability of dying to proportion of children dead. This ratio is set equal to the multiplier $k(i)$.

coefficients is provided for each of the four different families of model life tables in the Coale-Demeny system.

Step 4: calculation of probabilities of dying and of surviving. Estimates of the probability of dying, $q(x)$, are obtained for different values of exact age x as the product of the reported proportions dead, $D(i)$, and the corresponding multipliers, $k(i)$. Note that the value of x is not generally equal to that of i, because x is related, in broad terms, to the average age of the children of women in age group i.

Once $q(x)$ is estimated, its complement $l(x)$, the probability of surviving from birth to exact age x, is readily obtained as $l(x) = 1.0 - q(x)$.

Step 5: calculation of reference period. As explained earlier, when mortality is changing smoothly, the reference period, $t(x)$, is an estimate of the number of years before the survey date to which the child mortality estimates, $q(x)$, obtained in the previous step *refer*. The value of $t(x)$ is also estimated by means of an equation whose coefficients were estimated from simulated cases by using linear regression. The equation used in this case is presented in table 48 together with a set of values for its coefficients.

3. *A detailed example*

The data shown in table 49 were gathered by a survey carried out in Panama between August and October 1976. They are used to illustrate the method just described. However, before proceeding with the estimation of child mortality, a quick check of the consistency

TABLE 48. COEFFICIENTS FOR ESTIMATION OF THE REFERENCE PERIOD, $t(x)$,[a] TO WHICH THE VALUES OF $q(x)$ ESTIMATED FROM DATA CLASSIFIED BY AGE REFER

Mortality model (1)	Age group (2)	Index i (3)	Age x (4)	Parameter estimate (5)	Coefficients		
					$a(i)$ (6)	$b(i)$ (7)	$c(i)$ (8)
North	15-19	1	1	$q(1)$	1.0921	5.4732	−1.9672
	20-24	2	2	$q(2)$	1.3207	5.3751	0.2133
	25-29	3	3	$q(3)$	1.5996	2.6268	4.3701
	30-34	4	5	$q(5)$	2.0779	−1.7908	9.4126
	35-39	5	10	$q(10)$	2.7705	−7.3401	14.9352
	40-44	6	15	$q(15)$	4.1520	−12.2448	19.2349
	45-49	7	20	$q(20)$	6.9650	−13.9160	19.9542
South	15-19	1	1	$q(1)$	1.0900	5.4443	1.9721
	20-24	2	2	$q(2)$	1.3079	5.5568	0.2021
	25-29	3	3	$q(3)$	1.5873	2.6755	4.7471
	30-34	4	5	$q(5)$	1.9399	−2.2739	10.3876
	35-39	5	10	$q(10)$	2.6457	−8.4819	16.5153
	40-44	6	15	$q(15)$	4.0794	−13.8308	21.1866
	45-49	7	20	$q(20)$	7.1796	−15.3880	21.7892
East	15-19	1	1	$q(1)$	1.0959	5.5864	−1.9949
	20-24	2	2	$q(2)$	1.2921	5.5897	0.3631
	25-29	3	3	$q(3)$	1.5021	2.4692	5.0927
	30-34	4	5	$q(5)$	1.9347	−2.6419	10.8533
	35-39	5	10	$q(10)$	2.6197	−8.9693	17.0981
	40-44	6	15	$q(15)$	4.1317	−14.3550	21.8287
	45-49	7	20	$q(20)$	7.3657	−15.8083	22.3005
West	15-19	1	1	$q(1)$	1.0970	5.5628	−1.9956
	20-24	2	2	$q(2)$	1.3062	5.5677	0.2962
	25-29	3	3	$q(3)$	1.5305	2.5528	4.8962
	30-34	4	5	$q(5)$	1.9991	−2.4261	10.4282
	35-39	5	10	$q(10)$	2.7632	−8.4065	16.1787
	40-44	6	15	$q(15)$	4.3468	−13.2436	20.1990
	45-49	7	20	$q(20)$	7.5242	−14.2013	20.0162

Estimation equation:

$$t(x) = a(i) + b(i)\,(P(1)/P(2)) + c(i)\,(P(2)/P(3))$$

[a] Number of years prior to the survey.

TABLE 49. CHILDREN EVER BORN AND CHILDREN SURVIVING, BY SEX AND AGE OF MOTHER, PANAMA, 1976

Age group (1)	Total number of women (2)	Male children		Female children		Sex ratio of children ever born (7)
		Ever born (3)	Dead (4)	Ever born (5)	Dead (6)	
15-19	2 695	278	24	279	16	0.9964
20-24	2 095	1 380	77	1 253	53	1.1014
25-29	1 826	2 395	172	2 362	140	1.0140
30-34	1 605	3 097	236	2 988	199	1.0365
35-39	1 362	3 444	348	3 278	288	1.0506
40-44	1 128	3 274	394	3 093	292	1.0585
45-49	930	2 682	354	2 594	335	1.0339
TOTAL	11 643	16 550	1 605	15 847	1 323	1.0444

of the data presented is carried out by computing the sex ratios of the number of children ever born. Column (7) of table 49 shows these ratios. They are computed by dividing the number of male children ever born by the corresponding number of female children. As an example, for age group 20-24, the sex ratio is

$$1{,}380/1{,}253 = 1.1014,$$

and the overall sex ratio is

$$16{,}550/15{,}847 = 1.0444.$$

The sex ratios given in column (7) of table 49 fluctuate somewhat by age of mother but show no systematic trend, and the overall sex ratio is acceptably close to the expected value of 1.05. Furthermore, since some variation of the sex ratios by age is expected because of the relatively small sample being considered, it is concluded that this test shows no clear deficiency in the data.

Step 1: calculation of average parity per woman. Average parities $P(1)$, $P(2)$ and $P(3)$ are calculated by dividing the number of children ever born of each sex (appearing in columns (3) and (5) of table 49) by the total number of women (column (2) of that table). Thus, for example, $P_m(1)$, the mean number of male children ever borne by women aged 15-19 is

$$P_m(1) = 278/2{,}695 = 0.1032.$$

The complete sets of $P_m(i)$ and $P_f(i)$ values are shown in columns (3) and (4) of table 50.

Note that the values of $P(i)$ for both sexes combined are just the sum of $P_m(i)$ and $P_f(i)$, the mean number of male and female children, respectively, born to women of age group i.

TABLE 50. AVERAGE PARITY PER WOMAN, BY SEX OF CHILD AND AGE OF MOTHER, PANAMA, 1976

Age group (1)	Index i (2)	Average parity per woman: Males $P_m(i)$ (3)	Females $P_f(i)$ (4)	Both sexes $P_t(i)$ (5)
15-19	1	0.1032	0.1035	0.2067
20-24	2	0.6587	0.5981	1.2568
25-29	3	1.3100	1.2920	2.6020
30-34	4	1.9300	1.8620	3.7920
35-39	5	2.5286	2.4068	4.9354
40-44	6	2.9025	2.7420	5.6445
45-49	7	2.8839	2.7892	5.6731

Step 2: calculation of proportion of children dead, for each age group of mother. The values of this proportion, $D(i)$, are computed from table 49 by dividing the number of children dead of each sex, given in columns (4) and (6), by the children ever born of the corresponding sex, shown in columns (3) and (5). Thus, $D_m(1)$, the proportion of male children dead among those ever born to women aged 15-19 is

$$D_m(1) = 24/278 = 0.0863.$$

To calculate the $D(i)$ values for both sexes combined, the deaths have to be added and then divided by the total number of children ever born (sum of males and females). Hence, $D_t(1)$ for both sexes would be

$$D_t(1) = (24+16)/(278+279) = 0.0718.$$

Table 51 shows a complete set of the proportions of children dead.

TABLE 51. PROPORTIONS OF CHILDREN DEAD, BY SEX OF CHILDREN AND AGE OF MOTHER, PANAMA, 1976

Age group (1)	Index i (2)	Proportion of children dead: Males $D_m(i)$ (3)	Females $D_f(i)$ (4)	Both sexes $D_t(i)$ (5)
15-19	1	0.0863	0.0574	0.0718
20-24	2	0.0658	0.0423	0.0494
25-29	3	0.0718	0.0593	0.0656
30-34	4	0.0762	0.0666	0.0715
35-39	5	0.1010	0.0879	0.0946
40-44	6	0.1203	0.0944	0.1077
45-49	7	0.1320	0.1291	0.1306

Step 3: calculation of multipliers. The multipliers, $k(i)$, required to adjust the reported proportion dead, $D(i)$, for the effects of the age pattern of childbearing are calculated from the ratios $P(1)/P(2)$ and $P(2)/P(3)$, by using the equation and the coefficients listed in table 47. Thus,

$$k(i) = a(i) + b(i)\,P(1)/P(2) + c(i)\,P(2)/P(3).$$

It is assumed that the West family of model life tables is an adequate representation of mortality in Panama, so values of $a(i)$, $b(i)$ and $c(i)$ are taken from the bottom panel of table 47. Given the values of $P(1)$, $P(2)$ and $P(3)$ shown in table 50, values of $k(i)$ can be calculated for each sex and for both sexes combined. The full set of $k(i)$ values is shown in table 52. As an example, the multiplier for the male children of women aged 20-24 ($i = 2$) is

$$k_m(2) = 1.2563 + (-0.5381)(0.1032/0.6587) + (-0.2637)(0.6587/1.3100) = 1.0394.$$

TABLE 52. TRUSSELL'S MULTIPLIERS FOR CHILD MORTALITY ESTIMATION, WEST MODEL: PANAMA, 1976

Age group (1)	Index i (2)	Multipliers $k(i)$ for: Males (3)	Females (4)	Both sexes (5)
15-19	1	1.1026	1.0279	1.0663
20-24	2	1.0394	1.0481	1.0404
25-29	3	0.9850	1.0027	0.9938
30-34	4	0.9939	1.0147	1.0042
35-39	5	1.0109	1.0337	1.0221
40-44	6	0.9984	1.0219	1.0100
45-49	7	0.9909	1.0138	1.0022
$P(1)/P(2)$		0.1567	0.1730	0.1645
$P(2)/P(3)$		0.5028	0.4629	0.4830

Step 4: calculation of probabilities of dying and of survi-

TABLE 53. ESTIMATES OF PROBABILITIES OF DYING AND OF SURVIVING, BY SEX, DERIVED FROM CHILD SURVIVAL DATA CLASSIFIED BY AGE OF MOTHER. WEST MODEL: PANAMA, 1976

Age group (x)	Age (x)	Probabilities of dying q(x) and of surviving l(x)					
		Males		Females		Both sexes	
		q(x)	l(x)	q(x)	l(x)	q(x)	l(x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
15-19	1	0.0952	0.9048	0.0590	0.9410	0.0766	0.9234
20-24	2	0.0580	0.9420	0.0440	0.9560	0.0514	0.9486
25-29	3	0.0707	0.9293	0.0595	0.9405	0.0652	0.9348
30-34	5	0.0757	0.9243	0.0676	0.9324	0.0718	0.9282
35-39	10	0.1021	0.8979	0.0909	0.9091	0.0967	0.9033
40-44	15	0.1201	0.8799	0.0965	0.9035	0.1088	0.8912
45-49	20	0.1308	0.8692	0.1309	0.8691	0.1309	0.8691

ing. The estimated values of the probabilities of dying, $q(x)$, are now calculated by multiplying the $k(i)$ values appearing in table 52 by the corresponding proportions dead, $D(i)$, given in table 51. A complete set of $q(x)$ estimates is shown in table 53. As an example, the value of $q_f(5)$ is obtained as follows:

$$q_f(5) = k_f(4)D_f(4) = (1.0147)(0.0666) = 0.0676.$$

Since every $q(x)$ value is the probabilistic complement of the probability of surviving, $l(x)$, the latter value can be obtained by subtracting the former from 1.0. Thus,

$$l_f(5) = 1.0 - 0.0676 = 0.9324.$$

In table 53, every $q(x)$ value is accompanied by its corresponding $l(x)$ value.

Step 5: calculation of reference period. Since mortality is not likely to have remained constant in Panama until 1976, it is useful to know the reference period, $t(x)$, of each $q(x)$ estimate. The values of the ratios $P(1)/P(2)$ and $P(2)/P(3)$ that are needed to estimate $t(x)$ have already been computed in step 3. The form of the estimation equation and the values of the coefficients needed to estimate $t(x)$ are obtained from table 48, again assuming a West mortality pattern. The value of $t_f(3)$ is calculated here as an example:

$$t_f(3) = 1.5305 + (2.5528)(0.1730)$$

$$+ (4.8962)(0.4629) = 4.24.$$

Thus, the estimated $q_f(3)$ value of 0.0595 is similar to that corresponding to the period life table in operation 4.24 years before the date of the survey, which may itself be regarded as the average date of interview. Since the survey took place between August and October 1976, in rough terms $q_f(3)$ refers to mid-1972. The complete set of estimated $t(x)$ values is presented in table 54.

Note that the $t(x)$ values imply that the estimates of $q(1)$, $q(2)$, $q(3)$ and $q(5)$ obtained above refer to mortality conditions prevalent approximately one year, two and one-half years, four and one-half years and six and one-half years before the survey, respectively; thereafter, the estimated values of $t(x)$ increase by some two and one-half to three years per age group. These values appear to be quite consistent with the notion that because the estimate of $q(2)$, for example, is based mainly on information corresponding to women aged 20-24, whose childbearing experience is relatively recent, the $q(2)$ estimate should also refer to the recent experience of the population. The plausibility and consistency of the values of $t(x)$ is reassuring. They provide important information for the study of child mortality trends over time.

4. *Comments on the detailed example*

The calculation of the sex ratios of children ever born by age group of mother did not reveal any irregularities that could not be explained by the small numbers involved in most cases. Another way to assess the quality of the data on children ever born is by examining the behaviour of the average parities reported by women of each age group. Unless fertility rose at some time in the past, average parities should increase with age up to age group 45-49. According to this rough test, data for Panama again appear to be satisfactory, although the very small increase in parity from ages 40-44 to ages 45-49 is somewhat suspicious (the average number of male children actually declines slightly). It is of interest to examine the parities because any omission of children

TABLE 54. ESTIMATES OF THE REFERENCE PERIOD[a] TO WHICH THE ESTIMATED PROBABILITIES OF DYING REFER. WEST MODEL, PANAMA, 1976

Age group (x)	Age (x)	Parameter estimated	Reference period, t(x)[a]		
			Males	Females	Both sexes
(1)	(2)	(3)	(4)	(5)	(6)
15-19	1	q(1)	0.97	1.14	1.05
20-24	2	q(2)	2.33	2.41	2.37
25-29	3	q(3)	4.39	4.24	4.32
30-34	5	q(5)	6.86	6.41	6.64
35-39	10	q(10)	9.58	8.80	9.19
40-44	15	q(15)	12.43	11.41	11.92
45-49	20	q(20)	15.36	14.33	14.86

[a] Number of years prior to the survey.

ever born might be made up disproportionately of dead children, thus greatly affecting the proportion dead. In the case in hand, the parity data show no clear evidence of omission. The proportions of dead children increase rapidly with age of mother, especially above age 35, if one ignores the value for women aged 15-19 (this value is almost always out of line with subsequent values, probably because the children of young women are, in fact, subject to higher mortality risks); these proportions thus give no indication of increasing omission of dead children as age rises. The very rapid increase in the proportions dead with age of mother suggests that a combination of effects is in operation: an increasingly longer average exposure to the risk of dying of the children and considerably higher child mortality from 10 to 15 years before the survey.

One simple way of exploring the consistency of the mortality estimates obtained by this method is to convert them into mortality levels in the Coale-Demeny system, in order to compare the age pattern of the estimates obtained with that of the models. When estimates are available by sex, it is worth calculating the mortality levels associated with the $q(x)$ estimates for each sex. Table 55 shows the levels, as derived from the West model life tables by interpolating between the values of tables 236 and 237 (see annex VIII); levels for both sexes are not shown, since, being effectively averages of the levels for the two sexes individually, they would not contribute any more information.

TABLE 55. MORTALITY LEVELS IN THE WEST MODEL LIFE TABLES CONSISTENT WITH THE CHILDHOOD MORTALITY ESTIMATES, $q(x)$. PANAMA, 1976

Age x (1)	Males: West mortality level (2)	Males: Reference date (3)	Females: West mortality level (4)	Females: Reference date (5)
1	16.3	1975.7	18.1	1975.6
2	20.0	1974.4	20.2	1974.3
3	19.4	1972.3	19.2	1972.5
5	19.4	1969.8	19.0	1970.3
10	18.4	1967.1	18.1	1967.9
15	17.8	1964.3	18.1	1965.3
20	17.8	1961.3	17.0	1962.4

As an example, the level consistent with the estimated $q(2)$ for females is calculated here. According to table 53, $q_f(2)$ is equal to 0.0440 and the corresponding $l(2)$ is 0.9560. Since only $l(x)$ values are tabulated in table 236 in annex VIII (females, model West), this last value is used for interpolation purposes. In the column labelled "$l(2)$" in table 236, the two values that enclose the observed $l(2)$ are $l_{20}(2)=0.95392$ and $l_{21}(2)=0.96559$ (the $l(2)$ values at levels 20 and 21, respectively). Therefore, the interpolation factor θ is

$$\theta=(0.9560-0.95392)/(0.96559-0.95392)=0.178,$$

and because the distance between levels 20 and 21 is just one, the level consistent with $q_f(2)$ is 20.178 or 20.2 when rounded. All other values shown in table 55 are obtained in a similar way. For a detailed explanation of the procedure for linear interpolation, see annex IV.

An examination of the levels displayed in table 55 shows that, as mentioned above, the estimates of $q(1)$ imply relatively high mortality (they are associated with relatively low levels in the Coale-Demeny models) and should not be considered. Making some allowance for the random variation inherent in any measure derived from probability samples, the remaining estimates of level decline steadily as age of mother rises, strongly suggesting that child mortality has been falling. Furthermore, the estimates of level are fairly consistent for the two sexes, indicating that child mortality differentials by sex in Panama are rather similar to those embodied in the West model life tables. Note that in table 55, reference-date estimates calculated by subtracting the $t(x)$ estimates given in table 54 from 1976.7 (the average date of the survey) are also shown. These estimates, in conjunction with the mortality levels, can be used to determine the trend of child mortality through time, especially because neither the data nor the results of their analysis reveal any obvious problems.

C. ESTIMATION OF CHILD MORTALITY USING DATA CLASSIFIED BY DURATION OF MARRIAGE

1. *Data required*

The data required for this method are given below:

(*a*) The number of children ever born classified by sex (see the note below) and by the mother's five-year duration-of-marriage group. Duration of marriage is defined as the time elapsed since entry into first union, regardless of whether this union is legal;

(*b*) The number of children dead classified by sex (see the note given below) and by the mother's five-year duration-of-marriage group;

(*c*) The total number of ever-married women in each five-year marriage-duration group. (The term "ever-married" means, in this instance, having entered into at least one union.)

As in the case of data classified by mother's age, note should be taken that the classification of children ever born and children surviving by sex is desirable, but not necessary. If it is available, child mortality for each sex can be estimated separately; otherwise, estimates by sex may be imputed by assuming that a certain mortality model represents the mortality pattern of the population being studied. When a classification by sex is available, sex ratios may be used to ascertain the consistency of the data.

2. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of average parity per woman. Parity $P(1)$ refers to women whose first union has lasted between 0 and 5 exact years (that is, the 0-4 duration

group), $P(2)$ to women in the 5-9 duration category and $P(3)$ to those in category 10-14. In general,

$$P(i) = CEB(i)/MFP(i) \qquad \text{(C.1)}$$

where $CEB(i)$ is the number of children ever born reported by women belonging to duration group i and $MFP(i)$ is the total number of ever-married women in duration group i. Note that in this case the index i represents duration groups and not age groups. The value $i = 1$ is associated with the first duration group (of length 0-4), $i = 2$ with the second (5-9), and so on.

Step 2: calculation of proportion of children dead for each duration group of mother. This proportion, $D(i)$, is again defined as

$$D(i) = CD(i)/CEB(i) \qquad \text{(C.2)}$$

where $CD(i)$ is the total number of children dead reported by women in duration group i and $CEB(i)$ is the number of children ever born declared by those women.

Step 3: calculation of multipliers. The multipliers, $k(i)$, are obtained by substituting into equation (C.3) the appropriate average parity ratios calculated by using the average parities derived in step 1 and the coefficients shown in table 56. The equation for obtaining $k(i)$ is

$$k(i) = a(i) + b(i)[P(1)/P(2)] + c(i)[P(2)/P(3)]. \qquad \text{(C.3)}$$

Note should be taken that table 56 lists coefficients for duration-of-marriage groups up to 30-34 years in length. In practice, data are often only tabulated for groups up to 15-19 or 20-24 years; coefficients for longer duration periods have been included for the sake of completeness, even though they may not be used very often. It should also be noted that in all cases, the duration categories used must span exactly five years. Data referring to an open-ended duration interval, such as 20+ (20 years or more), should not be used to estimate child mortality.

Step 4: calculation of probabilities of dying and of surviving. Each probability of dying before exact age x, denoted by $q(x)$, is calculated as the product of the proportion of children dead among the ever born, $D(i)$, and the corresponding multiplier $k(i)$ obtained in step 3. Thus,

$$q(x) = k(i)D(i) \qquad \text{(C.4)}$$

TABLE 56 COEFFICIENTS FOR ESTIMATION OF CHILD MORTALITY MULTIPLIERS, TRUSSELL VARIANT, WHEN DATA ARE CLASSIFIED BY DURATION OF MARRIAGE

Model (1)	Duration group (2)	Index i (3)	Mortality ratio[a] $q(x)/D(i)$ (4)	Coefficient $a(i)$ (5)	Coefficient $b(i)$ (6)	Coefficient $c(i)$ (7)
North	0-4	1	$q(2)/D(1)$	1.2615	−0.5340	0.1232
	5-9	2	$q(3)/D(2)$	1.1957	−0.4103	−0.0930
	10-14	3	$q(5)/D(3)$	1.3067	−0.6303	−0.4618
	15-19	4	$q(10)/D(4)$	1.4701	0.1763	−0.7268
	20-24	5	$q(15)/D(5)$	1.5039	0.0039	−0.7071
	25-29	6	$q(20)/D(6)$	1.4798	−0.2487	−0.5582
	30-34	7	$q(25)/D(7)$	1.4373	−0.2317	−0.5047
South	0-4	1	$q(2)/D(1)$	1.3103	−0.5856	0.1367
	5-9	2	$q(3)/D(2)$	1.2309	−0.3463	−0.1073
	10-14	3	$q(5)/D(3)$	1.2774	0.0336	−0.3987
	15-19	4	$q(10)/D(4)$	1.3493	0.1366	−0.5403
	20-24	5	$q(15)/D(5)$	1.3592	−0.0315	−0.4944
	25-29	6	$q(20)/D(6)$	1.3512	−0.1939	−0.4899
	30-34	7	$q(25)/D(7)$	1.3498	0.1663	−0.4131
East	0-4	1	$q(2)/D(1)$	1.2299	−0.3998	0.0910
	5-9	2	$q(3)/D(2)$	1.1611	−0.2431	−0.0797
	10-14	3	$q(5)/D(3)$	1.2026	0.0171	−0.2992
	15-19	4	$q(10)/D(4)$	1.2773	0.1015	−0.4276
	20-24	5	$q(15)/D(5)$	1.3014	0.0219	−0.4195
	25-29	6	$q(20)/D(6)$	1.3160	−0.1630	−0.3751
	30-34	7	$q(25)/D(7)$	1.3287	−0.1523	−0.3925
West	0-4	1	$q(2)/D(1)$	1.2584	−0.4683	0.1060
	5-9	2	$q(3)/D(2)$	1.1841	−0.3006	−0.0692
	10-14	3	$q(5)/D(3)$	1.2446	0.0131	−0.3555
	15-19	4	$q(10)/D(4)$	1.3353	0.1157	−0.5245
	20-24	5	$q(15)/D(5)$	1.3875	−0.0193	−0.5472
	25-29	6	$q(20)/D(6)$	1.4227	0.1954	−0.5127
	30-34	7	$q(25)/D(7)$	1.4432	−0.1977	−0.5339

Estimation equations

$$k(i) = a(i) + b(i)[P(1)/P(2)] + c(i)[P(2)/P(3)]$$
$$q(x) = k(i)D(i)$$

[a] Ratio of probability of dying to proportion of children dead. This ratio is set equal to the multiplier $k(i)$.

for some pair (x, i) defined in table 56, from which the coefficients needed to calculate $k(i)$ were obtained. From the $q(x)$ values, their probabilistic complements, $l(x)$ (the probability of surviving from birth to exact age x) are easily obtained by subtraction from one, that is,

$$l(x) = 1.0 - q(x).$$

Step 5: calculation of reference period. As before, $t(x)$ is an estimate of the number of years before the survey to which the estimates of childhood mortality obtained in step 4 refer when mortality has been changing. The $t(x)$ values are obtained by using an equation whose form and coefficients for the case in which data are classified by marriage duration are presented in table 57.

3. *A detailed example*

Data on the number of children ever born and children surviving obtained during a survey carried out in Panama between August and October 1976 were tabulated not only by age of mother but by the time elapsed since her first union. The data classified by duration are summarized in table 58.

These data are used to illustrate the duration-based procedure for estimating mortality in childhood. Only data for the first five duration groups are given in table 58; data for longer duration groups, spanning five years each, are not available. Once again, as a consistency check, the sex ratios of the reported number of children ever born are examined. Just as in the case in which these data are classified by age, the values of these sex ratios for different marriage durations are expected to be reasonably stable and to be close to 1.05 (although some allowance must be made for the random variability inherent in the small numbers considered). The sex ratios are shown in column (7) of table 58. They were obtained by dividing the number of male children ever born by the corresponding number of female children. The sex ratio values shown in table 58 are not exactly constant, but, except for that referring to duration group 0-4, they all fall acceptably close to the expected figure. The large deviation shown by the sex ratio of the children born to women married only a few years (duration group 0-4) is probably due to the relatively small number of cases considered. Survival probabilities estimated from the data corresponding to this duration group may well be subject to similar biases and should be treated with reserve.

The steps of the computational procedure are given below.

Step 1: calculation of average parity per women. Average parities are computed in a way very similar to step 1 of the age version: each of the entries in columns (3) and (5) (male and female children ever born, respectively) of

TABLE 57. COEFFICIENTS FOR ESTIMATION OF THE REFERENCE PERIOD $t(x)$,[a] TO WHICH THE VALUES OF $q(x)$ ESTIMATED FROM DATA CLASSIFIED BY DURATION OF MARRIAGE REFER

Mortality model (1)	Duration group (2)	Index i (3)	Age x (4)	Parameter estimate (5)	Coefficients a(i) (6)	Coefficients b(i) (7)	Coefficients c(i) (8)
North	0-4	1	2	q(2)	1.0311	3.3349	−0.3281
	5-9	2	3	q(3)	1.6964	4.2147	−0.0160
	10-14	3	5	q(5)	1.4285	3.2687	4.4073
	15-19	4	10	q(10)	−0.0753	−1.0800	12.9281
	20-24	5	15	q(15)	−1.9749	−3.4773	21.3318
	25-29	6	20	q(20)	−2.1888	0.6124	23.9376
	30-34	7	25	q(25)	0.9613	4.4416	21.4661
South	0-4	1	2	q(2)	1.0202	3.3064	−0.3297
	5-9	2	3	q(3)	1.6601	4.5105	−0.0335
	10-14	3	5	q(5)	1.2146	3.4684	4.9524
	15-19	4	10	q(10)	0.6454	1.6045	16.6771
	20-24	5	15	q(15)	−2.9104	−4.1357	24.0072
	25-29	6	20	q(20)	−3.1641	1.2106	26.3515
	30-34	7	25	q(25)	0.4456	5.6384	23.2565
East	0-4	1	2	q(2)	1.0380	3.4213	−0.3545
	5-9	2	3	q(3)	1.6441	4.7042	0.0642
	10-14	3	5	q(5)	1.1068	3.3032	5.4464
	15-19	4	10	q(10)	−0.8678	−1.9683	15.5187
	20-24	5	15	q(15)	−3.2154	−4.1123	24.8624
	25-29	6	20	q(20)	−3.3885	1.8746	26.9798
	30-34	7	25	q(25)	0.4716	5.8775	23.7246
West	0-4	1	2	q(2)	1.0349	3.3714	−0.3390
	5-9	2	3	q(3)	1.6654	4.5855	0.0233
	10-14	3	5	q(5)	1.2709	3.3291	5.1402
	15-19	4	10	q(10)	−0.5370	−1.7679	14.6370
	20-24	5	15	q(15)	−2.4694	−3.9194	23.0999
	25-29	6	20	q(20)	−2.2107	1.3059	24.4479
	30-34	7	25	q(25)	1.5845	5.0415	20.6725

Estimation equation:

$$t(x) = a(i) + b(i)\,[P(1)/P(2)] + c(i)\,[P(2)/P(3)]$$

[a] Number of years prior to the survey.

TABLE 58. CHILDREN EVER BORN AND CHILDREN SURVIVING, BY SEX OF CHILD AND MARRIAGE DURATION OF MOTHER, PANAMA, 1976

Duration group (1)	Ever-married women (2)	Male children		Female children		Sex ratio of children ever born (7)
		Ever born (3)	Dead (4)	Ever born (5)	Dead (6)	
0-4	1 523	836	35	862	24	0.9698
5-9	1 717	2 303	124	2 204	110	1.0449
10-14	1 362	2 827	217	2 647	149	1.0680
15-19	1 225	3 103	251	3 003	220	1.0340
20-24	1 069	3 206	387	3 052	292	1.0505
TOTAL	6 896	12 275	1 014	11 768	795	1.0431

table 58 is divided by the corresponding entry in column (2), the number of ever-married women. Thus, for example, $P_m(1)$ for male children is

$$P_m(1) = 836/1{,}523 = 0.5489.$$

The average parities corresponding to all births (shown in table 59 under the label "Both sexes") can be obtained in the same way; or, alternatively, they can be obtained simply by summing the average numbers of male and female children ($P_m(i)$ and $P_f(i)$).

Thus, for both sexes, $P_t(2)$ would be

$$P_t(2) = (2{,}303 + 2{,}204)/1{,}717 = 2.6249$$

or, simply

$$P_t(2) = 1.3413 + 1.2836 = 2.6249.$$

Other values of the average parities are shown in table 59.

TABLE 59. AVERAGE PARITIES BY SEX OF CHILDREN AND MARRIAGE DURATION OF MOTHER, PANAMA, 1976

Duration group (1)	Index i (2)	Average parity per woman		
		Males $P_m(i)$ (3)	Females $P_f(i)$ (4)	Both sexes $P_t(i)$ (5)
0-4	1	0.5489	0.5660	1.1149
5-9	2	1.3413	1.2836	2.6249
10-14	3	2.0756	1.9435	4.0191
15-19	4	2.5331	2.4514	4.9845
20-24	5	2.9991	2.8550	5.8541

Step 2: calculation of proportion of children dead for each duration group of mother. This proportion, $D(i)$, is computed from table 58 by dividing the number of children dead (column (4) for males, column (6) for females) by the number of children ever born (column (3) for males, column (5) for females). When both sexes are considered, the number of male and female dead children has to be calculated by adding the figures in columns (4) and (6) and then dividing by the sum of male and female children ever born (columns (3) and (5)). The calculation of $D(2)$ for all cases is shown below:

$$D_m(2) = 124/2{,}303 = 0.0538$$

$$D_f(2) = 110/2{,}204 = 0.0499$$

$$D_t(2) = (124 + 110)/(2{,}303 + 2{,}204) = 0.0519$$

where the subindices indicate whether the value of $D(2)$ is for males (m), females (f) or both sexes combined (t). All values of $D(i)$ are given in table 60.

TABLE 60. PROPORTIONS OF CHILDREN DEAD, BY SEX OF CHILD AND MARRIAGE DURATION OF MOTHER, PANAMA, 1976

Duration group (1)	Index i (2)	Proportion of children dead		
		Males $D_m(i)$ (3)	Females $D_f(i)$ (4)	Both sexes $D_t(i)$ (5)
0-4	1	0.0419	0.0278	0.0348
5-9	2	0.0538	0.0499	0.0519
10-14	3	0.0768	0.0563	0.0669
15-19	4	0.0809	0.0733	0.0771
20-24	5	0.1207	0.0957	0.1085

Step 3: calculation of multipliers. The coefficients needed to compute the multipliers, $k(i)$, are given in table 56. The estimation equation has the form:

$$k(i) = a(i) + b(i)(P(1)/P(2)) + c(i)(P(2)/P(3)) \qquad \text{(C.5)}$$

where the independent variables used are $P(1)/P(2)$ and $P(2)/P(3)$. Once more, the West mortality pattern is selected. Because of the simple form of equation (C.5) the computation of the $k(i)$ multipliers is straightforward. Results are summarized in table 61; as an example, $k_m(3)$ for males is computed as

$$k_m(3) = 1.2446 + (0.0131)(0.4092) + (-0.3555)(0.6462) = 1.0202.$$

TABLE 61. MULTIPLIERS FOR THE PROPORTIONS OF CHILDREN DEAD TABULATED BY DURATION OF MARRIAGE, ASSUMING A WEST MORTALITY PATTERN, PANAMA, 1976

Duration group (1)	Index i (2)	Multipliers		
		Males $k_m(i)$ (3)	Females $k_f(i)$ (4)	Both sexes $k_t(i)$ (5)
0-4	1	1.1366	1.1233	1.1300
5-9	2	1.0035	0.9926	0.9982
10-14	3	1.0202	1.0156	1.0180
15-19	4	1.0437	1.0399	1.0419
20-24	5	1.0260	1.0176	1.0219
$P(1)/P(2)$		0.4092	0.4409	0.4247
$P(2)/P(3)$		0.6462	0.6605	0.6531

Step 4: estimation of probabilities of dying and of surviving. Estimates of $q(x)$, the probability of dying between birth and exact age x, are obtained by multiplying the

proportions dead, $D(i)$, obtained in step 2 by the $k(i)$ multipliers just calculated. Thus, $q(x)=k(i)D(i)$. One must be careful in matching the indices; the value of x corresponding to each i is given in table 56. Table 62 shows the final results for $q(x)$ and for $t(x)$, the probability of surviving.

TABLE 62. ESTIMATES OF PROBABILITIES OF DYING AND OF SURVIVING, BY SEX, DERIVED FROM CHILD-SURVIVAL DATA CLASSIFIED BY DURATION OF MARRIAGE, WEST MODEL: PANAMA, 1976

Duration group (1)	Age x (2)	Probabilities of dying, $q(x)$, and of surviving, $l(x)$: Males $q(x)$ (3)	Males $l(x)$ (4)	Females $q(x)$ (5)	Females $l(x)$ (6)	Both sexes $q(x)$ (7)	Both sexes $l(x)$ (8)
0-4	2	0.0476	0.9524	0.0312	0.9688	0.0393	0.9607
5-9	3	0.0540	0.9460	0.0495	0.9505	0.0518	0.9482
10-14	5	0.0784	0.9216	0.0572	0.9428	0.0681	0.9319
15-19	10	0.0844	0.9156	0.0762	0.9238	0.0803	0.9197
20-24	15	0.1238	0.8762	0.0974	0.9026	0.1109	0.8891

Step 5: calculation of reference period. Since mortality is very likely to have changed recently in Panama, the estimation of the reference period, $t(x)$, is appropriate. For this purpose, one requires the values of $P(1)/P(2)$ and $P(2)/P(3)$, which have already been calculated in step 3 (see table 61). The equation used to estimate $t(x)$ and the appropriate coefficients appear in table 57. The calculation of $t(x)$ is straightforward. As an illustration, $t_m(3)$ for males is computed below:

$$t_m(3) = 1.6654 + 4.5855(0.4092) + 0.0233(0.6462) = 3.56.$$

Values of $t(x)$ are shown in table 63.

4. *Comments on the detailed example*

As in the case of the age-based analysis, the child survival data of the survey conducted in Panama in 1976, when classified by duration of marriage, appear to be of acceptable quality. The sex ratios at birth of children ever born are close to the expected value of 1.05, the average parities increase monotonically with duration of marriage, and the proportions of children dead also increase with marital duration. The consistency of the final mortality estimates, both internal and with respect to the estimates obtained from the age-based method, can be conveniently assessed by finding the mortality level in the Coale-Demeny West family of model life

TABLE 63. ESTIMATES OF THE REFERENCE PERIOD, $t(x)$,[a] TO WHICH THE ESTIMATED PROBABILITIES OF DYING REFER, WEST MODEL: PANAMA, 1976

Duration group (1)	Age x (2)	Reference proportion (3)	Examples of reference period[a]: Males $t_m(x)$ (4)	Females $t_f(x)$ (5)	Both sexes $t_b(x)$ (6)
0-4	2	$q(2)$	1.38	1.42	1.40
5-9	3	$q(3)$	3.56	3.70	3.63
10-14	5	$q(5)$	5.89	6.07	5.98
15-19	10	$q(10)$	8.20	8.35	8.27
20-24	15	$q(15)$	10.85	11.06	10.95
$P(1)/P(2)$			0.4092	0.4409	0.4247
$P(2)/P(3)$			0.6462	0.6605	0.6531

[a] Number of years prior to the survey.

tables consistent with each estimate and then comparing these levels. Table 64 shows the mortality levels implied by the $q(x)$ estimates for each sex. They are obtained by interpolation using the tables referring to the West model in annex VIII.

TABLE 64. MORTALITY LEVELS OF THE WEST MODEL LIFE TABLES CONSISTENT WITH THE DURATION-BASED ESTIMATES OF CHILD MORTALITY, PANAMA, 1976

Age x (1)	Males: Mortality level (2)	Males: Reference date (3)	Females: Mortality level (4)	Females: Reference date (5)
2	20.8	1975.3	21.3	1975.3
3	20.5	1973.1	20.0	1973.0
5	19.2	1970.8	19.7	1970.6
10	19.3	1968.3	18.9	1968.3
15	17.6	1965.8	18.1	1965.6

The estimated mortality levels show a fairly coherent trend and reasonable consistency by sex. The average of the duration-based mortality levels is somewhat higher (by about half a level) than that obtained when the data were classified by age, but the reference period of the duration-based estimates is somewhat more recent, for any given value of x. Therefore, although the overall duration-based estimates indicate lower mortality than do the age estimates, their differences are very moderate. Given the instability of marriage in Panama and the resultant danger that the date of first union might be incorrectly reported or that unions may be frequently interrupted, the age-based estimates should probably be preferred in this instance.

D. Estimation of intersurvey child mortality using data for a hypothetical intersurvey cohort

1. *Data required*

The data required for this method are described below:

(*a*) The number of children ever born classified by five-year age group of mother (or by five-year duration group when women can be classified by the time elapsed since their first union) from two censuses or surveys taken five or 10 years apart;

(*b*) The number of children surviving (or its complement, the number of children dead) classified by five-year age or duration group of mother for the same two censuses or surveys;

(*c*) The total number of women in each five-year age group when data are classified by age, or the number of ever-married women belonging to each five-year duration group if data are classified by the time elapsed since the first union, from each of the surveys being considered.

Note that the classification of children by sex is desirable, but not necessary. When data are not classified by sex, sex differentials in childhood mortality may be imputed by using mortality models.

2. *Computational procedure*

The estimation procedure described here differs from those described in sections B and C only in the way in which the proportion of children dead and the average number of children ever born per woman (average parity) are calculated. Once the proportion dead, $D(i)$, and the average parity, $P(i)$, have been obtained, the calculation of multipliers $k(i)$ and the estimation of $q(x)$ (the probability of dying between birth and exact age x) proceed exactly as described in steps 3 and 4 of the computational procedures presented in subsections B.2 and C.2. Therefore, these steps are not described again. Furthermore, since the calculation of proportions of children dead and parities for hypothetical cohorts are essentially the same whether data are classified by age or by duration, the steps needed to perform these calculations are described for the age model only. When data are classified by duration, the same steps can be followed, using ever-married women instead of all women, and bearing in mind that the index i refers to duration groups rather than to age groups.

Note should be taken that, when considering hypothetical cohorts, the value of $t(x)$, the reference period, has no clear meaning and its estimation is unnecessary. In any case, the objective of using data for a hypothetical cohort is to obtain estimates of child mortality referring specifically to the intersurvey period, so there would be no point in estimating the $t(x)$ values.

The steps of the computational procedure are given below.

Step 1: calculation of average parity per woman. As described in step 1 of the computational procedures described in subsections B.2 and C.2, the average parities or average number of children ever born per woman in each age (duration) group are just the quotients of the observed number of children ever born, CEB, and the number of women in that age (duration) group, FP. In this case, average parities are calculated for each survey separately and the index j is used to indicate the survey to which they refer. Therefore, following the notation used in subsection B.2:

$$P(i,j) = CEB(i,j)/FP(i,j). \qquad \text{(D.1)}$$

Step 2: calculation of average number of children dead per woman. In this step, the average number of children dead per woman in each age group and for each of the surveys being used is calculated. Let $CD(i,j)$ be the number of children dead among those born to women in age group i and reported in survey j, and let $FP(i,j)$ be the total number of women in age group i at survey j. Then, the average number of children dead among women in age group i during survey j, denoted by $ACD(i,j)$, is just the quotient of $CD(i,j)$ and $FP(i,j)$, or

$$ACD(i,j) = CD(i,j)/FP(i,j). \qquad \text{(D.2)}$$

Step 3: estimation of proportion of children dead for a hypothetical cohort of women. Usually, the proportion of children dead is calculated as the number of dead children divided by the number of children ever born. However, when a hypothetical cohort of women is being considered, this proportion is calculated by dividing the average number of children dead by the average number of children ever born per woman of the hypothetical cohort. The average number of births per woman occurring to a true cohort between two surveys is the increment in the average parity of the cohort from one survey to the other, and the average number of children ever born for a hypothetical intersurvey cohort can be constructed by adding such increments (see chapter II, Section C). The average number of dead children for a hypothetical cohort of women can be obtained in a similar way, since the increment in the average number of dead children per woman for a true cohort between two surveys is a measure of the effect of mortality during the intersurvey period. However, when fertility has been changing, the average number of children dead for a hypothetical cohort of women cannot, strictly speaking, be obtained by summing the intersurvey increments of different female cohorts, since the intersurvey deaths include deaths both to children born between the surveys and to children born before the first survey; and the latter (number of children born before the first survey) will not be adequately represented by the parities of the hypothetical cohort which reflect the intersurvey fertility change. A procedure to estimate the appropriate average number of children dead for a hypothetical cohort under conditions of changing fertility is available.[12] Yet

[12] Hania Zlotnik and Kenneth H. Hill, "The use of hypothetical cohorts in estimating demographic parameters under conditions of changing fertility and mortality", *Demography*, vol. 18, No. 1 (February 1981), pp. 103–122.

unless fertility is falling unprecedently fast, the error introduced by using the simpler procedure described here is very small. Therefore, the former procedure is not described.

Thus, if mortality is changing but fertility has remained constant, the proportions of children dead for a hypothetical intersurvey cohort of women can be calculated by summing the intersurvey increments observed both in the average number of children dead and in average parities, and then dividing the sum of the former by the sum of the latter. In such a case, mortality estimates for the intersurvey period can be obtained by the procedures described in subsection B.2 (or subsection C.2 when the data are classified by duration of first marriage).

A detailed description of the calculation of the average parities and average numbers of children dead for a hypothetical cohort follows. If the length of the intersurvey period is n five-year intervals, the average number of children ever borne by women of age group i in the hypothetical cohort exposed to intersurvey fertility rates and denoted by $P(i, s)$ is

$$P(i, s) = P(i, 2) - P(i - n, 1) + P(i - n, s). \quad \text{(D.3)}$$

Similarly, the average number of children dead per woman of age group i in the hypothetical cohort, denoted by $ACD(i, s)$, is

$$ACD(i, s) = ACD(i, 2) - ACD(i - n, 1) + ACD(i - n, s). \quad \text{(D.4)}$$

Note that in both equations (D.3) and (D.4), if i is smaller than or equal to n, the hypothetical-cohort value is assumed to be equal to the value observed at the second survey. For estimation of child mortality, the hypothetical-cohort approach is of little value if the surveys are more than 10 years apart, since with a 15-year interval, the proportions of children dead for women under age 30 are estimated as equal to the reported proportions of children dead from the second survey; therefore, the estimates of $q(1)$, $q(2)$, and $q(3)$ will not reflect the complete intersurvey experience.

The hypothetical-cohort proportions dead, $D(i, s)$, are then obtained by dividing the average number of children dead obtained from equation (D.4) by the average parities obtained from equation (D.3), that is,

$$D(i, s) = ACD(i, s)/P(i, s). \quad \text{(D.5)}$$

Note that, since the calculations use average numbers per woman, it is not important, except because of the effects of sampling errors, whether the two data sets come both from censuses, both from surveys or one from each type of source.

Step 4: estimation of probability of dying. As stated earlier, the estimation of $q(x)$ here corresponds to steps 3 and 4 of the computational procedures described in subsections B.2 and C.2 (age and duration versions, respectively). Their application is exactly the same as given before, using the parities for the hypothetical cohort of women, $P(i, s)$, and the proportions dead, $D(i, s)$, also corresponding to the hypothetical cohort.

3. *A detailed example*

Data on children ever born and surviving are available for Thailand from a sample of the 1970 census and from two sample surveys held in 1974 and 1975. Weighted averages of the data from the two surveys that approximate those referring to a point exactly five years after the census date were used. All the basic data are shown in table 65.

Steps 1 and 2: calculation of average parity and of average number of children dead per woman. Average numbers of children ever born, $P(i, j)$, and dead, $ACD(i, j)$, per woman are obtained for each age group of women and for each survey by dividing the recorded totals by the relevant number of women. Thus, the average number of children dead per woman aged 25-29 in 1970 is obtained as

$$ACD(3, 1) = 274{,}526/1{,}141{,}937 = 0.2404.$$

Table 66 shows the results for all age groups.

Step 3: estimation of proportion of children dead for a hypothetical cohort of women. The interval between the 1970 census and the 1975 blended survey results is five years, so n in equations (D.3) and (D.4) is 1. Thus, for the first age group, 15-19, the average parity and the average number of children dead for the hypothetical

TABLE 65. NUMBER OF WOMEN, CHILDREN EVER BORN AND CHILDREN DEAD, BY AGE GROUP, THAILAND, 1970 AND 1975

Age group (i)	1970[a]			1975[b]		
	Number of women (1)	Children ever born (2)	Children dead (3)	Number of women (4)	Children ever born (5)	Children dead (6)
15-19	1 883 232	245 069	15 223	13 054	1 662	185
20-24	1 359 859	1 267 179	99 316	10 037	8 879	800
25-29	1 141 937	2 795 340	274 526	7 432	16 787	1 329
30-34	1 075 972	4 175 274	504 766	6 508	22 969	2 175
35-39	956 335	4 931 749	732 342	6 244	29 557	3 515
40-44	765 298	4 477 565	829 656	5 454	31 298	4 598
45-49	596 648	3 608 055	821 915	4 388	27 550	4 941

[a] Obtained from an expanded 2 per cent sample.

[b] Weighted average of the 1974 and 1975 rounds of the Survey of Population Change.

TABLE 66. AVERAGE NUMBER OF CHILDREN EVER BORN AND CHILDREN DEAD PER WOMAN, BY AGE GROUP OF MOTHER, THAILAND, 1970 AND 1975

Age group (1)	Index i (2)	1970 Average number of: Children ever born P(i, 1) (3)	1970 Average number of: Children dead ACD(i, 1) (4)	1975 Average number of: Children ever born P(i, 2) (5)	1975 Average number of: Children dead ACD(i, 2) (6)
15-19	1	0.1301	0.0081	0.1273	0.0090
20-24	2	1.0054	0.0730	0.8806	0.0598
25-29	3	2.4479	0.2404	2.1489	0.1701
30-34	4	3.8805	0.4691	3.5293	0.3649
35-39	5	5.1570	0.7658	4.7337	0.5629
40-44	6	5.8505	1.0841	5.7385	0.8431
45-49	7	6.0472	1.3776	6.2785	1.1260

cohort of women are put equal to the corresponding values from the second survey:

$$P(1, s) = P(1, 2) = 0.1273$$

and

$$ACD(1, s) = ACD(1, 2) = 0.0090.$$

Subsequent values are obtained by adding the cohort increments successively. Thus, for the second age group (20-24):

$$P(2, s) = P(2, 2) - P(1, 1) + P(1, s)$$

$$= 0.8806 - 0.1301 + 0.1273$$

$$= 0.8778$$

and

$$ACD(2, s) = ACD(2, 2) - ACD(1, 1) + ACD(1, s)$$

$$= 0.0598 - 0.0081 + 0.0090$$

$$= 0.0607.$$

Table 67 shows the results. (For a more detailed explanation of the procedure used to calculate average parities for a hypothetical cohort, see chapter II, subsection B.4 (c).)

Once the average parities and the average numbers of children dead have been calculated, bearing in mind the caveat about changing fertility, the proportion of children dead for the hypothetical cohort can be obtained using equation (D.5). Thus, according to the values taken from table 67, the proportion of children dead among those ever born to women aged 25-29 years, $D(3, s)$, is calculated as follows.

$$D(3, s) = ACD(3, s)/P(3, s)$$

$$= 0.1578/2.0213 = 0.0781$$

Column (5) of table 67 shows the full set of $D(i, s)$ values.

Step 4: estimation of probability of dying. Estimates of the probability of dying, $q(x)$, between birth and certain exact age x are obtained by using the procedure described in subsection B.2 with the North mortality pattern. The values of the ratios $P(1)/P(2)$ and $P(2)/P(3)$ are needed to estimate the appropriate multipliers, $k(i)$. These ratios are computed by using the parities listed in table 67 as illustrated below:

$$P(1, s)/P(2, s) = 0.1273/0.8778 = 0.1450,$$

$$P(2, s)/P(3, s) = 0.8778/2.0213 = 0.4343.$$

The multipliers, $k(i)$, are then obtained by using the equation appearing at the bottom of table 47 and the coefficients corresponding to the North mortality model, also shown in that table. The value of $k(3)$, for example, is obtained as

$$k(3) = 1.1884 + (0.0421)(0.1450)$$

$$+ (-0.5156)(0.4343) = 0.9706.$$

The complete set of multipliers used is shown in column (4) of table 68. The products of these multipliers and the proportions dead, $D(i, s)$, yield estimates of intersurvey child mortality. Thus, $q(5)$, for example, is obtained as

$$q(5) = k(4)D(4, s) = 0.0913.$$

All estimates of $q(x)$ are listed in column (6) of table 68. The mortality levels in the North model life tables to which each $q(x)$ estimate corresponds, calculated by assuming a sex ratio at birth of 105 males per 100 females, are shown in column (7). They were computed by interpolating within the values presented in table 242 (see annex IX), corresponding to model North and sex

TABLE 67. AVERAGE NUMBER OF CHILDREN EVER BORN AND CHILDREN DEAD PER WOMAN OF A HYPOTHETICAL INTERSURVEY COHORT, BY AGE OF MOTHER, THAILAND, 1970-1975

Age group (1)	Index i (2)	Average number of: Children ever born P(i, s) (3)	Average number of: Children dead ACD(i, s) (4)	Proportion of children dead D(i, s) (5)
15-19	1	0.1273	0.0090	0.0707
20-24	2	0.8778	0.0607	0.0692
25-29	3	2.0213	0.1578	0.0781
30-34	4	3.1027	0.2823	0.0910
35-39	5	3.9559	0.3761	0.0951
40-44	6	4.5374	0.4534	0.0999
45-49	7	4.9654	0.4953	0.0998

TABLE 68. CHILD MORTALITY ESTIMATES, THAILAND, PERIOD 1970-1975

Age group of women (i)	Index i (1)	Age x (3)	Multiplier k(i) (4)	Proportion of children dead D(i, i) (5)	Probability of dying q(x) (6)	Mean mortality level (7)
15-19	1	1	1.0638	0.0707	0.0752	16.7
20-24	2	2	1.0203	0.0692	0.0706	18.4
25-29	3	3	0.9706	0.0781	0.0758	18.6
30-34	4	5	1.0029	0.0910	0.0913	18.5
35-39	5	10	1.0639	0.0951	0.1012	18.5
40-44	6	15	1.0483	0.0999	0.1047	18.7
45-49	7	20	1.0269	0.0998	0.1025	19.3

ratio at birth equal to 1.05. For comparision, table 69 shows the child mortality estimates obtained by using only the 1975 data.

4. *Comments on the detailed example*

The data on children ever born and children surviving from the two surveys conducted in Thailand are not classified by sex of child, so one cannot check whether the reported sex ratios at birth are plausible. However, the availability of data for two points in time more than compensates for this lack, since parameter changes for true cohorts can be examined for plausibility. The average parity of a cohort of women should increase from the first point in time to the second by an amount consistent with the general age pattern of fertility; the average parities corresponding to true cohorts and shown in table 66 do increase in every case and by amounts that look reasonable, so the reporting of children ever born at the two time-points being considered seems consistent. The average number of children dead for true cohorts of women should also increase, whenever migration or selective mortality effects are not in operation; the data given in table 66 also pass this test, although the increases in average numbers of children dead for older women are very small. However, it can be concluded that, so far, these tests reveal no obvious shortcomings of the data.

TABLE 69. CHILD MORTALITY ESTIMATES, THAILAND, 1975

Age group of women (i)	Index i (2)	Age x (3)	Average parity per woman P(i) (4)	Proportion of children dead D(i) (5)	Multiplier k(i)[a] (6)	Probability of dying q(x) (7)	Reference period[b] (8)	Mean mortality level (9)
15-19	1	1	0.1273	0.0710	1.0370	0.0736	1.1	16.9
20-24	2	2	0.8836	0.0679	1.0272	0.0697	2.2	18.5
25-29	3	3	2.1459	0.0792	0.9832	0.0779	3.8	18.4
30-34	4	5	3.5293	0.1034	1.0167	0.1051	5.7	17.5
35-39	5	10	4.7337	0.1189	1.0782	0.1282	7.8	17.2
40-44	6	15	5.7383	0.1469	1.0615	0.1559	10.3	16.4
45-49	7	20	6.2785	0.1793	1.0381	0.1861	13.1	15.7

[a] Using $P(1)/P(2) = 0.1446$ and $P(2)/P(3) = 0.4098$.

[b] Number of years prior to the survey to which estimates refer.

The behaviour of the proportion dead among children ever born, $D(i, x)$, provides a further test of data quality; these proportions are expected to increase with age of mother, as the average exposure of her children to the risk of dying increases. Although the proportions dead reported at the two points in time being considered rise steadily with age of mother (leaving aside information for women aged 15-19), the proportions dead for the hypothetical cohort actually decline for age group 45-49, suggesting that there is some slight omission of dead children by older women, an omission masked in the original data sets by the effects of declining child mortality.

It is clear from the estimates of mortality level given in table 68 that the child mortality estimate based on information from women under 20 years of age is not reliable, a finding that is fairly common. The estimates for women in the age range from 20 to 44 are highly consistent, indicating that child mortality in Thailand between 1970 and 1975 was approximately equal to that of level 18.5 of the North model. The estimate obtained from data referring to women aged 45-49 indicates somewhat lower child mortality, probably because of the slight omission of dead children already detected. The consistency of the mortality estimates shown in table 68 is in marked contrast with that of the estimates presented in table 69, which show heavier mortality as age of mother rises, presumably because child mortality has been declining.

The intersurvey method of estimation described and illustrated here assumes roughly constant fertility. Fertility in Thailand was falling before 1970, and it continued to fall throughout the intersurvey period. In such circumstances, the estimated intersurvey child deaths exceed those expected on the basis of intersurvey births because they include the child deaths occurring during the intersurvey period to birth cohorts that were larger at the time of the first survey (or census in the case of Thailand) than those constructed for the hypothetical female cohort on the basis of intersurvey fertility. Child mortality will thus be overestimated somewhat by an amount that increases with age of mother, since the ratio of births during the intersurvey period to surviving children

at the beginning of the intersurvey period declines with age of mother. Thus, it is surprising to find that the estimated mortality levels given in table 68 tend to increase somewhat with age of mother in spite of the fact that the methodological bias just described would affect them in the opposite direction. Zlotnik and Hill[13] estimated that the magnitude of this bias in the case of Thailand is approximately −0.1 of a mortality level for the estimate derived from age group 25-29, and of about −0.2 of a mortality level for that derived from age group 30-34. The fact that such small biases are counteracted by other, yet undetected, flaws in the data make the average level, 18.5, an acceptable estimate of intersurvey child mortality.

E. Estimation of child mortality when the fertility experience of true cohorts is known

1. *Basis of method and its rationale*

As explained in section A, if fertility has been changing in the recent past, the observed parity ratios used as independent variables when estimating the multipliers, $k(i)$, may not reflect adequately the true experience of cohorts in the population; and, hence, the resulting multipliers may not be suited for mortality estimation purposes. A method proposed to circumvent the problems introduced by declining fertility consists of taking into account the experience of true cohorts when estimating the $k(i)$ multipliers, instead of basing their estimates on ratios of parities referring only to one point in time. This method is described below in detail.

As in the methods presented before, two types of cohorts may be considered: those defined according to age; and those defined according to the duration of first marriage or union. The estimation procedures used to analyze each of these types of data are very similar, so only the case where data are classified by age is described; the variations necessary to apply these procedures to data classified by duration of first marriage are pointed out as the need arises.

2. *Data required*

The following data are required for this method:

(*a*) The number of children ever born classified by five-year age (or duration) group of mother for two surveys five or 10 years apart;

(*b*) The number of children dead (or surviving) classified by five-year age (or duration) group of mother for the most recent survey being considered;

(*c*) The total number of women (or of ever-married women) classified by five-year age (or duration) groups for each one of the surveys being considered.

It is not necessary to have data on the number of children dead for both of the surveys being considered. If these data are available (children dead for both surveys), it is strongly recommended that the method described above in section D be used to estimate intersurvey child mortality, in spite of the fact that it does not make any explicit allowance for the effects of changing fertility.

As usual, it is helpful to have the data on children ever born and dead classified by sex. When this classification is not available, sex differentials in child mortality can only be imputed by using mortality models.

3. *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of average parity per woman. To calculate the average parity per woman, $P(i,j)$, let $CEB(i,j)$ be the number of children ever born to women in age (duration) group i at survey j, and let $FP(i,j)$ be the corresponding total number of women (ever-married women if data are classified by duration) of age (duration) group i. Then, as usual, the average number of children ever born per woman of age group i and survey j is calculated as

$$P(i,j) = CEB(i,j)/FP(i,j). \qquad \text{(E.1)}$$

Step 2: calculation of proportion of children dead reported at time of second survey. The values of this proportion, $D(i,2)$, are calculated only for the second survey or census. Thus, denoting the number of children dead to women of age group i from this survey by $CD(i,2)$, one has

$$D(i,2) = CD(i,2)/CEB(i,2). \qquad \text{(E.2)}$$

Step 3: calculation of multipliers. It is in this step that the use of cohort experience becomes relevant. The values of multipliers $k(i)$ are estimated by means of equations fitted to model cases by means of least-squares regression and whose independent variable is a ratio of parities referring to a birth cohort of women at two points in time. Therefore, if the surveys considered are five years apart, these parity ratios have the form $P(i-1,1)/P(i,2)$; while if the surveys are 10 years apart, the corresponding ratios would be $P(i-2,1)/P(i,2)$. The form of the equation to estimate the $k(i)$ multipliers, the values of the fitted coefficients and the form of the corresponding parity ratios are presented in tables 70-73. Tables 70 and 71 are to be used when the data are classified by age, whereas tables 72 and 73 are needed when the data are classified by duration of first marriage. The first table of each set (tables 70 and 72) is to be used when the intersurvey interval is five years, whereas the second table of each set (tables 71 and 73) is needed when the interval is 10 years. After selecting the appropriate table for the case at hand, the calculation of the $k(i)$ multipliers is straightforward, as is shown in the detailed examples.

Step 4: estimation of probability of dying. The estimated values of $q(x)$, the probability of dying between birth and exact age x, are obtained as the products of the observed proportions dead, $D(i,2)$, and the multipliers, $k(i)$, computed in step 3.

Step 5: estimation of reference period. The use of the experience of true cohorts to estimate the multipliers,

[13] *Ibid.*

TABLE 70. COEFFICIENTS FOR ESTIMATION OF THE MULTIPLIERS, $k(i)$, FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY AGE OF MOTHER AND THE INTERSURVEY INTERVAL IS FIVE YEARS

Mortality model (i)	Age group (x)	Index i (i)	Age x (x)	Parity ratio for two successive groups $P(i-1, t)/P(i, t)$ (5)	Coefficients a(i) (6)	Coefficients b(i) (7)
North	20-24	2	2	$P(1, 1)/P(2, 2)$	1.1635	−1.0530
	25-29	3	3	$P(2, 1)/P(3, 2)$	1.1833	−0.4924
	30-34	4	5	$P(3, 1)/P(4, 2)$	1.5408	−0.5210
	35-39	5	10	$P(4, 1)/P(5, 2)$	1.5425	−0.6137
South	20-24	2	2	$P(1, 1)/P(2, 2)$	1.2015	−1.0218
	25-29	3	3	$P(2, 1)/P(3, 2)$	1.2121	−0.4235
	30-34	4	5	$P(3, 1)/P(4, 2)$	1.2973	−0.4071
	35-39	5	10	$P(4, 1)/P(5, 2)$	1.4205	−0.4515
East	20-24	2	2	$P(1, 1)/P(2, 2)$	1.1614	−0.7298
	25-29	3	3	$P(2, 1)/P(3, 2)$	1.1524	−0.3159
	30-34	4	5	$P(3, 1)/P(4, 2)$	1.2240	−0.3184
	35-39	5	10	$P(4, 1)/P(5, 2)$	1.3253	−0.3682
West	20-24	2	2	$P(1, 1)/P(2, 2)$	1.1838	−0.8901
	25-29	3	3	$P(2, 1)/P(3, 2)$	1.1776	−0.3828
	30-34	4	5	$P(3, 1)/P(4, 2)$	1.2757	−0.3939
	35-39	5	10	$P(4, 1)/P(5, 2)$	1.4017	−0.4662

Estimation equations:

$$k(i) = a(i) + b(i)\, P(i-1, 1)/P(i, 2)$$
$$q(x) = k(i)\, D(i, 2)$$

TABLE 71. COEFFICIENTS FOR ESTIMATION OF THE MULTIPLIERS, $k(i)$, FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY AGE OF MOTHER AND THE INTERSURVEY INTERVAL IS 10 YEARS

Mortality model (i)	Age group (x)	Index i (i)	Age x (x)	Parity ratio for two successive groups $P(i-2, 1)/P(i, 2)$ (5)	Coefficients a(i) (6)	Coefficients b(i) (7)
North	25-29	3	3	$P(1, 1)/P(3, 2)$	1.0301	−1.1435
	30-34	4	5	$P(2, 1)/P(4, 2)$	1.1163	−0.4176
	35-39	5	10	$P(3, 1)/P(5, 2)$	1.2648	−0.3991
South	25-29	3	3	$P(1, 1)/P(3, 2)$	1.0795	−0.9481
	30-34	4	5	$P(2, 1)/P(4, 2)$	1.1203	−0.3211
	35-39	5	10	$P(3, 1)/P(5, 2)$	1.1972	−0.2918
East	25-29	3	3	$P(1, 1)/P(3, 2)$	1.0535	−0.7242
	30-34	4	5	$P(2, 1)/P(4, 2)$	1.0859	−0.2523
	35-39	5	10	$P(3, 1)/P(5, 2)$	1.1585	−0.2390
West	25-29	3	3	$P(1, 1)/P(3, 2)$	1.0579	−0.8796
	30-34	4	5	$P(2, 1)/P(4, 2)$	1.1054	−0.3139
	35-39	5	10	$P(3, 1)/P(5, 2)$	1.1914	−0.3043

Estimation equations:

$$k(i) = a(i) + b(i)\, P(i-2, 1)/P(i, 2)$$
$$q(x) = k(i)\, D(i, 2)$$

$k(i)$, makes allowance for changes in fertility, but it does nothing with respect to changes in mortality. Therefore, if there is evidence suggesting a mortality decline in the recent past, it is important to ascertain to which time period the $q(x)$ estimates obtained in step 4 really refer. The estimation of the reference period, $t(x)$, the number of years before the second survey to which the corresponding $q(x)$ estimate refers, is carried out by means of equations whose coefficients were estimated by using least-squares regression applied to data generated by model schedules. The estimated values of these coefficients are given in tables 74-77. The order of these tables parallels that used in presenting the tables needed to calculate the $k(i)$ multipliers. The first two tables are used when data are classified by age and the second two when data are classified by marriage duration. Within each set, the first table is used if the intersurvey period is five years and the second if it is 10 years. The use of these tables is illustrated in the next examples.

4. *Detailed examples*

This section presents two examples: that used in the previous section referring to Thailand and illustrating the estimation procedure applied to a five-year intersurvey interval; and the case of Brazil, where data on children ever born and children surviving have been collected by several of its decennial censuses.

(a) *Thailand, 1970-1975*

The basic data available for Thailand for the years

TABLE 72. COEFFICIENTS FOR ESTIMATION OF THE MULTIPLIERS $k(i)$, FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY DURATION OF MARRIAGE AND THE INTERSURVEY INTERVAL IS FIVE YEARS

Mortality model (1)	Duration group (2)	Index i (3)	Age x (4)	Parity ratio for two successive surveys $P(i-1, 1)/P(i, 2)$ (5)	Coefficients	
					$a(i)$ (6)	$b(i)$ (7)
North	5-9	2	3	$P(1, 1)/P(2, 2)$	1.2000	−0.5977
	10-14	3	5	$P(2, 1)/P(3, 2)$	1.3060	−0.4662
	15-19	4	10	$P(3, 1)/P(4, 2)$	1.4789	−0.5290
South	5-9	2	3	$P(1, 1)/P(2, 2)$	1.2159	−0.5626
	10-14	3	5	$P(2, 1)/P(3, 2)$	1.2797	−0.3843
	15-19	4	10	$P(3, 1)/P(4, 2)$	1.3564	−0.3915
East	5-9	2	3	$P(1, 1)/P(2, 2)$	1.3648	−0.4057
	10-14	3	5	$P(2, 1)/P(3, 2)$	1.2047	−0.2919
	15-19	4	10	$P(3, 1)/P(4, 2)$	1.2823	−0.3120
West	5-9	2	3	$P(1, 1)/P(2, 2)$	1.1882	−0.4803
	10-14	3	5	$P(2, 1)/P(3, 2)$	1.2455	−0.3499
	15-19	4	10	$P(3, 1)/P(4, 2)$	1.3408	−0.3857

Estimation equations:

$$k(i) = a(i) + b(i)\,P(i-1, 1)/P(i, 2)$$
$$q(x) = k(i)\,D(i, 2)$$

TABLE 73. COEFFICIENTS FOR ESTIMATION OF THE MULTIPLIERS $k(i)$, FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY DURATION OF MARRIAGE AND THE INTERSURVEY INTERVAL IS 10 YEARS

Mortality model (1)	Duration group (2)	Index i (3)	Age x (4)	Parity ratio for two successive surveys $P(i-2, 1)/P(i, 2)$ (5)	Coefficients	
					$a(i)$ (6)	$b(i)$ (7)
North	10-14	3	5	$P(1, 1)/P(3, 2)$	1.1650	−0.7209
	15-19	4	10	$P(2, 1)/P(4, 2)$	1.2697	−0.4084
South	10-14	3	5	$P(1, 1)/P(3, 2)$	1.1630	−0.5922
	15-19	4	10	$P(2, 1)/P(4, 2)$	1.2015	−0.3022
East	10-14	3	5	$P(1, 1)/P(3, 2)$	1.1162	−0.4502
	15-19	4	10	$P(2, 1)/P(4, 2)$	1.1589	−0.2409
West	10-14	3	5	$P(1, 1)/P(3, 2)$	1.1394	−0.5401
	15-19	4	10	$P(2, 1)/P(4, 2)$	1.1883	−0.2978

Estimation equations:

$$k(i) = a(i) + b(i)\,P(i-2, 1)/P(i, 2)$$
$$q(x) = k(i)\,D(i, 2)$$

TABLE 74. COEFFICIENTS NEEDED TO ESTIMATE THE REFERENCE PERIOD, $t(x)$,[1] FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY AGE AND THE INTERSURVEY INTERVAL IS FIVE YEARS

Age group (1)	Index i (2)	Age x (3)	Parity ratio for two successive surveys $P(i-1, 1)/P(i, 2)$ (4)	Coefficients	
				$a(i)$ (5)	$b(i)$ (6)
20-24	2	2	$P(1, 1)/P(2, 2)$	1.3999	5.9156
25-29	3	3	$P(2, 1)/P(3, 2)$	1.1637	6.4668
30-34	4	5	$P(3, 1)/P(4, 2)$	−0.4262	10.1371
35-39	5	10	$P(4, 1)/P(5, 2)$	−2.7596	14.6371

Estimation equation:

$$t(x) = a(i) + b(i)\,P(i-1, 1)/P(i, 2)$$

[1] Number of years prior to the survey to which estimates refer.

1970 and 1975 are presented in table 65. From those data, child mortality is estimated by using only the proportions of children dead reported at the second point in time and the parity changes experienced by the different female birth cohorts.

The steps of the calculations are given below.

Step 1: calculation of average parity per woman. The values of average parities, $P(i, j)$, have already been calculated in step 1 of the detailed example given in subsection D.3 and are given in table 66.

Step 2: calculation of proportion of children dead reported at the time of the second survey. The proportion of children dead at the time of the second survey (1975 in this case), denoted by $D(i, 2)$, is calculated directly from the

TABLE 75. COEFFICIENTS NEEDED TO ESTIMATE THE REFERENCE PERIOD, $t(x)$,[a] FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY AGE AND THE INTERSURVEY INTERVAL IS 10 YEARS

Age group (i)	Index i (i)	Age x (x)	Parity ratios for two successive surveys $P(i-2, 1)/P(i, 2)$ (3)	Coefficients a(i) (3)	b(i) (4)
25-29	3	5	$P(1, 1)/P(3, 2)$	3.2474	14.2086
30-34	4	5	$P(2, 1)/P(4, 2)$	3.6914	8.9412
35-39	5	10	$P(3, 1)/P(5, 2)$	3.4605	10.1997

Estimation equation:

$$t(x) = a(i) + b(i)\,P(i-2, 1)/P(i, 2)$$

[a] See table 74, footnote a.

TABLE 76. COEFFICIENTS NEEDED TO ESTIMATE THE REFERENCE PERIOD, $t(x)$,[a] FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY MARRIAGE DURATION AND THE INTERSURVEY INTERVAL IS FIVE YEARS

Duration group (i)	Index i (i)	Age x (x)	Parity ratios for two successive surveys $P(i-1, 1)/P(i, 2)$ (4)	Coefficients a(i) (3)	b(i) (4)
5-9	2	5	$P(1, 1)/P(2, 2)$	1.6812	4.3954
10-14	3	5	$P(2, 1)/P(3, 2)$	1.5051	6.4997
15-19	4	10	$P(3, 1)/P(4, 2)$	−0.4116	11.1290

Estimation equation:

$$t(x) = a(i) + b(i)\,P(i-1, 1)/P(i, 2)$$

[a] See table 74, footnote a.

TABLE 77. COEFFICIENTS NEEDED TO ESTIMATE THE REFERENCE PERIOD, $t(x)$,[a] FROM THE EXPERIENCE OF TRUE COHORTS WHEN DATA ARE CLASSIFIED BY MARRIAGE DURATION AND THE INTERSURVEY INTERVAL IS 10 YEARS

Duration group (i)	Index i (i)	Age x (x)	Parity ratios for two successive surveys $P(i-2, 1)/P(i, 2)$ (5)	Coefficients a(i) (5)	b(i) (6)
10-14	3	5	$P(1, 1)/P(3, 2)$	3.3781	10.5019
15-19	4	10	$P(2, 1)/P(4, 2)$	3.9324	8.7033

Estimation equation:

$$t(x) = a(i) + b(i)\,P(i-2, 1)/P(i, 2)$$

[a] See table 74, footnote a.

data given in table 65 by dividing the number of children dead by those ever born for each age group of mother. For example, $D(4, 2)$ is calculated as follows:

$$D(4, 2) = 2{,}375/22{,}969 = 0.1034.$$

All values of $D(i, 2)$ are shown in table 78.

Step 3: calculation of multipliers. The independent variables necessary to estimate the multipliers, $k(i)$, are the ratios of the observed average parities corresponding to true cohorts at the two points in time being considered. These ratios have the form $P(i-1, 1)/P(i, 2)$, since the intersurvey period is five years. The values of these ratios are shown in column (5) of table 78. For $i = 4$, the ratio is

$$P(3, 1)/P(4, 2) = 2.4479/3.5293 = 0.6936$$

where the values of $P(i-1, 1)$ and $P(i, 2)$ are obtained from table 66.

Once the necessary parity ratios are computed, the values of $k(i)$ are estimated by using the coefficients presented in table 70. As in the example given in subsection D.3, the mortality pattern selected is North. The

TABLE 78. ESTIMATION OF CHILD MORTALITY FROM 1975 DATA TAKING INTO ACCOUNT THE FERTILITY EXPERIENCE OF COHORTS, THAILAND

Age group (i)	Index i (1)	Age x (2)	Proportion of children dead $D(x, 2)$ (4)	Parity ratios for two successive surveys $P(i-1, 1)/P(i, 2)$ (5)	Multiplier k(i) (6)	Probability of dying q(x) (7)	Reference period t(x) (8)	Mortality level (9)
15-19	1	1	0.0710	-	-	-	-	-
20-24	2	2	0.0679	0.5477	1.0000	0.0684	2.3	18.8
25-29	3	3	0.0782	0.4679	0.9529	0.0755	4.2	18.6
30-34	4	5	0.1034	0.6936	0.9794	0.1013	6.6	17.4
35-39	5	10	0.1189	0.8198	1.0394	0.1236	9.2	16.8

resulting values of $k(i)$ are shown in table 78; $k(4)$ is calculated in detail below:

$$k(4) = 1.3408 + (-0.5210)(0.6936) = 0.9749.$$

Step 4: calculation of probability of dying. Estimates of the probability of dying, $q(x)$, are obtained as the products of the proportion of children dead, $D(i, 2)$, and the $k(i)$ multipliers. Note that this time no estimate of $q(1)$ is possible; consequently, there is no multiplier for $D(1, 2)$. The resulting values of $q(x)$ are shown in column (7) of table 78.

Step 5: calculation of reference period. Using the coefficients presented in table 74 and the parity ratios used in calculating $k(i)$, the values of the reference period, $t(x)$, are obtained by substituting them in the estimation equation shown at the bottom of table 74. For example,

$$t(2) = 1.3999 + (5.9156)(0.1477) - 2.3.$$

This value means that the estimate of $q(2)$ obtained in step 4 refers to approximately 2.3 years before the second survey. So it can be said that $q(2)$ roughly represents the child mortality prevalent in 1973, since the second data set being considered is supposed to represent 1975 experience. The complete set of $t(x)$ values is listed in table 78.

Column (9) of table 78 shows the levels of mortality in the North family of Coale-Demeny life tables consistent with the estimated $q(x)$ values. Once more, these levels suggest that child mortality has been declining. Their values are quite consistent with those shown in table 69, where child mortality estimates were obtained solely from the 1975 data. If anything, the new values suggest that the decline in mortality was more rapid.

(b) *Brazil, 1960-1970*

The data on children ever born and surviving collected for the whole of Brazil during the 1960 and 1970 censuses (both of which have exactly the same reference date, 1 September) are given in table 79. This wealth of data permits the application of most of the methods described in this chapter. Since the detailed application of these methods has already been described in the other examples presented so far, it is not repeated here, but results obtained by using each of them are shown.

TABLE 79 DATA ON CHILDREN EVER BORN AND SURVIVING, BRAZIL, 1960 AND 1970
(Thousands)

Age group (1)	1960				1970		
	Number of women (2)	Children ever born (3)	Corrected children ever born (4)	Children surviving (5)	Number of women (6)	Children ever born (7)	Children surviving (8)
15-19	3 723	467	453	398	5 306	630	557
20-24	3 244	3 574	3 467	2 984	4 309	4 182	3 653
25-29	2 701	6 864	6 658	5 624	3 264	7 923	6 817
30-34	2 266	8 612	8 354	6 889	2 862	10 808	9 133
35-39	1 991	9 782	9 489	7 607	2 571	12 343	10 229

Before proceeding to apply these methods, it must be pointed out that the question used in the 1960 census to obtain information on children ever born was not phrased properly; consequently, the data collected exhibit a definite bias. Specifically, the question asked of women of reproductive age was: "How many children have you ever had?", instead of "How many children, who have been born alive, have you ever had?" It is therefore likely that the 1960 information on children ever born includes both those born alive and those born dead. So, before the 1960 data are used to estimate child mortality, the number of children ever born have to be corrected for the inclusion of stillbirths. The correction made consists in multiplying the reported numbers of children ever born by 0.97, on the assumption that the incidence of stillbirths among all births is 0.03. In studies of similar data collected during the 1950 census of Brazil, the proportion of stillbirths used for the purpose of correction was 0.05.[14] This estimate was based on the proportion of stillbirths yielded by the 1940 census, during which a definite distinction was made between live births and stillbirths. According to this census, about 7 per cent of all births were stillbirths, but the estimate of 5 per cent was accepted on the grounds that women were not as likely to report all their stillbirths when only a question on children ever born was posed as when two questions, one about live births and another about stillbirths, were asked.

More information about the incidence of stillbirths among all births to the Brazilian population is now available. Two separate questions referring to these events were asked by the 1970 census and by several country-wide sample surveys carried out during the 1970s. All these data reveal that the proportion of reported stillbirths among all births has declined to roughly 0.05 (0.045 may be a more accurate figure); and therefore, a 5 per cent correction for the 1960 data seems too high, especially because of the known tendency among women to underreport the number of children they have ever had. As a compromise, a 3 per cent correction was accepted. The corrected numbers of children ever born by age group of mother for 1960 are shown in column (4) of table 79.

Table 80 shows the results of the steps needed to estimate child mortality from the information gathered in

[14] Giorgio Mortara, "A fecundidade da mulher e a sobrevivencia dos filhos no Brasil, segundo o Censo de 1950", *Contribuição para o Estudo da Demografia do Brasil* (Rio de Janeiro, Fundação Instituto Brasileiro de Geografia e Estatistica, 1970) and J. A. M. Carvalho, "Regional trends of fertility and mortality in Brazil", *Population Studies* Vol. XXVIII, No. 3 (November 1974), pp. 401-421.

TABLE 80. CHILD MORTALITY ESTIMATION USING MULTIPLIERS BASED ON THE WEST MORTALITY PATTERN, BRAZIL, 1960

Age group (1)	Index i (2)	Age x (3)	Average parity per woman P(i) [a] (4)	Proportion of children dead D(i) (5)	Multiplier k(i) [b] (6)	Probability of dying q(x) (7)	Reference period [c] R(x) (8)	Mean mortality level (9)
15-19	1	1	0.1217	0.1214	1.1656	0.1415	0.9	12.2
20-24	2	2	1.0686	0.1393	1.0808	0.1506	2.1	13.6
25-29	3	3	2.4650	0.1554	1.0112	0.1571	3.9	13.9
30-34	4	5	3.6865	0.1754	1.0134	0.1778	6.2	13.6

[a] Adjusted to exclude stillbirths.
[b] Based on $P(1)/P(2) = 0.1138$ and $P(2)/P(3) = 0.4335$.
[c] Number of years prior to the survey to which estimates refer

TABLE 81. CHILD MORTALITY ESTIMATION USING MULTIPLIERS BASED ON THE WEST MORTALITY PATTERN, BRAZIL, 1970

Age group (1)	Index i (2)	Age x (3)	Average parity per woman P(i) [a] (4)	Proportion of children dead D(i) (5)	Multiplier k(i) [b] (6)	Probability of dying q(x) (7)	Reference period [c] R(x) (8)	Mean mortality level (9)
15-19	1	1	0.1187	0.1159	1.1368	0.1294	1.0	13.0
20-24	2	2	0.9705	0.1265	1.0850	0.1373	2.1	14.3
25-29	3	3	2.4274	0.1396	1.0254	0.1432	3.8	14.6
30-34	4	5	3.7764	0.1550	1.0293	0.1596	5.9	14.5

[a] Based on $P(1)/P(2) = 0.1223$ and $P(2)/P(3) = 0.3998$
[b] See table 80, footnote c.

1960 (see subsection B.2). Note that the levels of mortality implied by the $q(x)$ estimates are fairly similar, except for that associated with $q(1)$, which is clearly out of line with the rest, probably because child mortality is not independent of mother's age (see subsection A.1).

Note that the raw data (table 79) provide the number of children surviving rather than the number dead. The latter, of course, are obtained by subtraction.

Table 81 shows the estimates of childhood mortality obtained solely from the 1970 information (see subsection B.2). Once more, the estimate of $q(1)$ is not consistent with the others. In general, the level of child mortality seems to have improved between 1960 and 1970.

It is worth remarking that when child mortality is estimated from a single data set, the estimated values of $q(1)$, $q(2)$, $q(3)$ and $q(5)$ refer approximately to one, two, four and six years before the census being considered. Reference dates for the usual child mortality estimates always display this pattern.

Tables 82 and 83 illustrate the steps necessary to obtain intercensal estimates of child mortality (see section D). The average number of children dead is obtained in this case by subtracting the number of children surviving from the total number of children ever born and dividing the result by the number of women in the relevant age group. Thus, for example,

$$ACD(3, 1) = (6,658 - 5,624)/2,701 = 0.3828;$$

$$ACD(2, 2) = (4,182 - 3,653)/4,309 = 0.1228.$$

Final results are shown in table 83. Note that the

TABLE 82. CHILD MORTALITY ESTIMATION FOR THE INTERCENSAL PERIOD 1960-1970, WITH 1960 DATA ON CHILDREN EVER BORN CORRECTED FOR INCLUSION OF STILLBIRTHS, BRAZIL

Age group (1)	Index i (2)	Average number of children ever born		Average number of children dead		
		1960 P(i, 1) [a] (3)	1970 P(i, 2) (4)	1960 ACD(i, 1) (5)	1970 ACD(i, 2) (6)	Intercensal period ACD(i, x) (7)
15-19	1	0.1217	0.1187	0.0148	0.0138	0.0134
20-24	2	1.0686	0.9705	0.1488	0.1228	0.1224
25-29	3	2.4650	2.4274	0.3828	0.3388	0.3578
30-34	4	3.6865	3.7764	0.6464	0.5853	0.5593
35-39	5	4.7610	4.8009	0.9441	0.8222	0.7771

[a] Adjusted to exclude stillbirths.

TABLE 83. INTERCENSAL ESTIMATES OF CHILD MORTALITY, BRAZIL, 1960-1970

Age group (1)	Index i (2)	Age x (3)	Average parity per woman P(i, x) (4)	Proportion of children dead D(i, x) (5)	Multiplier k(i) [a] (6)	Probability of dying q(x) (7)	Mean mortality level (8)
15-19	1	1	0.1187	0.1163	1.1172	0.1299	12.9
20-24	2	2	0.9705	0.1265	1.0849	0.1372	14.4
25-29	3	3	2.4244	0.1393	1.0256	0.1429	14.7
30-34	4	5	3.6783	0.1521	1.0296	0.1566	14.6
35-39	5	10	4.7927	0.1621	1.0460	0.1696	14.6

[a] Based on $P(1)/P(2) = 0.1223$ and $P(2)/P(3) = 0.4003$

TABLE 84. CHILD MORTALITY ESTIMATION TAKING INTO ACCOUNT THE EXPERIENCE OF TRUE COHORTS, BRAZIL, 1970

Age group (x)	Index i (3)	Age x (5)	Parity ratio for two successive cohorts $P(x-1, i)/P(x, i)$ (4)	Multiplier $k(i)$ (5)	Probability of dying $q(x)$ (6)	Reference period $t(x)$ (7)	West mortality level (8)
25-29	3	3	0.0501	1.0138	0.1413	4.0	14.7
30-34	4	5	0.2830	1.0166	0.1576	6.2	14.6
35-39	5	10	0.5134	1.0352	0.1773	8.7	14.5

[a] Number of years prior to the survey to which estimates refer.

mortality levels prevalent during the intercensal period almost coincide with those obtained solely from the 1970 data; and, excluding $q(1)$, they are fairly consistent.

Lastly, when estimates for 1970 are obtained by taking into account the fertility experience of true cohorts, almost the same mortality levels are obtained (see table 84). Such consistency is reassuring. It permits one to conclude that the average West mortality level prevalent during the period 1960-1970, as measured by child mortality, was approximately 14.5.

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Chapter IV

ESTIMATION OF ADULT SURVIVORSHIP PROBABILITIES FROM INFORMATION ON ORPHANHOOD AND WIDOWHOOD

A. BACKGROUND OF METHODS

1. *Nature and use of indirect information on adult mortality*

Chapter III describes how information on the survival of close relatives—information from mothers about the survival of their children—can be used to make indirect estimates of child mortality. The principle can be extended to information about the survival of other close relatives. This chapter describes procedures for obtaining estimates of adult mortality from information concerning the survival of parents and the survival of spouses. In both cases, a particular target person is known to have been alive at the time of some past event (the birth of the respondent in the case of mothers, the conception of the respondent in the case of fathers and marriage in the case of spouses); and some information is available about both the length of exposure to the risk of dying and about the age at which the exposure began. Sample surveys are the usual source of the necessary data, though on occasion, the necessary questions have been included in censuses. The great advantage of these methods of mortality estimation is that they rely upon information gathered by questions that are simple and easy to answer. The normal forms of the questions on orphanhood are "Is your father alive?" and "Is your mother alive?", the possible answers being yes, no or not known. For widowhood, all ever-married respondents are asked whether their first spouse is still alive, the possible answers again being yes, no or not known. It should be mentioned that the term "orphanhood" is used in a rather unusual sense in this chapter, since it is specific to a parent's sex. Thus, when describing methods based on proportions with a surviving mother, an orphaned respondent is one whose mother has died, regardless of the father's survival.

Preston[1] points out that data on child survival collected from all women provide information about the overall proportion of persons in a given population with a surviving mother. In a closed population, the number of surviving children reported by all women, regardless of age, should be equal to the number of people in the population with a surviving mother. If no direct information on orphanhood is available, an estimate of the level of adult mortality can be made iteratively by finding the level of mortality that, when combined with the observed age distribution and age-specific fertility rates, produces the same overall proportion orphaned as the proportion calculated from child survival data. If data on the survival of mother are available, the consistency of the reported number of respondents with a surviving mother and the total reported number of surviving children can be checked; a tendency to omit surviving children will reduce the latter figure, and a tendency to report incorrectly a surviving mother will increase the former, so typical errors in the two types of data should show up clearly.

Information on the survival of parents or spouses is an indicator only of adult mortality, since the exposure to risk of the target person begins in adulthood, at the birth, conception or marriage of the respondent. Thus, strictly speaking, such data should be used only to estimate survivorship probabilities from one adult age to another; the first group of techniques presented in this chapter permits the estimation of such conditional survivorship probabilities, and the methods in this group are described as "conditional". However, if an estimate of the level of child mortality is available, in the form of an estimate of $l(2)$, and some assumption can be made about the form of the relationship between child and adult mortality in the population under study, the information on child mortality can be combined with an indicator of adult mortality to estimate an unconditional survivorship probability, that is, the probability of surviving from birth to some adult age. The second group of methods presented in this chapter, described as "direct" methods, allows the estimation of such unconditional survivorship probabilities. In choosing between these two approaches, it should be remembered that the estimates of conditional survivorship are less dependent upon assumed models, but they are also more difficult to incorporate into a full life table, than are the direct estimates, which, in turn, are determined, to a considerable extent, by the estimates of child mortality used in their calculation.

Estimates of adult mortality derived from information on the survival of close relatives represent averages of the mortality experienced over the period during which the relatives were exposed to the risk of dying. If mortality has been changing in a regular way, each estimate has a time reference; that is, there is a time before the survey with a period life table having the survivorship probability that has been estimated. The number of years, t, before the survey that defines the period to which this life table refers will depend mainly upon the average exposure to risk of the target persons and upon

[1] Samuel H. Preston, "Estimating adult female mortality from reports on number of children surviving", University of Pennsylvania, Population Studies Center, 1980 (mimeographed).

the average age range of their exposure; in general, this number of years i will be somewhat less than half the average exposure to risk. It is possible to estimate the time period to which the survivorship probabilities derived from a single survey refer if certain assumptions about the regularity of the prevalent mortality trend are made. Brass and Bamgboye[2] propose such a method of estimation. In addition, when data are available from two surveys held five or 10 years apart, it is possible to derive estimates referring to the intersurvey period by reconstructing the experience of a hypothetical intersurvey cohort.

2. *Organization of this chapter*

All the estimation methods described in this chapter share two characteristics in common: they allow the estimation of probabilities of survivorship to ages in adulthood (beyond age 20); and they use data on the survival of close relatives (parents and spouses). The methods presented can be classified into several categories according to whether they permit the estimation of direct or conditional probabilities of survivorship

[2] William Brass and E. A. Bamgboye, "The time location of reports of survivorship: estimates for maternal and paternal orphanhood and the ever-widowed", Working Paper No. 81-1, London School of Hygiene and Tropical Medicine, Centre for Population Studies, 1981.

TABLE 65. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER IV

Section	Subsection and method	Type of input data	Estimated parameters
B. Estimation of conditional adult survivorship	B.2(b) Maternal orphanhood data: Brass method	Number of respondents with mother alive (or dead) classified by five-year age group Total number of respondents classified by five-year age group Number of respondents who did not know or did not state the survivorship status of their mothers Number of births occurring in a given year classified by five-year age group of mother	Probabilities of surviving from age 25 to ages 35, 40, 45, ..., 85 for females
	B.2(b) Paternal orphanhood data: Brass method	Number of respondents with father alive (or dead) classified by five-year age group Total number of respondents classified by five-year age group Number of respondents who did not know or did not declare the survivorship status of their fathers Number of births occurring in a year classified by five-year age group of father (or of mother, if nothing else is available)	Probabilities of surviving from age 32.5 to ages 45, 50, 55, ..., 90; or the probabilities of surviving from age 37.5 to ages 50, 55, 60, ... 95 (depending upon the value of M, the mean age of fathers at the birth of their children) for males
	B.2(c) Maternal orphanhood data: regression method	Number of respondents with mother alive (or dead) classified by five-year age group Total number of respondents classified by five-year age group Number of respondents who did not know or did not declare the survivorship status of their mother Number of births occurring in a given year classified by five-year age group of mother	Probabilities of surviving from age 25 to ages 35, 40, 45, ..., 75 for females The time period to which each estimated probability refers when mortality is changing
	B.2(d) Maternal orphanhood data from two surveys	Number of respondents with mother alive (or dead) classified by five-year age group, for two surveys five or 10 years apart Total number of respondents by five-year age group for the same two surveys Number of respondents who did not declare the survival status of their mother for the same two surveys Number of births occurring in a given year (preferably during the intersurvey period) classified by five-year age group of mother	Probabilities of surviving from age 25 to ages 35, 40, 45, ..., 75 for females during the intersurvey period

TABLE B3 (continued)

Section	Subsection and method	Type of input data	Estimated parameters
B. Estimation of conditional adult survivorship	B.3(d) Paternal orphanhood data from two surveys	Number of respondents with father alive (or dead) classified by five-year age group for two surveys five or 10 years apart Total number of respondents by five-year age group for the same two surveys Number of respondents who did not declare the survivorship status of their father, classified by five-year age group for the same two surveys Number of births occurring in a given year (preferably during the intersurvey period) classified by five-year age group of father (or of mother, if nothing else is available)	The probabilities of surviving from ages 32.5 to ages 45, 50, ..., 90, or the probabilities of surviving from age 37.5 to ages 50, 55, ..., 95 (depending upon the value of M, the mean age of fathers at the birth of their children) for males during the intersurvey period
	B.3(b) Widowhood data by age	Number of ever-married male (female) respondents with first spouse alive (or dead) classified by five-year age group Total number of ever-married male (female) respondents by five-year age group Number of ever-married male (female) respondents who did not declare or did not know the survivorship status of first spouse, by five-year age group Singulate mean ages at marriage for both males and females (see annex I)	The probabilities of surviving from age 20 to ages 25, 30, 35, ..., 55 (60) for females (males) The time period to which each estimate refers when mortality is changing
	B.3(c) Widowhood data by duration of first marriage	Number of ever-married male (female) respondents with first spouse alive (or dead) classified by five-year duration-of-marriage group Total number of ever-married male (female) respondents classified by five-year duration-of-marriage group Number of ever-married male (female) respondents who did not declare or did not know the survivorship status of first spouse, by five-year duration group Singulate mean age at marriage for females (males)	Probabilities of surviving from age 20 to ages 25, 30, ..., 50 for females (males) The time period to which each estimate refers when mortality is changing
	B.3(d) Widowhood data from two surveys	Number of ever-married male (female) respondents with spouse alive (or dead) classified by five-year age (duration) group for two surveys five or 10 years apart Total number of ever-married male (female) respondents classified by five-year age (duration) group from the same two surveys Number of ever-married male (female) respondents who did not state the survivorship status of their first spouse classified by five-year age (duration) group for the same two surveys Singulate mean ages at marriage for males and females for the two surveys	Probability of surviving from age 20 to ages 25, 30, ..., 50 for females (males) during the intersurvey period

TABLE 85 *(continued)*

Section	Subsection and method	Type of input data	Estimated parameters
C. Estimation of survivorship to adulthood from birth	C.2 Maternal orphanhood data	Number of respondents with mother alive (or dead) classified by five-year age group Total number of respondents by five-year age group Number of respondents who did not declare or did not know the survivorship status of their mother classified by five-year age group Number of births in a year classified by five-year age group of mother An estimate of $l(2)$ for females	$l(45)$, $l(50)$, ..., $l(75)$ for females
	C.3(b) Widowhood data by age	Number of ever-married male (female) respondents with first spouse alive (or dead) classified by five-year age group Total number of ever-married male (female) respondents by five-year age group Number of ever-married male (female) respondents who did not declare the survivorship status of first spouse, by five-year age group Singulate mean age at marriage for both males and females An estimate of $l(2)$ for females (males)	$l(25)$, $l(30)$, ..., $l(55)$ (or $l(60)$) for females (males)
	C.3(c) Widowhood data by duration of first marriage	Number of ever-married male (female) respondents with first spouse alive (or dead) classified by five-year duration-of-marriage group Total number of ever-married male (female) respondents classified by five-year duration group Number of ever-married male (female) respondents who did not declare the survivorship status of first spouse, by five-year duration group Singulate mean age at marriage for females (males) An estimate of $l(2)$ for females (males)	$l(20)$, $l(25)$, ..., $l(40)$ for females (males)

and according to the type of data each method requires: information on the orphanhood status; or data on the widowhood status of respondents. The organization of the following sections closely follows this classification. In order to aid the user in selecting the method best suited for a particular application, brief descriptions of each section follow (see also table 85):

Section B. Estimation of conditional adult survivorship. This section presents methods that allow the estimation of conditional probabilities of survivorship, that is, probabilities of surviving to age x given that the target population has already survived to age y, where y is less than x. This section is divided into two main subsections. In the first, methods using data on the orphanhood status of respondents are described; in the second, methods using data on the widowhood status of ever-married respondents are presented;

Section C. Estimation of survivorship to adulthood from birth. This section presents methods that permit the estimation of the probabilities of surviving from birth to certain ages x, all greater than 20. All the methods described require as input an estimate of $l(2)$, the probability of surviving from birth to exact age 2. The two main parts of this section present methods based on orphanhood and widowhood data, respectively.

B. ESTIMATION OF CONDITIONAL ADULT SURVIVORSHIP

1. *General characteristics*

The methods described in this section deal with the estimation of adult mortality from information about the

orphanhood or widowhood status of respondents. These methods may be called "conditional" because the estimated parameters are conditional probabilities of survival that do not, by themselves, define a complete life table. To derive life-table values from them more information is needed about child mortality; methods for combining information about child and adult mortality are described in chapter VI.

2. *Estimation of adult survivorship based on proportions not orphaned*

(a) *Basis of method and its rationale*

Data on the proportions of respondents whose mother (father) is still living can yield plausible estimates of adult mortality. The first method of estimation based on this type of data was proposed by Brass,[3] who established an equation relating the female probability of surviving from age 25 to age $25 + n$ to the proportions of respondents in two contiguous five-year age groups whose mother was still alive at the time of the interview. This equation has the form

$$l(25+n)/l(25) = W(n)S(n-5) + (1 - W(n))S(n) \qquad (B.1)$$

where $S(n)$ is the proportion of respondents aged from n to $n+4$ with mother alive, and $W(n)$ is a weighting factor employed to make allowance for typical age patterns of fertility and mortality. The set of $W(n)$ values that Brass proposed was estimated on the basis of data simulated by using a single mortality pattern (the African standard) and model fertility schedules of fixed shape but variable age locations. Each age location of the fertility schedule is associated with a particular value of M, the mean age of women (or men) at the birth of their children. Thus, the weights, $W(n)$, depend both upon n, the central point of the age groups being considered, and upon M.

More recently, Hill and Trussell[4] proposed another estimation procedure based on the following equation:

$$l(25+n)/l(25) = a(n) + b(n)M + c(n)S(n-5) \qquad (B.2)$$

where $a(n)$, $b(n)$ and $c(n)$ are coefficients estimated by using linear regression to fit equation (B.2) to data from 900 simulated cases. These cases were derived by using several fertility schedules generated by the Coale-Trussell[5] model and a variety of mortality schedules generated by the logit system[6] with the four different Coale-Demeny[7] mortality patterns as standards (see chapter I, subsections B.2 and B.4).

When both methods of estimation were applied to nearly 1,000 other cases simulated by using different fertility and mortality schedules, the method proposed by Brass was found to perform as well as or better than the regression method for n not exceeding 30 years, while the regression method produced substantially better results for higher values of n.

Even though this comparison was carried out only on simulated cases that are not subject to the type of errors commonly found in real data, it does reveal the probable strengths and weaknesses of the different methods. Since these methods seem to complement each other in the realm of theory, both of them are described here in the hope that they may also complement each other in practice.

Before proceeding with their description, some general features of methods that attempt the estimation of adult mortality based on the reported proportions of orphaned respondents by age and an indicator of the age pattern of fertility should be pointed out.

First, the estimated probabilities of survivorship do not refer to the entire population, since they reflect only the mortality experience of parents with surviving children. Furthermore, if questions on orphanhood are asked of the entire population, parents with several surviving children will tend to be overrepresented. Theoretically, this problem can be avoided by using an additional filter question that seeks to identify only one child per parent, such as the oldest surviving child or the first-born child. However, in practice, errors in the reporting of family-order status have been found to be so large that methods of analysis using data where the responses have been limited to one child per parent have not yielded better estimates than the original methods.[8]

Secondly, survivorship estimates based on reports by young respondents, and thus corresponding to small values of n (under 20), tend to be affected by misreporting of orphanhood status: young orphaned children are often adopted by relatives who report them as their own children. This phenomenon artificially inflates the proportion of young respondents having a surviving parent and biases upward the estimated survivorship probabilities of younger adults.

Lastly, the estimated probabilities of survival do not, strictly speaking, refer to specific time periods, since they represent average measures over the somewhat ill-defined intervals of exposure to the risk of dying of the target population. In cases where mortality has remained essentially constant, problems in the interpre-

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[3] W. Brass and K. H. Hill, "Estimating adult mortality from orphanhood", *Proceedings of the International Population Conference, Liège, 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 3, pp. 111-123.

[4] K. Hill and J. Trussell, "Further developments in indirect mortality estimation", *Population Studies*, vol. XXXI, No. 2 (July, 1977), pp. 313-333.

[5] Ansley J. Coale and T. James Trussell, "Model fertility schedules: variations in the age structure of childbearing in human populations", *Population Index*, vol. 40, No. 2 (April 1974), pp. 185-258.

[6] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, North Carolina, Carolina Population Center, Laboratories for Population Statistics, 1975).

[7] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966).

[8] Kenneth Hill, Hugo Behm and Augusto Solis, *La situación de la mortalidad en Bolivia* (La Paz, Instituto Nacional de Estadística, 1976).

tation of the estimates obtained do not arise since, in the absence of data errors, they should all imply the same, unchanging mortality level. However, because mortality has not remained constant during the recent past in most countries, the interpretation of survivorship probabilities derived from orphanhood data is not always straightforward. Interpretation and assessment are easier when the estimates obtained can be related to specific time periods. If one can assume that mortality has been changing regularly (for instance, by assuming that in the logit system the parameter indicating mortality level, α, has been changing linearly with respect to time) and that the adult mortality pattern of the target population is similar to that represented by the general standard (see chapter I, subsection B.4), it is possible to estimate the time t (as number of years before the survey) to which each survivorship probability refers. The estimation method used for this purpose has only recently been proposed by Brass and Bamgboye;[9] and although it has not yet been widely tested, its general relevance warrants its inclusion in this chapter.

(b) *Brass method*

(i) *Data required*

The following data are required for this method:

(*a*) The proportion of respondents with a surviving mother (father) in each five-year age group from n to $n+4$. This proportion is denoted by $S(n)$. The set of proportions $S(n)$ can be calculated when any two of the following items are available: (*a*) the number of respondents with mother (father) alive; (*b*) the number of respondents with mother (father) dead; (*c*) the total number of respondents whose mother's (father's) survival status is known. It is important to exclude from the calculations all respondents who did not know or did not declare their mother's (father's) survival status. In every case, data must be classified by five-year age group;

(*b*) The number of births in a given year classified by five-year age group of mother (father). This information is needed to estimate M, the mean age of mothers (fathers) at the birth of their children in the population being studied. The M to be estimated is not the mean age of the fertility schedule (also known as "mean age of childbearing"); it is, rather, a mean age of the fertility schedule weighted by the age distribution of the female (male) population. It may be regarded as an estimate of the average age difference between mother (father) and child in the population, thus being an indicator of the average age at which the target persons (parents) begin their exposure to the risk of dying.

(ii) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of mean age at maternity (paternity). The mean age of mothers (fathers) at the birth of a group of children (normally those born in the year before the survey) is denoted by M. If one uses the index i to denote the different five-year age groups in the reproductive life span of a woman and lets $B(i)$ be the number of births during a particular period to women in age group i, then

$$M = \sum_{i=1}^{7} a(i)B(i) / \sum_{i=1}^{7} B(i) \qquad \text{(B.3)}$$

where $a(i)$ is the mid-point of age group i. (The user should be reminded that, just as stated in chapters II and III, $i = 1$ represents age group 15-19, $i = 2$ is 20-24 and so on.) Note that if the births used to calculate M are those reported as occurring during the year preceding the survey and are tabulated by age of mother at the time of the survey, the true age group of the mother will be, on average, six months younger than stated, so that six months should be subtracted from the mid-point of each age group when calculating M.

The estimation of M for males is one of the additional problems associated with the estimation of male adult mortality from the proportions of respondents with a surviving father. Fertility questions are generally not asked of males, so the information from which the female M is estimated is usually not available for fathers. Births during the year preceding a survey are sometimes tabulated by age of husband, but this tabulation is generally limited to those cases in which the mother and her husband are enumerated in the same household. Calculating the male M from such a tabulation often biases its value upward because young fathers are more likely to be temporarily absent.

A more robust procedure for estimating M for males consists of adjusting the female M by using information on marital status. The median age of the currently married population can be calculated by sex, and the difference between the male and female medians can be added to the previously calculated female M to obtain an estimate of the male M. Medians are used to reduce the influence of older and, for the purpose in hand, largely irrelevant couples. In principle, the assumption of constant fertility can be relaxed by employing different values of M for different age groups of respondents, but in practice the data required to follow this approach are often lacking.

Step 2: calculation of weighting factors. For the value of M calculated above and for each value of n, the weighting factors, $W(n)$, are calculated by interpolating linearly (see annex IV) in table 86 (table 87 is used when fathers are being considered).

Step 3: calculation of survivorship probabilities. If the survivorship of mothers is being considered, the probabilities of surviving from age 25 to age $25 + n$ are calculated by using the equation

$$l_f(25+n)/l_f(25) = W(n)S(n-5) + (1.0 - W(n))S(n) \qquad \text{(B.4)}$$

where $S(n)$ is the proportion of respondents aged from n to $n+4$ who declared that their mothers were alive at the time of the interview.

[9] W. Brass and E. A. Bamgboye, *op. cit.*

TABLE 86. WEIGHTING FACTORS, $W(n)$, FOR CONVERSION OF PROPORTIONS OF RESPONDENTS WITH MOTHER ALIVE INTO SURVIVORSHIP PROBABILITIES FOR FEMALES

Age x	Mean age, M, for mothers at maternity								
	22 (1)	23 (2)	24 (3)	25 (4)	26 (5)	27 (6)	28 (7)	29 (8)	30 (9)
10	0.420	0.470	0.517	0.557	0.596	0.634	0.674	0.717	0.758
15	0.418	0.469	0.556	0.618	0.678	0.738	0.800	0.863	0.924
20	0.404	0.500	0.590	0.673	0.756	0.838	0.921	1.004	1.085
25	0.366	0.485	0.598	0.704	0.809	0.913	1.016	1.118	1.218
30	0.303	0.445	0.580	0.708	0.834	0.957	1.080	1.203	1.323
35	0.241	0.401	0.554	0.701	0.844	0.986	1.128	1.270	1.412
40	0.125	0.299	0.467	0.630	0.791	0.950	1.111	1.274	1.442
45	0.007	0.186	0.361	0.535	0.708	0.884	1.063	1.250	1.447
50	−0.190	−0.017	0.158	0.334	0.514	0.699	0.890	1.095	1.318
55	−0.368	−0.220	−0.059	0.101	0.270	0.456	0.645	0.856	1.083
60	0.466	−0.352	−0.217	−0.084	0.053	0.220	0.378	0.579	0.800

Estimation equation:

$$l_f(25+n)/l_f(25) = W(n)\,S(n-5) + (1-W(n))\,S(n)$$

TABLE 87. WEIGHTING FACTORS, $W(n)$, FOR CONVERSION OF PROPORTIONS OF RESPONDENTS WITH FATHER ALIVE INTO SURVIVORSHIP PROBABILITIES FOR MALES

Age x	Mean age, M, for fathers at paternity								
	28 (2)	29 (3)	30 (4)	31 (5)	32 (6)	33 (7)	34 (8)	35 (9)	36 (10)
(a) *From 32.5 years*									
10	0.192	0.258	0.322	0.388	0.455	0.521	0.587	0.650	0.714
15	0.151	0.243	0.336	0.429	0.522	0.613	0.702	0.790	0.877
20	0.043	0.166	0.287	0.406	0.523	0.638	0.750	0.861	0.969
25	−0.093	0.051	0.194	0.335	0.474	0.611	0.744	0.877	1.007
30	−0.327	−0.161	0.001	0.162	0.319	0.475	0.627	0.779	0.931
35	−0.640	−0.408	−0.211	−0.047	0.109	0.269	0.438	0.610	0.782
40	−0.856	−0.714	−0.554	−0.379	−0.203	−0.034	0.133	0.308	0.480
45	−1.120	−0.963	−0.806	−0.631	−0.495	−0.340	−0.183	−0.024	0.141
50	−1.162	−1.030	−0.900	−0.776	−0.651	−0.524	−0.396	−0.264	−0.128
55	−1.080	−0.943	−0.850	−0.758	−0.667	−0.576	−0.486	−0.397	−0.304

Estimation equation:

$$l_m(35+n)/l_m(32.5) = W(n)\,S(n-5) + (1.0-W(n))\,S(n)$$

(b) *From 37.5 years*

	36	37	38	39	40	41	42	43	44
10	0.384	0.460	0.537	0.613	0.687	0.758	0.827	0.897	0.969
15	0.378	0.484	0.588	0.690	0.790	0.888	0.984	1.079	1.174
20	0.324	0.455	0.582	0.708	0.833	0.954	1.075	1.195	1.318
25	0.164	0.315	0.465	0.613	0.759	0.904	1.051	1.197	1.346
30	−0.043	0.122	0.286	0.450	0.614	0.778	0.944	1.116	1.295
35	−0.359	−0.183	−0.015	0.152	0.321	0.496	0.677	0.863	1.062
40	−0.624	−0.473	−0.316	−0.157	0.003	0.168	0.342	0.529	0.722
45	−0.757	−0.631	−0.503	−0.372	−0.237	−0.099	0.047	0.208	0.393
50	−0.742	−0.650	−0.559	−0.471	−0.377	−0.280	−0.182	−0.069	0.063
55	−0.559	−0.541	−0.485	−0.423	−0.366	−0.308	−0.238	−0.149	−0.049

Estimation equation:

$$l_m(40+n)/l_m(37.5) = W(n)\,S(n-5) + (1.0-W(n))\,S(n)$$

In the case of fathers, the value 25 is replaced by the values 32.5 or 37.5 to allow for the fact that men are usually older than women at the birth of their children. The survivorship probabilities are estimated in this case by the following equations:

$$l_m(35+n)/l_m(32.5) = W(n)S(n-5)+(1.0-W(n))S(n) \qquad \text{(B.5)}$$

if the weighting factors $W(n)$ are obtained from table 87, part (a); and

$$l_m(40+n)/l_m(37.5) = W(n)S(n-5)+(1.0-W(n))S(n) \qquad \text{(B.6)}$$

if the weighting factors are obtained from table 87, part

TABLE 88. VALUES OF THE STANDARD FUNCTION FOR CALCULATION OF THE TIME REFERENCE FOR INDIRECT ESTIMATES OF ADULT SURVIVORSHIP

Age x (i)	Standard function Z(x) (ii)	Age x (iii)	Standard function Z(x) (iv)	Age x (v)	Standard function Z(x) (vi)	Age x (vii)	Standard function Z(x) (viii)	Age x (ix)	Standard function Z(x) (x)
26	0.090	36	0.092	46	0.149	56	0.274	66	0.452
27	0.090	37	0.093	47	0.160	57	0.289	67	0.473
28	0.090	38	0.095	48	0.171	58	0.305	68	0.495
29	0.090	39	0.099	49	0.182	59	0.321	69	0.518
30	0.090	40	0.104	50	0.193	60	0.338	70	0.542
31	0.090	41	0.109	51	0.205	61	0.356	71	0.568
32	0.090	42	0.115	52	0.218	62	0.374	72	0.595
33	0.090	43	0.122	53	0.231	63	0.392	73	0.622
34	0.090	44	0.130	54	0.245	64	0.411	74	0.650
35	0.091	45	0.139	55	0.259	65	0.431	75	0.678

(b). In both cases, $S(n)$ stands for the proportion of respondents in the age group from n to $n+4$ whose father was alive at the time of the interview. A decision has to be made whether to use the weights given in table 87, part (a), and hence equation (B.5); or those in table 87, part (b), and hence equation (B.6). In general, the choice depends upon the estimated value of M: if it is less than or equal to 36, part (a) should be used; if it is greater than 36, part (b) should be used.

Step 4: calculation of number of years before the survey to which the survivorship estimates refer. It can be shown that if the level of mortality is declining linearly on the logit scale (see chapter I, subsection B.4), the survivorship estimates obtained from orphanhood data are equal to those prevalent at specific time periods prior to the survey and that the time location of these periods is largely independent of the rate of the mortality change.[10] In addition, when female mortality in adulthood has a pattern similar to that embodied by the general standard (see chapter I, subsection B.4), the number of years before the survey to which each estimate derived from maternal orphanhood data refers, denoted by $t(n)$, can be estimated as

$$t(n) = n(1.0 - u(n))/2.0 \qquad \text{(B.7)}$$

where

$$u(n) = 0.3333 \ln({}_{10}S_{n-5}) + Z(M+n) + 0.0037(27 - M), \qquad \text{(B.8)}$$

and the value of $Z(M+n)$ is obtained by interpolating linearly in table 88.

Note that in this case ${}_{10}S_{n-5}$ represents the proportion of respondents in the age group from $n-5$ to $n+4$ with mother alive, and that n is the mid-point of the 10-year age group being considered.

When data on paternal orphanhood are being used, equations (B.7) and (B.8) become

$$t(n) = (n + 0.75)(1.0 - u(n))/2.0 \qquad \text{(B.9)}$$

and

$$u(n) = 0.3333 \ln({}_{10}S_{n-5}) + Z(M+n) + 0.0037(27 - M + 0.75) \qquad \text{(B.10)}$$

where $Z(x)$ is still obtained from table 88; ${}_{10}S_{n-5}$ now represents the proportion of respondents aged from $n-5$ to $n+4$ with father alive; M is the mean age of fathers at the time of their children's birth; n is the mid-point of the age group considered; and 0.75 or three quarters of a year have been added to make allowance for the fact that a father must have been alive at the time of conception, but not necessarily at the time of the birth of his offspring.

(iii) *A detailed example*

The data shown in tables 89 and 90 were collected by the National Demographic Survey carried out in Bolivia during 1975.[11] They are used here to illustrate the way in which the estimation of female adult mortality is carried out using data on the maternal orphanhood status of respondents.

Table 89 shows both the raw data gathered by the survey and the proportions of respondents whose mother was alive at the time of the interview. Note that in calcu-

TABLE 89. DATA ON MATERNAL ORPHANHOOD STATUS AND PROPORTIONS OF RESPONDENTS WITH MOTHER ALIVE, BOLIVIA, 1975

Age group of respondent (i)	Number of respondents with: Mother alive (ii)	Mother dead (iii)	Unknown maternal orphanhood status (iv)	Proportion with mother alive $S(n)$ (v)
15-19	5 560	448	6	0.9252
20-24	3 995	541	10	0.8807
25-29	2 886	769	8	0.7896
30-34	1 910	852	9	0.6915
35-39	1 661	1 234	11	0.5737
40-44	1 027	1 272	7	0.4467
45-49	855	1 556	6	0.3546
50-54	369	1 243	6	0.2289

[10] *Ibid.*

[11] K. Hill, H. Behm and A. Soliz, *op. cit.*

TABLE 90. CHILDREN BORN DURING THE 12 MONTHS PRECEDING THE SURVEY, BY AGE OF MOTHER AT TIME OF THE SURVEY, BOLIVIA, 1975

Age group of mother at time of interview (i)	Index i (2)	Mid-point of age group (adjusted by six months) a(i) (3)	Number of children born in past year B(i) (4)
15-19	1	17	136
20-24	2	22	409
25-29	3	27	483
30-34	4	32	320
35-39	5	37	259
40-44	6	42	96
45-49	7	47	50
TOTAL			753

luting the latter figure, the numbers of respondents who were classified in the category of "unknown maternal orphanhood status" were ignored. In the case of the 1975 survey in Bolivia, the numbers in the unknown category are very small and their inclusion in the denominators of the proportions with surviving mother would not have affected the final results. However, because greater levels of non-response may occur, it is important to exclude the non-responses when calculating the proportions with surviving mother. As an example, $S(50)$ is computed explicitly below:

$$S(50) = 369/(369 + 1,243) = 0.2289.$$

The computational procedure is described below.

Step 1: calculation of mean age at maternity. According to equation (B.3):

$$M = \sum_{i=1}^{7} a(i)B(i) / \sum_{i=1}^{7} B(i)$$

where $a(i)$ is the mid-point of age group i; and $B(i)$ is the number of births to women in age group i. However, to obtain a correct estimate of the mean age at maternity, M, the births used in equation (B.3) should be classified by age of mother at the time the birth occurred. In the present case, the age reported by a woman is that at the time of the interview and not that at the time of the birth; some allowance must be made for this fact. The simplest strategy is to assume that births are uniformly distributed in time and with respect to mother's age. Hence, on average, women were six months younger at the time of the birth they reported than at the time of the interview, so that, for example, the current age group spanning exact ages 15-20 may be considered to span exact ages from 14.5 to 19.5 at the time the births occurred. All subsequent age groups are similarly affected. Therefore, the central $a(i)$ values to be used in applying equation (B.3) are 17 instead of 17.5, 22 instead of 22.5, and so on, as shown in column (3) of table 90.

Applying equation (B.3), M is calculated as follows:

$$\sum_{i=1}^{7} a(i)B(i) = (17)(136) + (22)(409) + \cdots + (47)(50) = 50,526$$

and

$$\sum_{i=1}^{7} B(i) = 1,753$$

so

$$M = 50,526/1,753 = 28.8.$$

Step 2: calculation of weighting factors. Since data on maternal orphanhood are being considered, the appropriate weights, $W(n)$, are obtained by interpolating linearly (see annex IV) between the values given in table 86. The value of M is 28.8 years, so the weights in the columns labelled "28" and "29" are used as extremes in the interpolation. For all values of n, the interpolation factor Θ is

$$\Theta = (29 - 28.8)/(29 - 28) = 0.2.$$

Therefore, to obtain $W(35)$, for example, the values for $n = 35$ appearing under the columns for M equal to 28 and 29 of table 86 are weighted by Θ and $(1-\Theta)$, respectively, yielding

$$W(35) = 0.2(1.128) + 0.8(1.270) = 1.2416.$$

The complete set of $W(n)$ values corresponding to $M = 28.8$ is shown in column (2) of table 91.

Step 3: calculation of survivorship probabilities. Once the weighting factors corresponding to the observed M have been calculated, the values of $l_f(25+n)/l_f(25)$ are

TABLE 91. ESTIMATION OF FEMALE ADULT SURVIVORSHIP FROM PROPORTIONS OF RESPONDENTS WITH SURVIVING MOTHER, USING THE BRASS METHOD, BOLIVIA, 1975

Age x (1)	Weighting factor W(x) (2)	Proportion with mother surviving 5(x-25) (3)	Complement of weighting factor (1 - W(x)) (4)	Proportion with mother surviving 5(x) (5)	Female adult survivorship probability $l_f(25+x)/l_f(25)$ (6)	Life expectancy level (7)
20	0.9874	0.9252	0.0126	0.8807	0.925	18.4
25	1.0976	0.8807	−0.0976	0.7896	0.890	18.1
30	1.1784	0.7896	−0.1784	0.6915	0.807	16.0
35	1.2416	0.6915	−0.2416	0.5737	0.720	15.0
40	1.2414	0.5737	−0.2414	0.4467	0.604	13.9
45	1.2126	0.4467	−0.2126	0.3546	0.466	13.0
50	1.0540	0.3546	−0.0540	0.2289	0.361	14.3

computed as indicated by equation (B.4). Table 91 summarizes these computations. This table also shows the mortality level in the Coale-Demeny West family of model life tables associated with each survivorship ratio. These levels were calculated by interpolating linearly between the values given in table 210 (see annex VI). They provide a useful index for examining the consistency of the survivorship ratios themselves.

As an example, the calculation of the level consistent with the estimated $l_f(65)/l_f(25)$ is carried out below in detail. The value of $l_f(65)/l_f(25)$ according to table 91 is 0.604. The column labelled "65" of table 210 in annex VI is then used to locate the two values that bracket the estimated value. They are 0.58183, corresponding to level 13; and 0.60583, corresponding to level 14. Hence, the interpolation factor Θ is

$$\Theta = (0.60400 - 0.58183)/(0.60583 - 0.58183) = 0.92$$

and the estimated level is

$$(0.92)(14) + (0.08)(13) = 13.92.$$

The complete set of mortality levels is shown in column (7) of table 91.

Step 4: calculation of number of years before the survey to which the survivorship estimates refer. The reference period of each of the estimates obtained in the previous step, denoted by $t(n)$, is calculated from ${}_{10}S_{n-5}$, the proportion of respondents aged from $n-5$ to $n+4$ whose mother was alive at the time of the interview. These proportions are calculated by dividing the number of respondents with surviving mother in the two adjacent age groups from $n-5$ to $n-1$ and from n to $n+4$ by the sum of the number of respondents in both age groups who provided information on mother's survival. For example,

$${}_{10}S_{20} = (3,995 + 2,886)/(3,995 + 2,886 +$$

$$541 + 769) = 0.8401.$$

The complete set of ${}_{10}S_{n-5}$ values is shown in column (2) of table 92.

Because the data at hand refer to maternal orphanhood, equations (B.7) and (B.8) are used in calculating $t(n)$. In step 1, M was found to be equal to 28.8 years. Therefore, the values of $x = M + n$ range from 48.8 to 78.8, increasing in steps of five years. The standard function, $Z(x)$, for calculation of the reference period, is obtained for these values by linear interpolation within table 88. Note that because the table does not cover the range 78-79, the last Z value calculated is $Z(73.8)$. Thus, $Z(53.8)$ is obtained as

$$Z(53.8) = 0.2Z(53) + 0.8Z(54) = 0.2(0.231) +$$

$$0.8(0.245) = 0.242.$$

All $Z(x)$ values are given in column (4) of table 92. Using these values and substituting those of ${}_{10}S_{n-5}$, M and n in equation (B.8), one obtains the set of values for the correction function, $u(n)$, given in column (5) of the same table. To illustrate the procedure followed, $u(30)$ is calculated below:

$$u(30) = 0.3333 \ln {}_{10}S_{25} + Z(58.8) + 0.0037(-1.8)$$

$$= 0.3333 \ln(0.7474) + 0.318 - 0.0067$$

$$= 0.2143.$$

Then, using equation (B.7), $t(n)$ is calculated for each case. Continuing with the illustration of the case $n = 30$,

$$t(30) = 30(1 - 0.2143)/2 = 11.8.$$

The survey in Bolivia was carried out between 15 June and 31 October 1975, so the mid-point of that period may be considered its reference date. The simplest way of obtaining the decimal equivalent of the mid-period date is by transforming the end-points into decimal equivalents and finding their mean. Thus, 15 June 1975, is equivalent to 1975.45 (the number of days from 1 January to 15 June, both included, being 166, and 166/365 = 0.45) and 31 October 1975 is equivalent to 1975.83, so that the desired mean is 1975.64. Therefore, the $l_f(55)/l_f(25)$ estimate listed in table 91 refers approximately to 11.8 years before 1975.6, that is, to 1963.8. The complete set of reference dates and the West mortality levels associated with them are shown in columns (7) and (8), respectively, of table 92.

TABLE 92. ESTIMATION OF TIME REFERENCE PERIODS FOR SURVIVORSHIP ESTIMATES DERIVED FROM MATERNAL ORPHANHOOD DATA, BOLIVIA, 1975

Age n (1)	Proportion with mother surviving ${}_{10}S_{n-5}$ (2)	Length of exposure indicator $M+n$ (3)	Standard function $Z(M+n)$ (4)	Correction function $u(n)$ (5)	Reference period[a] $t(n)$ (6)	Reference date (7)	West mortality level (8)
20	0.9060	48.8	0.180	0.1405	8.6	1967.0	18.4
25	0.8401	53.8	0.242	0.1773	10.3	1965.3	18.1
30	0.7474	58.8	0.318	0.2143	11.8	1963.8	16.9
35	0.6313	63.8	0.407	0.2471	13.2	1962.4	15.0
40	0.5175	68.8	0.513	0.2868	14.3	1961.3	13.9
45	0.3996	73.8	0.644	0.3316	15.0	1960.6	13.0

[a] Number of years prior to the survey to which survivorship estimates refer.

It is worth mentioning that the estimates obtained from maternal orphanhood data refer, when adult mortality has been changing steadily, to periods between 8 and 15 years prior to the survey. When judged with respect to these estimated dates, the mortality levels obtained in the previous step seem less satisfactory than at first sight. The estimates imply that female adult mortality in Bolivia fell by an average of one mortality level per annum during the period 1960-1965. It is difficult to believe that such rapid gains in life expectancy could actually have taken place. Further, the similarity of the levels corresponding to the two most recent periods is suspect. It suggests that even up to age groups 20-24 and 25-29 the adoption effect may be biasing the survivorship estimates upward. To what extent such an effect is also operating at ages 30-34 and 35-39 is difficult to ascertain, but the possibility that the estimates for 1962-1964 may also be subject to an undesirable positive bias must be borne in mind. From these brief comments it transpires that the data for Bolivia may not be as consistent as one would desire, and that further analysis and additional, independent evidence may be necessary to establish with a greater degree of certainty the mortality levels to which the female population of Bolivia has been subject.

(c) *Method based on an equation fitted by using regression*

(i) *Data required*

The data required to apply this method in the case of maternal orphanhood are exactly the same as those needed for the Brass method (see subsection B.2 (b) (i); however, no regression method has been developed for the estimation of male mortality from data on paternal orphanhood. The data required are:

(*a*) The proportion of respondents whose mother was alive at the time of the interview, classified by five-year age group. The reported proportion in the age group from n to $n+4$ is denoted by $S(n)$;

(*b*) The number of children born during a given year, classified by five-year age group of mother. These data allow the estimation of M, the mean age of mothers at the birth of a particular group of children.

(ii) *Computational procedure*

The following steps are required in the computational procedure.

Step 1: calculation of mean age at maternity. This step is identical to that described in subsection B.2 (b) (ii). It is not described again here.

Step 2: calculation of survivorship probabilities. Using the values of the coefficients $a(n)$, $b(n)$ and $c(n)$ shown in table 93, female survivorship probabilities are obtained by substituting these values in the following equation:

$$l_f(25+n)/l_f(25) = a(n) + b(n)M + c(n)S(n-5) \qquad \text{(B.11)}$$

where M is the mean age of mothers at the birth of their children; and $S(n-5)$ is the proportion of respondents in the age group from $n-5$ to $n-1$ whose mother was alive at the time of the interview.

TABLE 93. COEFFICIENTS FOR ESTIMATION OF FEMALE SURVIVORSHIP PROBABILITIES FROM AGE 25 FROM PROPORTIONS WITH SURVIVING MOTHER

Age n	Coefficient $a(n)$	Coefficient $b(n)$	Coefficient $c(n)$
20	-0.1798	0.00476	1.0505
25	0.2267	0.00737	1.0291
30	-0.1108	0.01032	1.0281
35	-0.4259	0.01413	1.0473
40	-0.5366	0.01900	1.0818
45	-0.6676	0.02256	1.1229
50	-0.6981	0.02344	1.1454

Estimation equation:

$$l_f(25+n)/l_f(25) = a(n) + b(n)M + c(n)S(n-5)$$

Note should be taken that this method permits the estimation of female survivorship only. No regression method has been developed to estimate male survivorship, so if paternal orphanhood data are available, the original Brass method has to be used.

It should also be pointed out that the conditional probabilities of survivorship obtained by using this method do not, by themselves, define a complete life table, unless one can assume that the mortality pattern of the population studied is identical to that of some model set of life tables. When information on child mortality is available, the techniques described in chapter VI can be used to construct a life table that does not depend upon such a restrictive assumption.

Step 3: calculation of number of years before the survey to which the survivorship estimates refer. In cases where adult mortality has been changing regularly during the 15 or 20 years preceding the survey (that is, the change has been linear on the logit scale), estimates of the time to which each of the survivorship probabilities obtained in the previous step refers, denoted by $t(n)$, can be calculated by means of the following equations:

$$u(n) = 0.3333 \ln S(n-5) + Z(M+n-2.5) + 0.0037(27-M) \qquad \text{(B.12)}$$

and

$$t(n) = (n-2.5)(1.0 - u(n))/2.0 \qquad \text{(B.13)}$$

where $S(n-5)$ is the proportion of respondents aged from $M-5$ to $M-1$ whose mother was alive at the time of the survey; M is the mean age of mothers at the time of the birth of their children; $n-2.5$ is a rough indicator of the mean age of respondents; and $Z(x)$ is a standard function of age whose values in any given case can be obtained by interpolating between the values listed in table 88. Strictly speaking, the estimates yielded by equations (B.12) and (B.13) when the $Z(x)$ values of table 88 are used as input are based on the assumption that female adult mortality in the population being studied has a similar pattern to that embodied by the general standard (see chapter I, subsection B.4). In practice,

however, the impact of deviations from this assumption is unlikely to be large, at least in comparison with the effects on the final estimates of deviations from the assumed linear trend followed by adult mortality changes over time.

(iii) *A detailed example*

The application of the regression method is illustrated below by again using the data for Bolivia presented in subsection B.2 (b) (iii) (see table 89).

Step 1: calculation of mean age at maternity. Refer to step 1 of subsection B.2 (b) (iii), where the mean age of mothers at maternity, M, was estimated as 28.8 years for Bolivian women.

Step 2: calculation of survivorship probabilities. Table 94 summarizes the calculations needed to estimate these probabilities. It shows that the application of equation (B.11) is straightforward and that the estimation process is very simple. Once again, the mortality level consistent with each survivorship ratio in the West family of model life tables has been computed by interpolating in table 210 in annex VI (see step 3 of subsection B.2 (b) (iii)). Note that for a value of n under 40, the survivorship estimates obtained by this method are very similar to those obtained by the Brass procedure (see table 91). For values of n of 40 and over, however, the differences increase with n; for these ages, the regression-based estimates would probably be preferable.

Step 3: calculation of number of years before the survey to which the survivorship estimates refer. As stated by equations (B.12) and (B.13), the values of this reference period, denoted by $t(n)$, depend in this case upon the observed proportions not orphaned in each five-year age group, $S(n-5)$. They are shown in column (2) of table 95. The values of $Z(x)$ for x ranging from 46.3 to 76.3 in steps of five years are obtained by interpolating linearly between the values listed in table 88. For example, $Z(66.3)$ is obtained as

$$Z(66.3) = 0.7Z(66) + 0.3Z(67) =$$

$$0.7(0.452) + 0.3(0.473) = 0.458.$$

Since the values listed in table 88 do not cover the age range from 76 to 77, linear extrapolation was used to calculate $Z(76.3)$. The complete set of $Z(x)$ values is shown in column (4) of table 95. Using equation (B.12) and these values as input, the set of $u(n)$ values can be calculated. For example,

$$u(40) = 0.3333 \ln S(35) + Z(66.3) + 0.0037(27.0 - 28.8)$$

$$= 0.3333 \ln (0.5737) + 0.458 - 0.0067$$

$$= 0.2661,$$

so that

$$t(40) = (37.5)(1.0 - 0.2661)/2.0 = 13.8.$$

The complete set of $t(n)$ values, the number of years prior to the survey to which each survivorship estimate refers, is shown in column (6) of table 95. Since the survey reference date is 1975.6, these $t(n)$ estimates can be transformed to dates by subtraction, the results being shown in column (7) of table 95. The West mortality levels implied by each survivorship estimate are also shown

TABLE 94 CALCULATION OF FEMALE SURVIVORSHIP PROBABILITIES FROM AGE 25 USING PROPORTIONS WITH SURVIVING MOTHERS AND THE REGRESSION METHOD, BOLIVIA, 1975

Age n (1)	Intermediate value $u(n) + 0.0037(28.8)$ (2)	Coefficient $c(n)$ (3)	Proportion with mother surviving $S(n-5)$ (4)	Female adult survivorship probability $l_f(25+n)/l_f(25)$ (5)	West mortality level (6)
20	−0.0427	1.0505	0.9252	0.929	18.7
25	−0.0144	1.0291	0.8807	0.892	18.2
30	−0.0021	1.0287	0.7896	0.810	16.2
35	−0.0017	1.0473	0.6915	0.722	15.1
40	−0.0065	1.0818	0.5737	0.612	14.2
45	−0.0179	1.1228	0.4467	0.484	13.7
50	−0.0230	1.1454	0.3546	0.383	15.3

TABLE 95 ESTIMATION OF TIME REFERENCE PERIODS FOR SURVIVORSHIP PROBABILITIES DERIVED FROM MATERNAL ORPHANHOOD DATA USING THE REGRESSION METHOD, BOLIVIA, 1975

Age n (1)	Proportion with mother surviving $S(n-5)$ (2)	Length of exposure interval $(n+n-2.5)$ (3)	Standard function $Z(n+n-2.5)$ (4)	Correction factor $u(n)$ (5)	Reference period $t(n)$ [a] (6)	Reference date (7)	West mortality level (8)
20	0.9252	46.3	0.152	0.1194	7.7	1967.9	18.7
25	0.8807	51.3	0.209	0.1600	9.5	1966.1	18.2
30	0.7896	56.3	0.279	0.1936	11.1	1964.5	16.2
35	0.6915	61.3	0.361	0.2314	12.5	1963.1	15.1
40	0.5737	66.3	0.458	0.2661	13.8	1961.8	14.2
45	0.4467	71.3	0.576	0.3007	14.9	1960.7	13.7
50	0.3546	76.3	0.714	0.3618	15.2	1960.4	15.3

[a] Number of years prior to the survey to which survivorship estimates refer.

in table 95 to facilitate comparisons and assessment. Note that, as already pointed out in subsection B.2 (b) (iii), the pace of the mortality change implied by these estimates between 1960 and 1966 seems implausibly rapid. It is also clear that the relatively low mortality associated with values of n of 20, 25 and 50 are suspect, that for 50 because it does not conform with the trend of the rest, and those for 20 and 25 because they imply a fairly important decline in mortality from mid-1964 to 1966, compared with a very moderate decline in previous years. It is very likely that the survivorship estimates for n values of 20 and 25 are biased upward by the adoption effect and that the survivorship ratio for n of 50 is distorted both by overreporting of age and by misreporting of maternal survival.

(d) *Use of data from two surveys*

One of the problems faced when estimating adult survivorship probabilities from data on the survival of parents is that if mortality has been changing, the estimates refer to some fairly distant point in the past. If, however, information of orphanhood has been collected by two censuses or surveys and the period between them is a multiple of five years, cohorts from the first census can be identified at the second, and survivorship probabilities applicable to the intersurvey period can be estimated from the constructed proportions not orphaned for a hypothetical intersurvey cohort of respondents. It is assumed, of course, that migration does not affect either set of proportions not orphaned and that no relationship exists between the mortality of the respondents and that of their mothers. If the period between the two surveys is not a multiple of five years, a set of proportions not orphaned for a suitable point in time can, in some circumstances, be estimated by interpolation between or extrapolation beyond the observed sets.

(i) *Data required*

The following data are required for this method:

(*a*) The proportion of respondents with surviving mother in each five-year age group from n to $n+4$ for two points five or 10 years apart, denoted by $S(n, 1)$ for the first survey and $S(n, 2)$ for the second;

(*b*) The number of births in a year classified by five-year age group of mother from one of the surveys (and preferably from both) or for some year in the intersurvey period.

(ii) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of mean age at maternity. See the description of step 1 given in subsection B.2 (b) (ii). If the data required for the calculation of the mean age, M, are available from both surveys, M can be calculated for both, and the average can be used in the analysis. If the required data are available for only one of the two surveys, or for some intermediate point, then the value of M obtained for that point should be used.

Step 2: calculation of proportions not orphaned for a hypothetical intersurvey cohort of respondents. The concept of a hypothetical cohort has already been discussed, and other uses of it appear in chapters II and III. The procedure followed in calculating the proportions not orphaned among a hypothetical intersurvey cohort of respondents depends upon the length of the interval between the surveys, which is denoted by T. The value of T should be a multiple of five, so that the survivors of a standard five-year age group at the earlier survey can be identified as belonging to a standard five-year age group at the second survey. If one denotes by $S(n, 1)$ the proportion of persons in the age group from n to $n+4$ whose mother was alive at the time of the first survey and by $S(n+T, 2)$ the equivalent proportion at the second survey, then $S(n+T, 2)$ is the proportion not orphaned at the second survey among the survivors of those whose proportion not orphaned at the first survey was $S(n, 1)$. Using this notation, and assuming that T is a multiple of five, the proportions not orphaned for a hypothetical intersurvey cohort, $S(n, s)$, are obtained as follows:

$$S(n, s) = S(n, 2) \quad \text{for } n < T; \tag{B.14}$$

$$S(n, s) = S(n-T, s)\, S(n, 2)/S(n-T, 1) \quad \text{for } n \geq T. \tag{B.15}$$

TABLE 96. CALCULATION OF PROPORTIONS NOT ORPHANED FOR A HYPOTHETICAL INTERSURVEY COHORT. CONSTRUCTED EXAMPLE

Age group (x)	Age n (x)	Proportions not orphaned: First survey $S(n, 1)$ (3)	Second survey $S(n, 2)$ (4)	Intervening $S(n, s)$: Five-year interval (5)	Ten-year interval (6)
0-4	0	0.960	0.930	0.970[a]	0.970[a]
5-9	5	0.925	0.935	0.945	0.935[a]
10-14	10	0.881	0.895	0.914	0.904
15-19	15	0.790	0.805	0.835	0.814
20-24	20	0.692	0.720	0.761	0.739
25-29	25	0.536	0.605	0.666	0.623
30-34	30	0.447	0.474	0.550	0.506
35-39	35	0.355	0.387	0.476	0.420
40-44	40	0.223	0.274	0.367	0.310
45-49	45	0.109	0.159	0.262	0.148

[a] Proportions taken directly from second survey.

Step 3: calculation of survivorship probabilities. Once the proportions not orphaned for the hypothetical cohort have been calculated, survivorship probabilities are obtained from them and from the mean age at maternity, M, estimated in step 1 by using either the Brass or the regression method (see subsections B.2 (b) (ii) and B.2 (c) (ii)). The survivorship probabilities obtained from hypothetical-cohort data should reflect adult mortality levels during the intersurvey period, so there is no need in this case to estimate the reference period, $t(n)$, to which each estimate refers.

(iii) *A detailed example*

The only difference in the applications just described and the intersurvey method lies in the calculation of the proportions with surviving mother in a hypothetical cohort. Thus, a constructed example of the way in which to calculate these proportions (step 2) is given in table 96. Column (3) shows the proportions not orphaned at the first survey, classified by five-year age group; and column (4) shows the corresponding proportions at the second survey.

Column (5) shows the hypothetical-cohort proportions not orphaned for an intersurvey period of five years. To illustrate, the value for age group 15-19, 0.835, is obtained as follows (from equations (B.14) and (B.15)):

$$S(15, s) = 0.914(0.805/0.881) = 0.835.$$

It may be mentioned that this value may also be obtained by

$$S(15, s) = S(0, 2)S(5, 2)S(10, 2)S(15, 2)/$$

$$S(0, 1)S(5, 1)S(10, 1) =$$

$$(0.970)(0.935)(0.895)(0.805)/$$

$$(0.960)(0.925)(0.881) = 0.835.$$

Column (6) shows the hypothetical-cohort proportions not orphaned for an intersurvey period of 10 years. The value for age group 20-24, 0.739, is obtained as follows (from equations (B.14) and (B.15)):

$$S(20, s) = S(10, s)S(20, 2)/S(10, 1) =$$

$$(0.904)(0.720/0.881) = 0.739.$$

Again this value may be obtained by

$$S(20, s) = S(0, 2)S(10, 2)S(20, 2)/S(0, 1)S(10, 1)$$

$$= (0.970)(0.895)(0.720)/(0.960)(0.881) = 0.739.$$

Intersurvey adult survivorship probabilities, allowing for both mortality and fertility change (if M can be calculated from data at the beginning and at the end of the intersurvey period) can now be obtained by applying either the Brass or the regression method to the hypothetical-cohort proportions not orphaned (see subsections B.2 (b) (ii) and B.2 (c) (ii)).

3. *Estimation of adult survivorship based on proportions widowed*

(a) *Basis of method and its rationale*

The proportions of ever-married persons classified by age whose first spouse is still alive can be used to estimate adult survivorship probabilities in much the same way as the proportions not orphaned can be so used. In the case of widowhood, an additional possibility exists because the ever-married respondents can be classified by duration of marriage. In order to ensure that there shall be only one person at risk of dying per respondent and to minimize the problems arising because of remarriage, the data collected should refer only to the survival of the first spouse of each respondent. Hence, the measure of duration required is the time elapsed since the respondent's first union. Classification of the data by duration of first marriage has the methodological advantage of establishing directly the length of exposure to the risk of dying of the spouse; and it may have a very real practical advantage in some parts of the world, where duration of marriage appears to be more accurately reported than age. When data are classified by age, the length of exposure to the risk of dying has to be estimated from the current age of the respondent and a measure of the average age at marriage of all respondents.

The estimation of adult mortality based on information about widowhood status has several advantages over that based on the orphanhood status of respondents. Since only first marriages are being considered, there is in all cases only one respondent for each target person. Further, no type of adoption effect is likely to affect this type of data, so proportions not widowed for the shorter average exposure periods may produce acceptable survivorship estimates referring to more recent periods which are generally those of greater interest. Lastly, experience suggests that because the most reliable information about survival of first spouse is provided by women, these methods provide a fairly good means of estimating male adult mortality, the estimation of which from data on paternal orphanhood is the weakest both from a methodological point of view and for reasons related to data quality.

However, this estimation method shares several of the disadvantages associated with those based on orphanhood. Once more, the universe to which the estimated survival probabilities refer is not the entire population: in this case, they refer only to the ever-married portion of it. Yet, in countries where almost universal marriage is the rule, the biases that may be introduced by assuming that mortality risks among the never-married are similar to those estimated for the ever-married are likely to be small. A second disadvantage of this method is that it assumes that mortality and nuptiality have remained constant in the recent past and that the survival of the respondent is independent of that of his or her spouse, assumptions that are not likely to hold strictly in practice. In addition, a disadvantage specific to widowhood data classified by age is that the length of exposure to the risk of dying of a respondent's first spouse has to be

estimated from the respondent's age and from a summary measure of age at first marriage, thus introducing yet another level of approximation. Lastly, in terms of data quality, it is likely that in cases where the respondent has remarried and shares a household with the new spouse, the interviewer may neglect to ask explicitly the question about survivorship of the first spouse.

There is little that the analyst can do to minimize the negative effects of some of these disadvantages on the final estimates of survivorship. With respect to data quality, it is obviously important to make sure that the interviewer understands the concepts being used and avoids mistakes such as that cited above. The assumption of constant mortality has been relaxed by the work of Brass and Bamgboye,[12] who propose a method to estimate the time reference of the survivorship probabilities derived from data on widowhood. Of course, the change of mortality assumed in deriving reference period estimates does not represent all possible patterns of change, but it is general enough to be adequate in most circumstances.

The assumption of constant nuptiality in the past can be relaxed if the mean ages of both respondents and their spouses at first marriage are estimated for each age or marriage-duration group of respondents. In such a case, the $SMAM_m$ and $SMAM_f$ values appearing in equations (B.17)-(B.26) should be replaced by age or duration-specific values (according to the respondents' age group being used). This approach is possible if reliable data on the retrospective marriage history of the population under study are available.

This introductory section may be concluded by pointing out that the widowhood methods presented in this chapter represent recent revisions of the procedures first proposed by Hill[13] and later revised by Hill and Trussell.[14]

(b) *Widowhood data classified by age*

(i) *Data required*

This method seeks to estimate male (female) probabilities of surviving from age 20 to age n from the following data:

(a) The observed singulate mean ages at marriage for men ($SMAM_m$) and for women ($SMAM_f$). For a description of the way in which to estimate these ages from the proportions single classified by age, see annex I. To calculate the proportions single in each age group, the basic data required are: the total male (female) population; and the male (female) population single, both classified by five-year age group;

(b) The proportion of ever-married women (men) in each five-year age group whose first husband (wife) was alive at the time of the interview. The proportion of respondents aged from n to $n+4$ with first spouse alive is denoted by $NW(n)$. Note that the set of proportions non-widowed, $NW(n)$, can be obtained by having as input any two of the following: (a) the number of respondents whose first spouse was alive at the time of the interview classified by five-year age group; (b) the number of respondents whose first spouse was dead classified by five-year age group; and (c) the total number of ever-married respondents who declared the survivorship status of their first spouse, also classified by five-year age group. Since category (c) is supposed to be the sum of the first two, it is important to make sure that it excludes all respondents who did not know or did not declare the survivorship status of their first spouse.

(ii) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of proportions not widowed. The proportion of female (male) respondents not widowed in the age group from n to $n+4$, denoted by $NW(n)$, is equal to the ratio of female (male) respondents aged from n to $n+4$ whose first husband (wife) was alive at the time of the interview and the total number of ever-married female (male) respondents aged from n to $n+4$ who declared the survivorship status of their first husband (wife). Since the latter figure equals the sum of the female (male) respondents aged from n to $n+4$ with first husband (wife) alive and those of the same age group with first husband (wife) dead, knowledge of only these two categories of respondents is sufficient to calculate the desired proportions.

In order to estimate survivorship probabilities for males, the method requires that the proportions of females not widowed be known, while knowledge of the male proportions not widowed is necessary to estimate female survivorship probabilities.

Step 2: calculation of singulate mean age at marriage for both males and females. For a description of the procedure required to calculate the singulate mean age at marriage, $SMAM$, see annex I. Both values are necessary when estimating either male or female survivorship probabilities from widowhood data classified by age. Subindices f and m are used to distinguish the values referring to females from those referring to males.

Step 3: calculation of survivorship probabilities. The relationship between the variables calculated in the previous steps and certain probabilities of survivorship was established by using least-squares regression to fit the following equations:

$$l_m(n)/l_m(20) = a(n) + b(n)\,SMAM_f + c(n)\,SMAM_m + d(n)\,NW_f(n-5)$$

$$l_f(n)/l_f(20) = a(n) + b(n)\,SMAM_f + c(n)\,SMAM_m + d(n)\,NW_m(n) \qquad \text{(B.16)}$$

to data generated by using model mortality and nuptiality schedules (see chapter I). The estimated regression coefficients $a(n)$, $b(n)$, $c(n)$ and $d(n)$ are shown, for each value of n, in tables 97 and 98. Also shown in each

[12] W. Brass and E. A. Bamgboye, *op. cit.*

[13] K. Hill, "Estimating adult mortality levels from information on widowhood", *Population Studies*, vol. XXXI, No. 1 (March 1977), pp. 75-84.

[14] K. Hill and J. Trussell, *loc. cit.*

TABLE 97. COEFFICIENTS FOR ESTIMATION OF CONDITIONAL MALE SURVIVORSHIP PROBABILITIES FROM DATA ON THE WIDOWHOOD STATUS OF FEMALE RESPONDENTS

Age n (1)	Coefficient $a(n)$ (2)	$b(n)$ (3)	$c(n)$ (4)	$d(n)$ (5)
25	0.1082	−0.00209	0.00072	0.9136
30	−0.0284	−0.00465	0.00151	1.0422
35	−0.0139	−0.00638	0.00233	1.0831
40	0.0041	−0.00784	0.00295	1.0596
45	0.0152	−0.00953	0.00611	1.0324
50	0.0087	−0.01189	0.00925	1.0144
55	−0.0169	−0.01515	0.01353	1.0111
60	−0.0590	−0.01940	0.01880	1.0291

Estimation equation

$$l_m(n)/l_m(20) = a(n) + b(n)SMAM_f + c(n)SMAM_m + d(n)NW_f(n-5)$$

TABLE 98. COEFFICIENTS FOR ESTIMATION OF CONDITIONAL FEMALE SURVIVORSHIP PROBABILITIES FROM DATA ON THE WIDOWHOOD STATUS OF MALE RESPONDENTS

Age n (1)	Coefficient $a(n)$ (2)	$b(n)$ (3)	$c(n)$ (4)	$d(n)$ (5)
25	−0.0208	0.00052	−0.00137	1.0451
30	−0.2135	0.00104	−0.00329	1.2791
35	−0.1896	0.00162	−0.00492	1.2884
40	−0.1290	0.00236	−0.00624	1.2483
45	−0.0713	0.00340	−0.00742	1.2005
50	−0.0327	0.00502	−0.00660	1.1590
55	−0.0139	0.00749	−0.01019	1.1297

Estimation equation:

$$l_f(n)/l_f(20) = a(n) + b(n)SMAM_f + c(n)SMAM_m + d(n)NW_m(n)$$

table is the estimation equation to be used in each case. Note that according to table 97 male survivorship probabilities are estimated by using as input the proportions of non-widowed females, while according to table 98 female survivorship probabilities are estimated from the observed proportions of non-widowed males.

Step 4: calculation of number of years before the survey to which survivorship estimates refer. In cases where mortality has been changing regularly (that is, when the change can be assumed to be linear on the logit scale) during the 15 or 20 years preceding the survey, estimates of the time, denoted by $t(n)$, to which each of the survivorship probabilities obtained in the previous step refers can be calculated by using the following equations:

$$u_m(n) = 0.3333 \ln NW_f(n-5) + Z(SMAM_m + n - 2.5 - SMAM_f) + 0.0037(27.0 - SMAM_m) \quad \text{(B.17)}$$

$$t_m(n) = (n - 2.5 - SMAM_f)(1.0 - u_m(n))/2.0 \quad \text{(B.18)}$$

or

$$u_f(n) = 0.3333 \ln NW_m(n) + Z(SMAM_f + n + 2.5 - SMAM_m) + 0.0037(27.0 - SMAM_f) \quad \text{(B.19)}$$

$$t_f(n) = (n + 2.5 - SMAM_m)(1.0 - u_f(n))/2.0 \quad \text{(B.20)}$$

Equations (B.17) and (B.18) are to be used when the estimated survivorship probabilities refer to males, while equations (B.19) and (B.20) are to be used when they refer to females. Note that the form of these sets of two equations is essentially the same, with the subindices indicating the sex category reversed. In both cases (males and females), the quantity $n + 2.5 - SMAM$ is used as an indicator of the mean duration of first marriage of respondents aged from n to $n+4$, and the values of the standard function $Z(x)$ are obtained by interpolating linearly between those listed in table 88. As the detailed example illustrates, the use of these equations is simple; and the timing estimates they yield, though not exact, are adequate indicators of the reference periods to which survivorship probabilities apply, if the change in mortality has been fairly steady and smooth.

(iii) *A detailed example*

Considered here is the case of ever-married women who were questioned about the survival status of their first husband during the National Demographic Survey of Bolivia, carried out between 15 June and 31 October 1975.[15] Data gathered by this survey have already been used to illustrating the application of the estimation method based on the orphanhood status of respondents.

The steps of the computational procedure are given below.

Step 1: calculation of proportions not widowed. Table 99 shows the raw data gathered by the survey and the pro-

[15] K. Hill, H. Behm and A. Soliz, *op. cit.*

TABLE 99. EVER-MARRIED FEMALE POPULATION AND NUMBER OF WOMEN WHOSE FIRST HUSBAND WAS DEAD AT TIME OF INTERVIEW, BY AGE GROUP, BOLIVIA, 1975

Age group (1)	Ever-married female population (2)	Women whose first spouse was dead (3)	Proportion not widowed (female respondents) (4)
20-24	1 187	24	0.9798
25-29	1 515	41	0.9729
30-34	1 276	62	0.9514
35-39	1 494	124	0.9170
40-44	1 115	141	0.8735
45-49	1 202	217	0.8195
50-54	774	228	0.7054
55-59	612	213	0.6520

portions of ever-married women whose husbands were still alive at the time of the interview, $NW_f(n)$. In computing these proportions, the denominator used is the number of ever-married women rather than all women and there were no cases of non-response. The proportions $NW_f(n)$ shown in column (4) are obtained by dividing the number of women with first husband dead (appearing in column (3) of table 99) by the number of ever-married women reporting first husband's survivorship status (listed in column (2)) and subtracting the result from 1.0. Thus, $NW_f(25)$ is calculated as

$$NW_f(25) = 1.0 - (41/1{,}515) = 1.0 - 0.0271 = 0.9729.$$

Step 2: calculation of singulate mean age at marriage for both males and females. For a detailed explanation of the way in which to calculate the necessary *SMAM* values, see annex I. In the case of Bolivia in 1975, these values are: $SMAM_m$ = 25.3 years; and $SMAM_f$ = 23.2 years.

Step 3: calculation of survivorship probabilities for males. Using the coefficients shown in table 97 and applying the version of equation (B.16) appearing in this table with $SMAM_f$ = 23.2 years, $SMAM_m$ = 25.3 years, and the observed proportions not widowed, $NW_f(n)$, the survivorship probabilities listed in column (6) of table 100 are obtained. As an example, $l_m(50)/l_m(20)$ is calculated below:

$$l_m(50)/l_m(20) = 0.0087 - 0.01189(23.2) + 0.00925\,(25.3)$$
$$+ 1.0144\,(0.8195)$$
$$= 0.798.$$

Also shown in table 100 (column (7)) are the mortality levels to which each estimated survivorship probability corresponds in the Coale-Demeny West family of model life tables. These levels are obtained by interpolating linearly between the values listed in table 226 (see annex VII). See step 3 of subsection B.2 (b) (iii) for a detailed example of the interpolation procedure used.

Step 4: calculation of number of years before the survey to which the male survivorship probabilities refer. Since male survivorship is being estimated, equations (B.17) and (B.18) are used to calculate this reference period, denoted by $t(n)$. Table 101 shows the necessary input

TABLE 100. ESTIMATION OF MALE SURVIVORSHIP PROBABILITIES FROM INFORMATION ON SURVIVAL OF FIRST HUSBAND, BOLIVIA, 1975

Age group (1)	Age n (2)	Intermediate value $a(n) + b(n)(23.2) + c(n)(25.3)$ (3)	Coefficient $d(n)$ (4)	Proportion not widowed $NW_f(n-5)$ (5)	Adult male survivorship probability $l_m(n)/l_m(20)$ (6)	Male mortality level (7)
20-24	25	0.0779	0.9136	0.9798	0.973	14.3
25-29	30	−0.0966	1.0822	0.9729	0.956	16.3
30-34	35	−0.0999	1.0831	0.9514	0.931	16.6
35-39	40	−0.0779	1.0596	0.9170	0.894	15.7
40-44	45	−0.0513	1.0324	0.8735	0.850	15.4
45-49	50	−0.0331	1.0144	0.8195	0.798	15.3
50-54	55	−0.0261	1.0111	0.7054	0.687	13.3
55-59	60	−0.0334	1.0291	0.6520	0.638	14.7

TABLE 101. ESTIMATION OF TIME REFERENCE PERIODS FOR MALE SURVIVORSHIP PROBABILITIES DERIVED FROM INFORMATION ON SURVIVAL OF FIRST HUSBAND, BOLIVIA, 1975

Age n (1)	Proportion not widowed $NW_f(n-5)$ (2)	Length of exposure indicator $u_m(n)$ (3)	Standard function $Z_m(n)$ (4)	Correction function $v_m(n)$ (5)	Reference period[a] $t_m(n)$ (6)	Reference date (7)	Male mortality level (8)
30	0.9729	29.6	0.090	0.0871	2.0	1973.6	16.3
35	0.9514	34.6	0.091	0.0807	4.3	1971.3	16.6
40	0.9170	39.6	0.102	0.0794	6.6	1969.0	15.7
45	0.8735	44.6	0.135	0.0962	8.7	1966.9	15.4
50	0.8195	49.6	0.189	0.1289	10.6	1965.0	15.3
55	0.7054	54.6	0.253	0.1430	12.6	1963.0	13.3
60	0.6520	59.6	0.331	0.1947	13.8	1961.8	14.7

[a] Number of years prior to the survey to which survivorship estimates refer.

information and some of the intermediate results. The observed proportions of females who were not widowed, $NW_f(n-5)$, are listed in column (2); and the values of $(SMAM_m + n - 2.5 - SMAM_f)$ for each n are denoted by $x_m(n)$ and are listed in column (3). The function Z is calculated for each of these $x_m(n)$ values by interpolating in table B8. Note that the data for the first value of n, 25, have been excluded from table 101 because the tabulated values of Z do not cover the value of $x(25) = 25.6$; and although extrapolation is possible, it leads to an estimate of $l(25)$ that is negative, a clearly unacceptable outcome. However, this result indicates that in a population where the female singulate mean age at marriage is 23 and that of males is 25, an estimate of the male probability of surviving from age 20 to age 25 derived from reports of ever-married females aged 20-24 is likely to be fairly poor, since a sizeable proportion of the women in this age group are not yet married and the mean exposure to the risk of dying of their husbands is relatively short. Such an indication is corroborated, in this case, by the fact that the mortality level associated with $l_m(25)/l_m(20)$ is quite clearly out of line with the subsequent estimates (see table 100).

Returning to the calculation of $Z(x)$, the procedure followed in the case of $n = 45$ is illustrated. First:

$$x_m(45) = 25.3 + 45.0 - 2.5 - 23.2 = 44.6$$

then

$$Z_m(44.6) = 0.4(0.130) + 0.6(0.139) = 0.135.$$

Therefore, using equation (B.17),

$$a_m(45) = 0.3333 \ln NW_f(40) + Z(44.6) + 0.0037(1.7)$$

$$= 0.3333 \ln(0.8735) + 0.135 + 0.0063$$

$$= 0.0962;$$

and according to equation (B.18),

$$t_m(45) = (45.0 - 2.5 - 23.2)(1.0 - 0.0962)/2.0$$

$$= (9.65)(0.9038) = 8.7.$$

That is, the estimate of $l_m(45)/l_m(20)$, which corresponds to a West mortality level of 15.4, refers approximately to 8.7 years before the survey; and since the latter has as reference date 1975.6 (see step 4 of subsection B.2 (b) (iii), the reference date for $l_m(45)/l_m(20)$ is the end of 1966, as shown in column (7) of table 101.

The mortality levels associated with the survivorship probabilities obtained in the previous step are listed in column (8) of table 101 to allow a quick assessment of the results obtained. It is worth taking note that, disregarding the last estimate ($l_m(60)/l_m(20)$), the adult mortality trend followed by the other estimates seems rapid though not impossible, implying a gain of some three mortality levels (or approximately 7.5 years in expectation of life) during 10 years or so. If the estimate associated with $l_m(55)/l_m(20)$, referring to approximately 1963, is also disregarded, the improvement in adult mortality is more modest (about one level) and perhaps somewhat more plausible. Unfortunately, the mortality levels that refer to periods comparable to those already estimated for Bolivia from maternal orphanhood data (see tables 92 and 95) do not quite match the latter; and although they refer to different sexes (the orphanhood estimates refer to female mortality, while those derived here correspond to males), the lack of consistency observed suggests that at least one of these sets of estimates may be seriously deficient. Although at this stage it seems more likely, given the trend they imply, that the estimates derived from orphanhood information are the deficient set, further evidence is required to make a definite decision.

(c) *Widowhood data classified by duration of first marriage*

(i) *Data required*

Male (or female) probabilities of surviving from age 20 to age n can also be estimated from data on survival of first spouse tabulated by duration of marriage (measured as the time elapsed since first union). The specific data required for the application of this variant of the method are listed below:

(*a*) The observed singulate mean age at marriage, $SMAM_m$ for males (females). This age is estimated from the proportion of single males (females) in each five-year age group. For a detailed description of the procedure used in estimating the singulate mean age at marriage, see annex I. Observe that when widowhood data are classified by duration of first marriage, only one value of $SMAM$ is needed in each case: if male mortality is being estimated from data from female respondents, only the value of $SMAM$ for males is required; and vice versa;

(*b*) The proportion of ever-married women (men) whose first husbands (wives) were alive at the time of the interview, classified by five-year duration-of-marriage group. These proportions are again denoted by $NW(k)$, where k now represents the lower limit of each duration group, that is, $NW(k)$ is the proportion not widowed among respondents first married from k to $k + 4$ years before the survey. As in the case of data classified by age, the proportions not widowed can be calculated from any two of the following: (*a*) the number of respondents whose first spouse was alive at the time of the interview classified by five-year duration group; (*b*) the number of respondents whose first spouse was dead at the time of the interview classified by five-year duration group; (*c*) the total number of ever-married respondents who declared the survivorship status of their first spouse, classified by five-year duration group. Since category (*c*) is supposed to be the sum of the first two, it is important that it exclude all respondents who did not know or did not declare the survivorship status of their first spouse.

(ii) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of proportions not widowed among

female (male) respondents. The proportion of female (male) respondents not widowed in the duration group from k to $k + 4$, denoted by $NW(k)$, is equal to the ratio of the number of female (male) respondents first married from k to $k + 4$ years earlier whose first husband (wife) was alive at the time of the interview divided by the total number of ever-married female (male) respondents in the same duration group who declared the survivorship status of their first husband (wife).

Note that male survivorship probabilities are estimated from the proportions of non-widowed females, whereas female survivorship probabilities are estimated from male proportions not widowed.

Step 2: calculation of singulate mean age at marriage for males (females). The necessary value of *SMAM* is calculated according to the procedure described in annex I. Note that the value of *SMAM* required is that corresponding to the sex category for which survivorship probabilities are being estimated.

Step 3: calculation of survivorship probabilities for males (females). When male mortality is being estimated, the probability of survival from age 20 is related to the singulate mean age at marriage for males and the proportion of women in a given duration-of-marriage group with a surviving first husband. The following equation is used:

$$l_m(n)/l_m(20) = a(n) + b(n)\,SMAM_m + c(n)\,NW_f(n-25) \qquad \text{(B.21)}$$

where $a(n)$, $b(n)$ and $c(n)$ are the coefficients shown in table 102; and $NW_f(n-25)$ is the proportion of women in the duration group from $n-25$ to $n-21$ having a surviving first husband. As in the case of the method based on data classified by age, the coefficients of equation (B.21) were estimated by using least-squares regression to fit this equation to data generated from model mortality and nuptiality schedules (see chapter I).

An equivalent relation is used to estimate female survival probabilities, namely,

$$l_f(n)/l_f(20) = a(n) + b(n)\,SMAM_f + c(n)\,NW_m(n-25) \qquad \text{(B.22)}$$

TABLE 102. COEFFICIENTS FOR ESTIMATION OF CONDITIONAL MALE SURVIVORSHIP PROBABILITIES FROM THE WIDOWHOOD STATUS OF FEMALE RESPONDENTS, CLASSIFIED BY DURATION OF MARRIAGE

Age n (1)	Duration group (2)	Coefficient $a(n)$ (3)	Coefficient $b(n)$ (4)	Coefficient $c(n)$ (5)
25	0-4	−1.0084	0.00039	1.9989
30	5-9	−0.3214	0.00098	1.2978
35	10-14	−0.1990	0.00202	1.1503
40	15-19	−0.1695	0.00353	1.0849
45	20-24	−0.1798	0.00557	1.0477
50	25-29	0.2187	0.00814	1.0278

Estimation equation:
$l_m(n)/l_m(20) = a(n) + b(n)SMAM_m + c(n)NW_f(n-25)$

where the male proportion not widowed and the female *SMAM* replace the female proportion not widowed and the male *SMAM* of equation (B.21). Note, however, that the estimated regression coefficients to be used also change; those listed in table 102 are to be employed in estimating male survivorship, while those given in table 103 should be employed in estimating the survivorship probabilities for females.

TABLE 103. COEFFICIENTS FOR ESTIMATION OF CONDITIONAL FEMALE SURVIVORSHIP PROBABILITIES FROM THE WIDOWHOOD STATUS OF MALE RESPONDENTS, CLASSIFIED BY DURATION OF MARRIAGE

Age n (1)	Duration group (2)	Coefficient $a(n)$ (3)	Coefficient $b(n)$ (4)	Coefficient $c(n)$ (5)
25	0-4	−1.2183	0.00141	2.1900
30	5-9	−0.3944	0.00147	1.3642
35	10-14	−0.2443	0.00184	1.2061
40	15-19	−0.2060	0.00279	1.1480
45	20-24	−0.2073	0.00428	1.1138
50	25-29	−0.2349	0.00639	1.1025

Estimation equation:
$l_f(n)/l_f(20) = a(n) + b(n)\,SMAM_f + c(n)\,NW_m(n-25)$

Step 4: calculation of number of years before the survey to which male survivorship probabilities refer. In cases where mortality has been changing regularly during the 15 or 20 years preceding the survey (that is, when the change in mortality has been roughly linear on the logit scale), estimates of the time to which each of the survivorship probabilities obtained in the previous step refers can be calculated using the following equations:

$$u_m(n) = 0.3333 \ln NW_f(n-25) + Z(n - 22.5 + SMAM_m) + 0.0037(27 - SMAM_m) \qquad \text{(B.23)}$$

$$t_m(n) = (n - 22.5)(1.0 - u_m(n))/2.0 \qquad \text{(B.24)}$$

or

$$u_f(n) = 0.3333 \ln NW_m(n-25) + Z(n - 22.5 + SMAM_f) + 0.0037(27 - SMAM_f) \qquad \text{(B.25)}$$

$$t_f(n) = (n - 22.5)(1.0 - u_f(n))/2.0. \qquad \text{(B.26)}$$

Equations (B.23) and (B.24) are used when the estimated survivorship probabilities refer to males, while equations (B.25) and (B.26) are used when they refer to females. Note that the form of these sets of equations is essentially the same, with the subindices indicating sex category reversed. In both cases (males and females), the quantity $n - 22.5$ is the mid-point of the duration group to which the respondents belong, and the values of the function $Z(x)$ are obtained by interpolating linearly between those listed in table 88.

(iii) *A detailed example*

The use of data on the survivorship of first spouse classified by duration of marriage to estimate adult survivorship probabilities is illustrated by using data gath-

ered by the National Demographic Survey carried out in Panama between August and October 1976. Table 104 shows ever-married males classified by time since first marriage and survival of first spouse. The survivorship probabilities to be estimated refer to females. Note that in table 104 the number of men who said they had been married, but who did not know the duration of their marriage, constituted a sizeable proportion of the total (slightly over 10 per cent). This level of non-response suggests that the data may not be reliable: in a country where some 10 per cent of the married male population does not declare or does not know the duration of their marriage, it is unlikely both that duration will be correctly reported by the rest and that those of unknown duration have the same characteristics as those who declared a duration. One possible treatment for the cases of non-response is to divide them among the different durations according to the proportions implied by the known cases. This procedure would be based on the assumption that, for a given survival of first spouse (dead or alive), all persons are equally likely to fail to state the duration of their union. However, it seems reasonable to suppose that people belonging to unions that have lasted longer are more likely to have forgotten the date when the union began, so a more complicated redistribution procedure should probably be used. Yet, such a procedure would require that an assumption be made about how the probability of not stating a date of first union changes according to the time elapsed since the union began. Because no information is available on this matter, the likelihood of introducing substantial biases in the final results by making incorrect assumptions is high. In view of this possibility, it seems safer to ignore completely the cases of unknown duration when carrying out this analysis, while bearing their existence in mind at the point of interpreting the final results.

The computational procedure for this example is given below.

Step 1: calculation of proportions not widowed among male respondents. Table 104 shows all the data categories that can be derived from a simple question on the survivorship status of the first wife of each ever-married male respondent. For some duration groups, the number of respondents who did not declare the survivorship status of first wife constitutes a sizeable proportion of those who declared that their first wives were dead. Again, this characteristic of the data renders their quality suspect, since if one were to assume that all cases of non-response were, in fact, cases of dead first wives, the survivorship estimates obtained would be substantially different, especially at lower marriage durations. Strictly speaking, however, there is no basis for making such an assumption; and in the rest of this analysis, the respondents who did not declare the survivorship status of their first wife are ignored. Therefore, to calculate the proportion of respondents not widowed, $NW_m(k)$ in duration group 10-14, for example, the number with first wife alive is divided by the sum of those with first wife alive and those with first wife dead, as shown below:

$$NW_m(10) = 1{,}096/(1{,}096 + 23) = 0.9794.$$

Note that $NW_m(10)$ can also be obtained by dividing the number with first wife alive by the difference between all possible respondents and those whose wives' survival status is not known. Thus,

$$NW_m(10) = 1{,}096/(1{,}127 - 8) = 0.9794.$$

The full set of $NW_m(k)$ values is shown in column (7) of table 104.

Step 2: calculation of singulate mean age at marriage for females. For a detailed description of the procedure to be used in calculating the values of *SMAM* for females, see annex I. In this case, $SMAM_f$ was found to equal 22.01 years.

Step 3: calculation of survivorship probabilities for females. Equation (B.22) is used to calculate the female probabilities of surviving from age 20 to age n. These probabilities are calculated by using as input the proportions not widowed among ever-married respondents, the female singulate mean age at marriage, and the coefficients listed in table 103. Table 105 shows the details of the calculations. Note that the final estimates of survivorship probabilities are not, in this case, very different from the original proportions of respondents with a surviving first wife. The West mortality level associated with each survivorship probability has been estimated by interpolating linearly between the values shown in table 222 in annex VII (see step 3 of subsection

TABLE 104. DATA ON SURVIVORSHIP STATUS OF FIRST WIFE, MALE RESPONDENTS, PANAMA, 1976

Duration group (t)	Index k	Number of male respondents: Wife first wife alive	Wife first wife dead	Status of first wife not declared	Total	Proportion not widowed $NW_m(k)$
	(2)	(3)	(4)	(5)	(6)	(7)
0-4	0	1 342	4	2	1 348	0.9970
5-9	5	1 419	15	4	1 438	0.9895
10-14	10	1 096	23	8	1 127	0.9794
15-19	15	960	34	7	1 001	0.9658
20-24	20	808	49	6	863	0.9428
25+	-	2 070	526	50	2 646	-
Unknown		594	169	212	975	-

TABLE 105. ESTIMATION OF ADULT FEMALE SURVIVORSHIP PROBABILITIES FROM MALE DATA ON SURVIVAL OF FIRST SPOUSE, BY DURATION OF MARRIAGE, PANAMA, 1976

Duration group (i)	Age x (2)	Proportion ever widowed (male respondents) (3)	Estimation equation $a(x) + b(x)\,SMAM_f + c(x)P_m(x-25)$ (4)	Adult female survivorship probability, $l_f(n)/l_f(25)$ (5)	West mortality level (6)
0-4	25	0.9970	−1.2183 + 0.0014(22.01) + 2.1900(0.9970)	0.9962	22.2
5-9	30	0.9895	−0.5944 + 0.00147(22.01) + 1.3642(0.9895)	0.9878	21.3
10-14	35	0.9794	−0.2443 + 0.00184(22.01) + 1.2061(0.9794)	0.9775	21.0
15-19	40	0.9658	−0.2060 + 0.00279(22.01) + 1.1480(0.9658)	0.9641	20.8
20-24	45	0.9428	−0.2073 + 0.00428(22.01) + 1.1188(0.9428)	0.9417	20.4

B.2 (b) (iii) for a detailed example of the procedure for calculating them). These levels imply, in general, that mortality among females in Panama was very low during the early 1970s.

Step 4: calculation of number of years before the survey to which the male survivorship probabilities refer. The West mortality levels shown in column (6) of table 105 suggest that mortality has not remained constant in Panama, since the reports of respondents who have been married for a longer period of time are associated with lower levels (that is, higher mortality) than are those associated with the reports of respondents married for a shorter and more recent period. It is important, therefore, to obtain some indication about the time periods, denoted by $t(n)$, to which the estimated survivorship probabilities refer. Equations (B.25) and (B.26) are therefore used to estimate $t_f(n)$. Table 106 shows some of the intermediate values required for their calculation and the final $t_f(n)$ estimates. The values of $Z_f(x)$ are obtained by interpolating linearly between the values listed in table 88, and $x_f(n)$ is used to denote the quantity $n - 22.5 + SMAM_f$. Since the survey took place between August and October of 1976, the mid-point of this period, corresponding in decimal form to 1976.7, is used as its reference date (see step 4 of subsection B.2 (b) (iii) for an example of how to calculate the decimal equivalent of this mid-point). Hence, by subtracting the estimated $t_f(n)$ values from 1976.7, the date to which each survivorship probability refers can be obtained. These reference dates are shown in column (7) of table 106; and to facilitate comparison, the West mortality levels obtained in the previous step are listed in column (8) of the same table. Without any other evidence, the trend of the dated survivorship estimates implying an improvement in adult female mortality of about a quarter of a level per annum seems plausible, although the estimated levels are relatively high. Because of the deficiencies noted earlier in this data set, the possibility that the reports on widowhood status of male respondents may understate true widowhood levels must be explored further before the estimates shown in table 105 can be fully accepted.

(d) *Use of widowhood data from two surveys*

Mortality estimates from widowhood share with those from orphanhood the problem of referring to periods located a substantial length of time before the survey, and the time reference of the estimates can only be identified when mortality has been changing regularly. A method for obtaining estimates of mortality that refer to a well-defined period of time by using orphanhood data from two surveys has been suggested earlier. Strictly speaking, a parallel approach cannot be used with widowhood data classified by age of respondent because age groups do not define marriage cohorts; and the change in proportions not widowed from one age group to the next is generally deflated by the influx of newly married respondents whose exposure to the risk of dying while married is below the average of the group. However, the same phenomena affect the proportions not widowed by age group at each survey; thus, as long as age patterns of marriage remain more or less constant, the proportions not widowed for a hypothetical intersurvey cohort will be representative of the intersurvey mortality experience. These proportions can be calculated following the method described in the case of

TABLE 106. ESTIMATION OF TIME REFERENCE PERIODS FOR FEMALE SURVIVORSHIP PROBABILITIES DERIVED FROM MALE WIDOWHOOD DATA, PANAMA, 1976

Age x (1)	Proportion ever widowed $_nW_m(x-25)$ (2)	Length of exposure indicator $x_f(n)$ (3)	Standard function $Z_f(x)$ (4)	Correction function $a_f(x)$ (5)	Reference period $t_f(n)$ (6)	Reference date (7)	West mortality level (8)
25	0.9970	24.5	0.090	0.1075	1.1	1975.6	22.2
30	0.9895	29.5	0.090	0.1049	3.4	1973.3	21.3
35	0.9794	34.5	0.091	0.1025	5.6	1971.1	21.0
40	0.9658	39.5	0.102	0.1089	7.8	1968.9	20.8
45	0.9428	44.5	0.135	0.1338	9.7	1967.0	20.4

[a] Number of years prior to the survey to which survivorship estimates refer.

orphanhood data (see subsection B.2 (d)) and can then be analysed by using the method described in subsection B.3 (b), with average male and female singulate mean ages at marriage during the intersurvey period as additional inputs (these average ages can be calculated as the mean of those corresponding to the beginning and end of the intersurvey period).

On the other hand, when widowhood data are classified by time elapsed since first marriage, five-year duration groups do represent marriage cohorts; and proportions not widowed for a hypothetical marriage cohort representing intersurvey experience may be calculated from two sets of observed proportions not widowed classified by duration and obtained from surveys or censuses five or 10 years apart. The calculation of the proportions not widowed representing the experience of a hypothetical intersurvey cohort is carried out exactly as described in step 2 of subsection B.2 (d) (ii) for the case of proportions not orphaned, with duration groups substituted throughout for age groups. Because the complete computational procedure to be followed in this case is very similar to that followed in the case of data on orphanhood and because no data sets on widowhood by duration from two surveys are as yet available, a detailed description of the application of this method is omitted.

C. Estimation of survivorship to adulthood from birth

1. *General characteristics of methods*

In this section, an extension of the conditional methods discussed in section B is presented. The methods that are described allow the estimation of probabilities of survival from birth, that is, of probabilities that are no longer conditional. These probabilities are estimated by means of equations that include $l(2)$, the probability of surviving from birth to exact age 2, as an independent variable, to take account of mortality in childhood. In practice, the value of $l(2)$ used is almost invariably obtained by applying the child mortality estimation techniques described in chapter III.

Since the methods to be presented here are extensions of the conditional methods of adult survivorship estimation described above, they are based on very similar assumptions. In fact, all the comments made in section B about the validity of the estimates of conditional survivorship probabilities apply also to the validity of estimates of direct probabilities; and because the calculation of the latter values demands extra information, for example, $l(2)$, certain other assumptions are necessary.

The methods presented here are based on equations fitted by least-squares regression to simulated data, so the main assumption made is that the models used in simulating data represent reality adequately. An additional assumption necessary in this case is that the relationship between adult and child mortality embodied in the model life tables used during simulation cover the range of actual experience. Therefore, estimates derived by these methods are likely to be unreliable whenever the experience of the population studied falls outside the range covered by the models used. The estimation of adult survivorship from birth is particularly sensitive to the relationship between child and adult mortality, and to the pattern of the mortality schedule between ages 5 and 25. This sensitivity can be reduced by including in the estimation equation an independent variable allowing for known variations in mortality between ages 2 and 20 or 25. The model cases to which the estimation equations were fitted were generated by using mortality schedules based on the logit system, using as standards the four life tables for females at level 16 of the Coale-Demeny models. Therefore, the probability of surviving from age 2 to age 20 (or 25 in the case of orphanhood) in the appropriate standard life table is included as an independent variable in the estimation equation in order to allow for the age pattern of mortality between childhood and early adulthood. It remains true, however, that estimates of survivorship from birth based on orphanhood or widowhood data from younger respondents are overwhelmingly determined by the estimation of $l(2)$, while the proportions not orphaned or not widowed have a relatively low predictive value,[16] a fact that may affect the consistency of the estimates obtained from data corresponding to younger respondents.

It has been stressed earlier in this chapter that orphanhood and widowhood data provide estimates of mortality referring to points in the past and that if the same type of data is available from two surveys five or 10 years apart, the experience of a hypothetical intersurvey cohort can be constructed to obtain estimates of mortality referring exactly to the intersurvey period. The methods of analysis outlined in the next sections can also be applied to the constructed proportions not orphaned or not widowed of a hypothetical intersurvey cohort, using as input either an estimate of $l(2)$ obtained also from the constructed experience of a hypothetical cohort and thus referring to the same period (see chapter III, section D); or, failing that, an estimate of $l(2)$ obtained from women aged 20-24 at the second survey and thus referring more or less to the mid-point of the intersurvey period. Once the proportions not orphaned or not widowed for the hypothetical intersurvey cohort have been obtained, the analysis follows exactly the procedures described below in subsections C.2 (c), C.3 (b) (ii) and C.3 (c) (ii), but the mortality estimates obtained have a well-defined reference period (the intersurvey period), so that estimates of $t(x)$ are not required.

2. *Estimation of female survivorship from birth to adult ages on the basis of proportions with surviving mother*

(a) *Basis of method and its rationale*

Regression techniques were used to relate the proportions of respondents whose mothers were still alive at the time of the interview to the survivorship probabilities experienced by these women. It was found that probabilities of survivorship from birth, $l(n)$, could be

[16] K. Hill and J. Trussell, *loc. cit.*

estimated with sufficient accuracy if the probability of surviving from birth to age 2 for females—$l_f(2)$—were taken into account among the independent variables determining the desired $l(n)$ values and if an indicator of the relationship between adult and child mortality, denoted by RS and equal to the ratio of $l(25)$ to $l(2)$ in the standard that best represents the experience of the population in question, were also included in the fitted equation. As in the case of the conditional regression method based on information about maternal orphanhood (see subsection B.2 (c)), the other independent variables used are M, the mean age of mothers at the birth of their children, and $S(n)$, the proportion of respondents in the age group from n to $n+4$ whose mother is still alive. The form of the fitted equation is

$$l_f(25+n) = a(n) + b(n)M + c(n)\,l_f(2)S(n-5) + d(n)RS \qquad \text{(C.1)}$$

where $a(n)$, $b(n)$, $c(n)$ and $d(n)$ are coefficients whose values for each n are listed in table 107.

These coefficients were obtained by using least-squares regression to fit equation (C.1) to data simulated on the basis of selected fertility and mortality schedules. The fertility schedules used were derived from the Coale-Trussell models (see chapter I, subsection E.1) and the mortality schedules were obtained by using the logit system, using as standards the level 16 female life tables from each regional family of the Coale-Demeny models (see chapter I, subsections B.2 and B.4).

It is worth pointing out that this method allows only the estimation of female adult mortality, since the coefficients shown in table 107 were derived exclusively from simulations referring to females. Furthermore, these simulations do not take into account any trends in mortality; that is, the same life table was used in each simulated case to generate $l_f(2)$ and $l_f(25+n)$ for n ranging from 20 to 50. Therefore, use of the coefficients listed in table 107 implies the tacit assumption of an unchanging mortality level for a period of at least 40 years. Since, in practice, this assumption is very likely to be violated, one cannot interpret the estimates obtained by applying this method as being exact period measures of mortality, especially because the use of $l_f(2)$ as an independent variable introduces yet another timing problem. In fact, probably the most satisfactory way of using this method is by calculating first the time period to which the estimates of survivorship would refer, were they determined mainly by the proportions with surviving mother (as in the conditional method) and had mortality followed a regular pattern of change (linear in the logit scale), and then using as input for equation (C.1) child mortality estimates referring to corresponding periods in order to obtain dated probabilities of surviving from birth to adult ages. In practice, however, it will seldom be possible to obtain child mortality estimates for the exact dates to which the conditional survivorship probabilities refer, and period averages will have to be used instead. In such cases, the user must bear in mind that the survivorship estimates obtained from equation (C.1) reflect a mixture of mortality levels with somewhat different reference dates.

(b) *Data required*

The following data are required for this method:

(*a*) The proportion of respondents with a surviving mother in each five-year age group from n to $n+4$, denoted by $S(n)$. See subsection B.2 (b) (i) for some comments on the calculation of these proportions;

(*b*) The number of births in a given year, classified by five-year age group of mother. This information is used to estimate M, the mean age of mothers of a particular group of births at the time of such births;

(*c*) An estimate or estimates of the probability of surviving from birth to exact age 2 for females, denoted by $l_f(2)$. Such estimates are usually obtained from information on the number of children ever born and surviving classified by age of mother (see chapter III).

(c) *Computational procedure*

The computational procedure includes the steps described below.

Step 1: calculation of mean age at maternity. See step 1 in subsection B.2 (b) (ii).

Step 2: calculation of reference periods of the conditional survivorship probabilities estimated from information on

TABLE 107. COEFFICIENTS FOR ESTIMATION OF FEMALE SURVIVORSHIP PROBABILITIES FROM BIRTH ON THE BASIS OF PROPORTIONS OF RESPONDENTS WITH MOTHER ALIVE

Age n	Coefficient $a(n)$	Coefficient $b(n)$	Coefficient $c(n)$	Coefficient $d(n)$
20	−1.2982	0.0033	1.2275	1.0422
25	−1.2508	0.0051	1.1934	0.9821
30	−1.2385	0.0077	1.1623	0.9334
35	−1.2669	0.0115	1.1423	0.8885
40	−1.3217	0.0166	1.1432	0.8506
45	−1.3541	0.0225	1.1678	0.8489
50	−1.2655	0.0274	1.1932	0.4365

Estimation equation:

$$l_f(25+n) = a(n) + b(n)M + c(n)\,l_f(2)S(n-5) + d(n)RS$$

maternal orphanhood. The procedure described in step 3 in subsection B.2 (c) (ii) should be followed.

Step 3: estimation of probability of surviving from birth to age 2 for females. Reference should be made to the methods described in chapter III. Ideally, estimates of the probability of surviving to age 2, denoted by $l_f(2)$, should be obtained for each of the reference periods estimated in step 2. Only estimates of $l(2)$ for females are needed.

Step 4: estimation of survivorship probabilities from birth. Estimation of this probability, $l(a)$, the true core of the procedure, is straightforward enough once the mean age at maternity, M, and appropriate estimates of $l_f(2)$ have been obtained. It involves only substitution in equation (C.1) using the coefficients given in table 107. Care must be taken to match correctly these coefficients with the proportions with surviving mother, $S(a)$, for different values of a. It is also necessary at this point to calculate the value of the standard ratio, RS, which is the probability of survival from age 2 to age 25 in the female level 16 life table of the Coale-Demeny family that best represents the population being studied. That value is

$$RS = l_f(25)/l_f(2). \qquad (C.2)$$

For convenience, the four values of RS associated with the Coale-Demeny models are shown in table 108. The user must select the appropriate model and then use the corresponding value of RS as input for equation (C.1). Note that the use of other standards, though not covered by the model cases, would not result in major biases, as long as RS is calculated for the standard being used.

TABLE 108. STANDARD RATIO VALUES TO BE USED IN ESTIMATION OF SURVIVORSHIP PROBABILITIES FROM BIRTH ON THE BASIS OF DATA ON MATERNAL ORPHANHOOD, COALE-DEMENY MODELS

Coale-Demeny family of model life tables (1)	Estimated ratio $RS = l_f(25)/l_f(2)$ (2)
North	0.90198
South	0.92581
East	0.93568
West	0.92346

(d) *A detailed example*

To compare the performance of the different methods of adult mortality estimation, the case of Bolivia presented in subsection B.2 (b) (iii) is again used. The raw data collected by the National Demographic Survey in 1974 are shown in table 89. Data on female children ever born and surviving are given below in table 109; results are summarized in table 110.

TABLE 109. FEMALE CHILDREN EVER BORN AND SURVIVING ACCORDING TO THE NATIONAL DEMOGRAPHIC SURVEY, BOLIVIA, 1975

Age group (1)	Total number of women (2)	Number of daughters: Ever born (3)	Number of daughters: Surviving (4)
15-19	3 069	170	150
20-24	2 404	1 175	974
25-29	1 932	2 343	1 867
30-34	1 469	2 857	2 200
35-39	1 619	4 213	3 149
40-44	1 192	3 447	2 596
45-49	1 283	3 771	2 742

TABLE 110. ESTIMATION OF CHILD MORTALITY FOR FEMALES, BOLIVIA, 1975

Age group of mother (1)	Age x (2)	Probability of dying $q_f(x)$ (3)	West mortality level (4)	Reference date (t_f) (5)
20-24	2	0.1863	11.68	1973.5
25-29	3	0.2080	10.87	1971.8
30-34	5	0.2360	10.58	1969.7
35-39	10	0.2631	10.37	1967.3
40-44	15	0.2541	11.35	1964.6
45-49	20	0.2787	11.20	1961.6

The steps of the procedure are given below.

Step 1: calculation of mean age at maternity. Reference should be made to the first step in subsection B.2 (b) (iii), where the mean age at maternity, M, was calculated in detail for Bolivian females. Its value was found to equal 28.8 years.

Step 2: calculation of reference periods of the conditional survivorship probabilities estimated from information on maternal orphanhood. This step is identical to that presented as step 3 in subsection B.2 (c) (iii). Therefore, it is not repeated here. However, the reference dates estimated there (see table 95) are again given in column (3) of table 111 for immediate reference.

Step 3: estimation of probability of surviving from birth to age 2 for females. The data on female children ever born

TABLE 111. ESTIMATION OF FEMALE SURVIVORSHIP PROBABILITIES FROM BIRTH USING INFORMATION ON ORPHANHOOD BY AGE OF RESPONDENT, BOLIVIA, 1975

Age group (1)	Proportion with mother surviving $S(a-x)$ (2)	Reference date (3)	West level for $l_f(2)$ (4)	Probability of surviving to age 2 $(l_f(2))$ (5)	Age x (6)	Female adult survivorship probability $l_f(25+x)$ (7)	West level for $l_f(25+x)$ (8)
15-19	0.9252	1967.9	11.4	0.8012	20	0.6692	13.9
20-24	0.8807	1966.1	10.2	0.7973	25	0.6410	14.1
25-29	0.7896	1964.5	10.1	0.7954	30	0.5752	13.4
30-34	0.6915	1963.1	9.9	0.7913	35	0.5098	13.1
35-39	0.5737	1961.8	9.8	0.7893	40	0.4318	12.9
40-44	0.4467	1960.7	9.7	0.7872	45	0.3407	12.8
45-49	0.3346	1960.4	9.6	0.7851	50	0.2774	14.1

and surviving, given in table 109, were gathered by the National Demographic Survey of Bolivia in 1975. This is the first such data set available for the country, as questions on this subject had not been asked previously. As explained in chapter III, these data can be used to obtain an estimate of $l_f(2)$, the probability of surviving from birth to age 2, for females. The procedure followed is that described in subsection B.2 of chapter III and its detailed application is not presented here. For the purpose at hand it suffices to consider the results obtained. Table 110 summarizes these results. The usual estimates of the probability of dying, $q_f(x)$, yielded by the method are listed in column (3); column (4) shows the mortality levels consistent with the estimated $q_f(x)$ estimates in the West family of Coale-Demeny model life tables, while column (5) shows the reference date of each estimate (obtained by subtracting each reference period value, $t(i)$, for the child mortality estimates, from 1975.6, the reference date for the 1975 survey).

Two characteristics of the child mortality estimates given in table 110 should be pointed out. First, the estimates corresponding to the earliest periods are derived from data for women aged 40-44 and 45-49 at the time of the survey; and, secondly, these estimates are precisely those which do not follow, in terms of mortality levels, the trend implied by the others. Given the most common flaws of data on children ever born and surviving (see chapter III, subsection A.1), it is likely that the estimates referring to 1961.6 and 1964.6 may be too high (in terms of mortality levels) because of the tendency of older women to omit some of their dead children. Such doubts about the quality of these estimates do not recommend them for use in other estimation procedures.

Unfortunately, as the reference dates listed in column (3) of table 111 show, most of the conditional survivorship estimates derived from data on maternal orphanhood refer to the period 1960-1965, so that child mortality estimates for the same period are desirable. In order to obtain such estimates, the trend implied by the $q_f(2)$, $q_f(3)$, $q_f(5)$ and $q_f(10)$ estimates given in table 110 is extrapolated into the past. There are many possible ways of extrapolating this trend, but the simplest was selected, mainly because the use of more sophisticated techniques is not warranted by the approximate nature of the timing estimates.

A plot of the first four mortality levels against time would show that they define a fairly linear trend. Therefore, using the first and fourth to determine the trend line, its slope (change in level per annum) is calculated as

$$\Theta = (11.04 - 10.37)/(1973.5 - 1967.3) = 0.108$$

so that, for 1960.7, for example, the extrapolated mortality level is

$$10.37 + 0.108(1960.7 - 1967.3) = 9.7.$$

The complete set of West mortality levels as defined by these extrapolated child mortality estimates and corresponding to the periods to which the orphanhood-based estimates refer is shown in column (4) of table 111. Column (5) shows the $l_f(2)$ values that these estimates imply. These values were obtained by interpolating linearly between the $l(2)$ values listed in table 236 (see annex VIII). For example, for the estimated level of 9.7 obtained above, the $l_f(2)$ value lies between that corresponding to level 9 (0.77271) and that corresponding to level 10 (0.79340). Using linear interpolation (see annex IV), the desired $l_f(2)$ for level 9.7 is calculated as follows:

$$l_f(2) = 0.3(0.77271) + 0.7(0.79340) = 0.7872.$$

Step 4: estimation of survivorship probabilities from birth. Using the values of M and $l_f(2)$ given above and the values for RS for model West given in table 108, the final step in this procedure is rather simple. Essentially, for each value of n, substitution of the values of M, $l_f(2)$, $S(n-5)$ and RS in equation (C.1) is carried out, using the coefficients given in table 107. Column (7) of table 111 shows the estimates of $l_f(25+n)$ obtained in this way. As an example, $l(50)$ and $l(60)$ are calculated below in detail.

$$\begin{aligned} l_f(50) = & -1.2508 + 0.0051(28.8) + \\ & 1.1934(0.7973)(0.8807) + 0.9821(0.92346) \\ = & \ 0.6410; \end{aligned}$$

$$\begin{aligned} l_f(60) = & -1.2669 + 0.0135(28.8) + \\ & 1.3423(0.7913)(0.6915) + 0.8885(0.92346) \\ = & \ 0.5098. \end{aligned}$$

Note that because different estimates of $l_f(2)$ are available for each time period, the values of this variable also changes with n, while that of RS remains constant throughout.

In order to assess the estimated probabilities of survival from birth to adult ages, the West mortality levels consistent with them were calculated. Again, this calculation involves linear interpolation between the values listed in table 236 in annex VIII. Note that because the estimated values are probabilities of survival from birth, the life-table values themselves are used as the basis of interpolation (that is, no special tabulations of the life tables are needed). As an example, the level consistent with $l_f(60) = 0.5098$ is found by identifying those values in the columns labelled "$l(60)$" of table 236 (see annex VIII) which enclose the observed value. In this case, the enclosing values are 0.50587 and 0.54215, corresponding to levels 13 and 14, respectively. Hence, the level corresponding to 0.5098 is

$$13.0 + (0.50980 - 0.50587)/(0.54215 - 0.50587) = 13.1.$$

The set of levels given in column (8) of table 111 shows that the estimates from $l_f(55)$ to $l_f(70)$ are fairly

consistent, implying similar mortality levels and a plausible trend of decline. The levels associated with $l_f(45)$, $l_f(50)$ and $l_f(75)$ are rather high and out of line with the others, so they need to be interpreted with caution.

At this point, it is worth comparing these estimates with those obtained earlier using the conditional methods. Refer, for example, to table 95, where the mortality levels displayed refer to the same periods as those shown in table 111. Note that the use of a child mortality indicator in the estimation process has had the effect of lowering substantially the adult mortality levels implied by the orphanhood data (table 111). The large difference between the levels given in table 95 (determined mainly by information on maternal orphanhood) and those presented in table 110 (determined by the data on child survivorship) implies one or more of three possibilities: that model West is not a good representation of the mortality pattern of the female population of Bolivia; that orphanhood is underreported, leading to underestimates of adult mortality; or that child survivorship is underreported and the child mortality estimates obtained are too high. Although the evidence presented here is not sufficient to decide among these three possibilities or between different combinations of them, other evidence suggests that maternal orphanhood in Bolivia was indeed underreported and that the estimated mortality levels presented in table 95 are too high. In this case, therefore, it seems that the estimates shown in table 111 are more acceptable, mainly because they are determined to a considerable extent by the child mortality estimates they incorporate.

3. *Estimation of survivorship from birth to adult ages on the basis of proportions not widowed*

(a) *Basis of method and its rationale*

The methods discussed below are an extension of those presented in subsection B.3. Once more, data on proportions with surviving first spouse, tabulated either by the age or the duration of first marriage of the respondent, are used to estimate survival probabilities. In the present case, however, the incorporation of some information on the level of mortality prevalent at younger ages (in the form of estimates of $l(2)$) allows the estimation of probabilities of survival from birth in a way that parallels the method just described (see subsection C.2), which uses data on survival of mother to estimate survivorship probabilities from birth.

Both the age and duration methods that are presented here use as input an estimate of $l(2)$ (the probability of survival from birth to age 2) for the sex whose survivorship probabilities are to be estimated. An estimate of $l(2)$ is usually obtained from responses to questions regarding children ever born and children surviving. Methods used in carrying out this estimation are described in chapter III. If an estimate of $l(2)$ is available only for both sexes combined, suitable estimates for each sex can be obtained by assuming that the sex differentials embodied in the Coale-Demeny model life tables are an adequate representation of those prevalent in the population being considered. As in the orphanhood case, the explicit use of $l(2)$ coupled with the derivation of estimation equations from data generated under the assumption of constant mortality further complicates the determination of the time location to which the estimates refer. When there is evidence supporting the existence of a mortality change, appropriate $l(2)$ values should be calculated for each of the periods to which the conditional survivorship estimates derived from widowhood data refer, in order to ensure that the survival probabilities from birth estimated according to the method described here shall also refer to well-defined periods.

(b) *Widowhood data classified by age*

An equation relating the value of probabilities of survival from birth to the values of the singulate mean ages at marriage for males and females, the value of $l(2)$ (the probability of surviving from birth to exact age 2), the proportion with surviving first spouse and an indicator of the relationship between child and adult mortality, RSW, was fitted by least-squares regression to data from 900 simulated cases. These cases were generated using model nuptiality, fertility and mortality schedules. The mortality schedules were obtained from the logit system using female level 16 model life tables from each of the four Coale-Demeny families as standards (see chapter I, subsections B.2 and B.4), while the nuptiality and fertility schedules were derived from the models proposed by Coale and McNeil[17] and Coale and Trussell,[18] respectively (see chapter I, section D and subsection E.1). The simulation carried out also makes some allowance for the age distribution at first marriage of spouses according to the age group to which the respondent belonged at the time of first marriage.

The equations used to estimate the probabilities of surviving to adult ages have very similar forms:

$$l_m(n) = a(n) + b(n)\,SMAM_f + c(n)\,SMAM_m + d(n)\,NW_f(n-5) + e(n)\,l_m(2) + f(n)\,RSW$$

$$l_f(n) = a(n) + b(n)\,SMAM_f + c(n)\,SMAM_m + d(n)\,NW_m(n) + e(n)\,l_f(2) + f(n)\,RSW \quad \text{(C.3)}$$

where $a(n)$, $b(n)$, $c(n)$, $d(n)$, $e(n)$ and $f(n)$ are coefficients that depend both upon sex and upon age; $SMAM_f$ and $SMAM_m$ are the singulate mean ages at marriage for females and males, respectively; $NW(n)$ is the proportion of ever-married respondents aged from n to $n+4$ whose first spouse was alive at the time of the interview; $l(2)$ is the probability of surviving from birth to age 2 for the spouses; and RSW is the ratio of $l(20)$ and $l(2)$ in the standard (the female level 16 model life table from the Coale-Demeny family that best approxi-

[17] Ansley J. Coale and Donald R. McNeil, "The distribution by age of the frequency of first marriage in a female cohort", *Journal of the American Statistical Association*, vol. 67, No. 340 (December 1972), pp. 743-749.

[18] A. J. Coale and T. J. Trussell, *loc. cit.*

mates the pattern of the mortality experienced by the population in question). Note that although the model cases were based on Coale-Demeny standards, the use of other standards, with their implied values of *RSW*, should not result in important biases.

Table 112 shows the values of the coefficients that are to be used in estimating male adult mortality. Note that in this case the value of $l(2)$ appearing in equation (C.3) should be the probability of surviving from birth to age 2 for males, while $NW(n)$ should be the proportion of ever-married female respondents whose first husbands were alive at the time of the interview. The *RSW* values listed in table 114 were calculated for female level 16 life tables from the Coale-Demeny families because it was those life tables which were used as standards when generating the simulated cases. If some other standard is regarded as more suitable, its *RSW* value should be calculated and used in equation (C.3).

The values of the coefficients that are to be used in estimating female mortality are shown in table 113. When these values are used, $l(2)$ should refer to females and $NW(n)$ should be the proportion of male respondents aged from n to $n+4$ whose first wives were alive at the time of the interview. *RSW* is again selected from the values listed in table 114 or calculated especially if some other standard is to be used.

TABLE 112. COEFFICIENTS FOR ESTIMATION OF MALE SURVIVORSHIP FROM BIRTH FROM PROPORTIONS OF WOMEN WITH SURVIVING FIRST HUSBAND CLASSIFIED BY AGE

Age n	Coefficients					
	$a(n)$ (2)	$b(n)$ (3)	$c(n)$ (4)	$d(n)$ (5)	$e(n)$ (6)	$f(n)$ (7)
25	−2.0441	−0.00364	0.00125	1.5870	1.1666	1.3952
30	−2.6765	−0.00595	0.00201	1.3842	1.1140	1.3032
35	−2.8864	−0.00708	0.00281	1.2018	1.0668	1.2386
40	−2.1993	0.00795	0.00403	1.0747	1.0230	1.1979
45	−2.0958	−0.00906	0.00581	0.9814	0.9805	1.2138
50	−2.0299	−0.01084	0.00844	0.9253	0.9267	1.2336
55	−1.9541	−0.01347	0.01203	0.8993	0.8330	1.2094
60	−1.8377	−0.01704	0.01651	0.9037	0.7581	1.1505

Estimation equation:

$$l_m(n) = a(n) + b(n)\,SMAM_f + c(n)\,SMAM_m + d(n)\,NW_f(n-5) + e(n)\,l_m(2) + f(n)\,RSW$$

TABLE 113. COEFFICIENTS FOR ESTIMATION OF FEMALE SURVIVORSHIP FROM BIRTH FROM PROPORTIONS OF MEN WITH SURVIVING FIRST WIFE CLASSIFIED BY AGE

Age n	Coefficients					
	$a(n)$ (2)	$b(n)$ (3)	$c(n)$ (4)	$d(n)$ (5)	$e(n)$ (6)	$f(n)$ (7)
25	−3.3410	0.00099	−0.00262	1.9929	1.1399	1.3161
30	−2.9062	0.00137	−0.00432	1.6824	1.1039	1.2615
35	−2.5558	0.00183	−0.00556	1.4558	1.0565	1.1989
40	−2.2860	0.00242	−0.00641	1.2837	1.0138	1.1456
45	−2.1048	0.00327	−0.00713	1.1534	0.9732	1.1290
50	−1.9451	0.00461	−0.00790	1.0648	0.9220	1.0890
55	−1.7821	0.00666	−0.00906	1.0044	0.8577	1.0267

Estimation equation:

$$l_f(n) = a(n) + b(n)\,SMAM_f + c(n)\,SMAM_m + d(n)\,NW_m(n) + e(n)\,l_f(2) + f(n)\,RSW$$

TABLE 114. VALUES OF THE STANDARD RATIO *RSW*, INDICATING THE RELATIONSHIP BETWEEN CHILD AND ADULT MORTALITY IN ESTIMATION OF SURVIVORSHIP PROBABILITIES FROM BIRTH ON THE BASIS OF WIDOWHOOD DATA: COALE-DEMENY MODELS

Coale-Demeny family of model life tables (1)	Standard ratio $RSW = l_f(2)/l_m(2)$ (2)
North	0.91932
South	0.94010
East	0.95022
West	0.94181

(i) *Data required*

The following data are required for this method:

(*a*) the proportions of ever-married male (female) respondents whose first spouses were alive at the time of the interview, classified by five-year age groups from n to $n+4$. These proportions are denoted by $NW(n)$. See subsection B.3 (*b*) (i) for a discussion of the type of raw input data necessary to calculate them;

(*b*) Proportions single classified by five-year age group and by sex. This information is needed to compute the singulate mean age at marriage, *SMAM*, for each sex (see annex I);

(*c*) Information on children ever born and surviving, by sex of child and by five-year age or marriage duration group of mother (the classification by sex is useful but not essential). This information is used to estimate $l(2)$ by using the methods described in chapter III.

(ii) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of singulate mean ages at marriage for males and females. A detailed description of the procedures followed in calculating these parameters is presented in annex I.

Step 2: calculation of reference periods for conditional survivorship probabilities obtained from information on widowhood status. This step is identical to that presented as step 3 in subsection B.3 (b) (ii). Therefore, its description is not repeated here.

Step 3: estimation of probability of surviving from birth to age 2. These methods are described in chapter III. Ideally, estimates of the probability of surviving to age 2, $l(2)$, should be obtained for each of the reference periods estimated in step 2. The $l(2)$ estimates are necessary only for the sex of the respondents' spouses. If data on child survivorship are not available by sex, models can be used to approximate acceptable sex differentials.

Step 4: estimation of survivorship probabilities from birth. Estimation of these probabilities, $l(n)$, is carried out by substituting into equation (C.3) the *SMAM* and $l(2)$ values computed in the previous steps, as well as the proportions not widowed, the selected *RSW* value from table 114 and the appropriate coefficients taken from tables 112 or 113. It is important to keep in mind that when male adult mortality is being estimated, $l(2)$ values for males must be used, while $NW(n)$ should be the proportion of female respondents not widowed. In contrast, when female mortality is being estimated, the respondents are male and values of $l(2)$ for females must be used.

(iii) *A detailed example*

The use of this method is illustrated by estimating female probabilities of survival from birth for Bolivia from data on the incidence of widowhood among male respondents. The data used were collected by the National Demographic Survey carried out in Bolivia during 1975. Columns (3) and (4) of table 115 show, respectively, the number of ever-married male respondents with first wife still alive and the number whose first wife was dead, classified by five-year age group. The denominators needed to calculate the proportions of respondents with surviving first wife for each age group are the sum of these two quantities, thus eliminating cases of non-response from both numerator and denominator; they are shown in column (5) under the heading "ever-married male population". Column (6) shows the required proportions of those not widowed, computed by dividing the entries in column (3) by those in column (5). For example, $NW_m(30)$ is calculated as follows:

$$NW_m(30) = 1{,}088/(1{,}088+39) = 1{,}088/1{,}127 = 0.9654.$$

Step 1: calculation of singulate mean ages at marriage for males and females. The values of the singulate mean ages at marriage have already been presented in subsection B.3 (b) (iii). They are: $SMAM_m = 25.3$; and $SMAM_f = 23.2$.

Step 2: calculation of reference periods of conditional survivorship probabilities obtained from information on widowhood status. Using equations (B.19) and (B.20), the reference periods for the conditional probabilities of female survival calculated from the proportions of male respondents with first wife alive are estimated. Table 116 shows several intermediate results and the final reference-period estimates both in terms of years before the survey, denoted by $t_f(n)$, and in terms of actual dates (decimal equivalents in terms of years). The dates are calculated by subtracting each $t_f(n)$ from 1975.6, the reference date of the survey. The West mortality levels consistent with each of the conditional survivorship probabilities are also shown for future reference. No detailed example of the calculation of $t_f(n)$ is presented here because such an example has already been given for the case of the estimation of male adult mortality in step 4 in subsection B.3 (b) (iii).

Step 3: estimation of probability of surviving from birth to age 2 for females. Since the process of estimating this probability, $l_f(2)$, for Bolivian females has already been described in step 3 in subsection C.2 (d), the reader is referred to that section for a discussion of the procedure for selecting a trend line defining changes in child mortality. In this case, the same trend line is used to estimate the West mortality levels prevalent during the more recent period to which the conditional survivorship

TABLE 115. DATA ON WIDOWHOOD STATUS OF MALES OBTAINED IN NATIONAL DEMOGRAPHIC SURVEY, BOLIVIA, 1975

Age group (1)	Age n (2)	Number of respondents: Not widowed (3)	Number of respondents: Widowed (4)	Ever-married male population (5)	Proportion not widowed $NW_m(n)$ (6)
20-24	20	713	5	718	0.9930
25-29	25	1 208	22	1 230	0.9821
30-34	30	1 088	39	1 127	0.9654
35-39	35	1 154	54	1 208	0.9553
40-44	40	980	77	1 057	0.9272
45-49	45	1 000	98	1 098	0.9107
50-54	50	651	92	743	0.8762
55-59	55	452	107	559	0.8086

TABLE 116 ESTIMATION OF TIME REFERENCE PERIODS FOR THE CONDITIONAL SURVIVORSHIP PROBABILITIES FOR FEMALES, BOLIVIA, 1975

Age x	Proportion not widowed NW_x/n	Length of exposure fulcrum $x_f(x)$	Standard function $Z_f(x)$	Correction function $c_f(x)$	Reference period $t_f(x)$	Reference date	Conditional female survivorship probability $l_f(x)/l_f(20)$	Mean mortality level
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
25	0.9821	25.4	0.090	0.0899	1.0	1974.6	0.9830	16.8
30	0.9654	30.4	0.093	0.0925	3.3	1972.3	0.9622	16.6
35	0.9553	35.4	0.091	0.0898	5.6	1970.0	0.9543	18.2
40	0.9272	40.4	0.106	0.0949	7.8	1967.8	0.9253	17.5
45	0.9107	45.4	0.143	0.1259	9.7	1965.9	0.9132	18.4
50	0.8762	50.4	0.198	0.1680	11.3	1964.3	0.8817	18.3
55	0.8086	55.4	0.265	0.2083	12.7	1962.9	0.8155	17.1

[a] Number of years prior to the survey to which survivorship estimates refer.

probabilities derived from widowhood refer. The estimated levels are shown in column (5) of table 117, and the $l_f(2)$ values associated with them are given in column (6).

Step 4: estimation of female survivorship probabilities from birth. Using the coefficients given in table 113, the estimates of female survivorship probabilities from birth, $l_f(n)$, are calculated by substituting the values of *SMAM* obtained in step 1, the $l_f(2)$ values obtained in step 3, the values of $NW_m(n)$ and the value of *RSW* corresponding to the West standard (female level 16) in equation (C.3). The value of *RSW* is taken from table 114 and is equal to 0.94181. As an example, $l_f(35)$ is calculated below:

$$l_f(35) = -2.5558 + 0.00183(23.2) - 0.00556(25.3)$$
$$+ 1.4558(0.9553) + 1.0565(0.8071)$$
$$+ 1.7989(0.94181)$$
$$= 0.7186.$$

Column (7) of table 117 shows the complete set of $l_f(n)$ estimates and column (8) shows the levels they imply in the West family of Coale-Demeny model life tables (obtained by interpolating linearly between the values shown in table 236 in annex VIII).

A comparison of these mortality levels with those presented in table 116 indicates that the estimates of conditional probabilities of survivorship associated with the latter imply substantially lower mortality (that is, higher levels) than that implied by the $l_f(n)$ values obtained by taking into account child mortality. The immediate reason for this difference is that the child mortality estimates employed imply very high mortality (that is, low levels); and hence, their use in the calculation of $l_f(n)$ reduces the mortality levels associated with the latter. As in the case of the data on orphanhood, the existence of such differences implies one or more of three possibilities: that the West model does not adequately represent the age pattern of mortality in Bolivia, that child mortality is overestimated or that male respondents overreport the survivorship of their first wives. Although the evidence presented here does not permit us to establish which of these mechanisms is in operation, a more comprehensive analysis of Bolivian data indicates that the information on male widowhood severely underestimates mortality. The weakness of the widowhood data in this particular case is also suggested by the fact that the mortality levels associated both with the conditional survivorship probabilities and with those estimated from birth (columns (9) and (8) of tables 116 and 117, respectively) fail to increase as age of respondent decreases (that is, as one moves towards the present in terms of reference dates). Taken at face value, these estimates would imply that female adult mortality in Bolivia has increased through time. The small likelihood of such an event immediately makes their accuracy questionable, but it is the comparison between

TABLE 117 ESTIMATION OF FEMALE SURVIVORSHIP FROM BIRTH USING DATA ON MALE WIDOWHOOD STATUS, BOLIVIA, 1975

Age group	Age x	Proportion not widowed $NW_m(x)$	Reference date	West level for $l_f(2)$	Probability of surviving to age x $l_f(x)$	Female survivorship from birth $l_f(x)$	West level for $l_f(x)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25-29	25	0.9821	1974.6	11.2	0.8167	0.7434	12.8
30-34	30	0.9654	1972.3	10.9	0.8110	0.7222	13.0
35-39	35	0.9553	1970.0	10.7	0.8071	0.7186	13.7
40-44	40	0.9272	1967.8	10.4	0.8012	0.6894	13.7
45-49	45	0.9107	1965.9	10.2	0.7973	0.6803	14.3
50-54	50	0.8762	1964.3	10.0	0.7934	0.6541	14.4
55-59	55	0.8086	1962.9	9.9	0.7913	0.6010	14.1

these levels and those implied by the estimates of child mortality that best illustrates the inconsistencies in the data. Hence, the Bolivian case is a good example of how the application of a variety of estimation methods allows the assessment of data quality.

(c) *Widowhood data classified by duration of marriage*

When the proportion of ever-married respondents whose first spouse is alive is tabulated by sex and by the time elapsed since their first union or marriage (in the broadest sense of this term) the estimation of $l(n)$ becomes somewhat simpler since the period of exposure to the risk of dying is directly known and does not need to be estimated from information on age. This simplification allows the set of independent variables used as predictors of $l(n)$ in equation (C.3) to be reduced. Thus, when male mortality is to be estimated from the proportion of females not widowed whose first marriage took place between k and $k+4$ years ago, denoted by $NW_f(k)$, the equivalent of equation (C.3) becomes

$$l_m(n) = a(n) + b(n)\, l_m(2) + c(n)\, NW_f(n-20) + d(n)\, SMAM_m \qquad \text{(C.4)}$$

where $l_m(2)$ is the probability of surviving from birth to exact age 2 years among males; $SMAM_m$ is the male singulate mean age at marriage; and $NW_f(n-20)$ is the proportion of female respondents whose first marriages occurred between $n-20$ and $n-16$ years before the interview and whose first husbands were still alive at the time of the interview. The equation to be used to estimate female mortality is

$$l_f(n) = a(n) + b(n)\, l_f(2) + c(n)\, NW_m(n-20) + d(n)\, SMAM_f \qquad \text{(C.5)}$$

where $SMAM_f$ is the female singulate mean age at marriage; $NW_m(n-20)$ is now the proportion of male respondents whose first wives were alive at the time of the interview and whose first marriages occurred between $n-20$ and $n-16$ years before the interview; and $l_f(2)$ is the probability of surviving from birth to age 2 for females.

In both cases, $a(n)$, $b(n)$, $c(n)$ and $d(n)$ stand for the values of coefficients obtained by using least-squares regression to fit equations (C.4) and (C.5) to simulated cases. Their values are listed in tables 118 and 119. Table 118 refers to male mortality and the values listed in it should be used in conjunction with equation (C.4), while table 119 refers to female mortality and its values should be used with equation (C.5).

Even though this method is simpler, from a theoretical standpoint, than that using data on widowhood status classified by age, it may, in practice, produce poorer results than the latter method in countries where a sizeable proportion of the ever-married population live in consensual unions rather than in formal marriages because, in this case, it may be difficult for many respondents to establish unambiguously the date on which their first union began. In particular, members of relatively unstable unions may have a tendency to report the survival of the current partner rather than that of the first, a phenomenon that would reduce artificially the observed proportions widowed.

A problem that the duration-based method shares

TABLE 118. COEFFICIENTS FOR ESTIMATION OF MALE SURVIVORSHIP FROM BIRTH FROM PROPORTIONS OF WOMEN WITH SURVIVING FIRST HUSBAND, CLASSIFIED BY DURATION OF FIRST MARRIAGE

Age n (x)	Duration group (x)	Coefficient $a(n)$ (3)	Coefficient $b(n)$ (4)	Coefficient $c(n)$ (5)	Coefficient $d(n)$ (6)
20	0-4	−3.4875	0.9607	3.4884	0.00077
25	5-9	−1.5427	0.9387	1.5458	0.00131
30	10-14	−1.1558	0.9165	1.1521	0.00224
35	15-19	−0.9867	0.8942	0.9703	0.00346
40	20-24	−0.8978	0.8701	0.8638	0.00499

Estimation equation:

$$l_m(n) = a(n) + b(n)\, l_m(2) + c(n)\, NW_f(n-20) + d(n)\, SMAM_m$$

TABLE 119. COEFFICIENTS FOR ESTIMATION OF FEMALE SURVIVORSHIP FROM BIRTH FROM PROPORTIONS OF MEN WITH SURVIVING FIRST WIFE, CLASSIFIED BY DURATION OF FIRST MARRIAGE

Age n (x)	Duration group (x)	Coefficient $a(n)$ (3)	Coefficient $b(n)$ (4)	Coefficient $c(n)$ (5)	Coefficient $d(n)$ (6)
20	0-4	4.0224	0.9386	4.0102	0.00263
25	5-9	1.6857	0.9083	1.7107	0.00189
30	10-14	1.2271	0.8801	1.2701	0.00203
35	15-19	1.0284	0.8519	1.0780	0.00277
40	20-24	0.9168	0.8242	0.9675	0.00392

Estimation equation

$$l_f(n) = a(n) + b(n)\, l_f(2) + c(n)\, NW_m(n-20) + d(n)\, SMAM_f$$

with that based on data classified by age is that the coefficients presented in tables 118 and 119 were derived from cases simulated under the assumption of constant mortality. When mortality has been changing, the estimates of survivorship derived exclusively from the proportions of respondents with first spouse alive will, in general, refer to different time periods. As discussed in subsection C.3 (a), estimates of $l(2)$ for the same reference periods as the conditional survivorship probabilities that may be estimated from the proportions with surviving first spouse should be used in equations (C.4) and (C.5).

With respect to the reference period of the estimates derived from widowhood information, the duration version of these estimation methods has an advantage over the age version when widowhood data classified by duration of first marriage are available from two surveys five or 10 years apart. In this case, proportions not widowed for a hypothetical intersurvey marriage cohort can be constructed following the procedure described for orphanhood data in subsection B.2 (d), and $l(x)$ estimates referring specifically to the intersurvey period can be obtained directly from equations (C.4) and (C.5) by using as input estimates of $l(2)$ obtained either for another hypothetical intersurvey cohort (see chapter III, section D) or from the second survey and referring roughly to the intersurvey period, and the values of *SMAM*, referring also to the intersurvey period. A hypothetical-cohort approach to the calculation of *SMAM* is feasible, as is shown in annex I, although, unless nuptiality patterns are changing very rapidly, the use of the *SMAM* estimates obtained from either survey or of the average of both should be acceptable.

(i) *Data required*

In order to estimate male (female) survivorship probabilities, the following data are needed:

(*a*) Estimates of $l(2)$, the probability of surviving from birth to age 2 for males (females), for different time periods. These estimates are usually obtained from information on children ever born and surviving by using the methods described in chapter III;

(*b*) The singulate mean age at marriage for males (females). Refer to annex I for a description of how to calculate this parameter;

(*c*) The proportion of female (male) ever-married respondents whose first spouses were alive at the time of the interview, classified by duration of marriage (strictly speaking, classification should be according to the time elapsed since their first union). See subsection B.3 (c) (i) for a discussion of the possible types of data from which these proportions may be calculated.

(ii) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of singulate mean age at marriage for spouses of respondents. Refer to annex I for a detailed description of the procedure to be followed in calculating this parameter.

Step 2: calculation of reference periods of conditional survivorship probabilities obtained from information on widowhood status. This step is identical to that presented as step 4 in subsection B.3 (c) (ii). Therefore, its description is not repeated here.

Step 3: estimation of probability of surviving to age 2 for sex of the spouses of respondents. Reference should be made to the methods described in chapter III. Ideally, these probabilities, denoted by $l(2)$, should be obtained for each of the reference periods estimated in step 2. These estimated probabilities are required only for the sex of the respondents' spouses. When data on child mortality by sex are not available, models can be used to approximate adequate sex differentials.

Step 4: estimation of survivorship probabilities from birth. Estimation of survivorship probabilities, $l(x)$, is carried out by substituting in equation (C.4) or (C.5), depending upon the sex of respondents, the *SMAM* and $l(2)$ values obtained in previous steps, the observed proportions of respondents with first spouse alive, $NW(x)$, and the appropriate coefficients. It is important to match the proportions $NW(x)$ determining a survivorship estimate for a certain period with the $l(2)$ value referring to the same period.

(iii) *A detailed example*

As an example of the estimation of adult mortality using information on the proportion of respondents whose first spouse was alive at the time of the interview, classified by duration of marriage, the case of Panama is again used. In this case, the question on widowhood was asked of both men and women during the Demographic Survey in 1976. The answers obtained were tabulated by duration of first union and have already been presented for male respondents in table 104. The number of respondents of known duration who do not know whether their first wife is alive or dead is, in most instances, rather small; and they are excluded from both numerator and denominator in calculating the proportions not widowed. As mentioned in subsection B.3 (c) (iii), however, these data contain a substantial number of cases of unknown marital duration. The problem posed by these cases of unknown duration cannot be dealt with so easily, and the reader is referred to the discussion in subsection B.3 (c) (iii). In this example, the proportions of male respondents whose first wives were alive at the time of the interview, shown in column (3) of table 120, have been calculated, excluding the cases of unknown duration entirely.

The computational procedure entails the following steps.

Step 1: calculation of singulate mean age at marriage for females. According to the survey in 1976, the singulate mean age at marriage for females was 21.95 years (see annex I).

Step 2: calculation of reference periods of female survivorship probabilities obtained from information on widowhood status. This step was carried out in detail in subsection B.3 (c) (iii) (step 4), so only the results obtained are quoted here. Column (4) of table 120 shows the reference dates associated with the conditional survivorship

TABLE 120. ESTIMATION OF FEMALE SURVIVORSHIP PROBABILITIES FROM BIRTH USING DATA ON WIDOWHOOD STATUS OF MALE RESPONDENTS, CLASSIFIED BY DURATION OF MARRIAGE, PANAMA, 1976

Duration group (1)	Age n (2)	Proportion ever widowed $(_nW_m(n-20))$ (3)	Reference date (4)	West level for $l_f(2)$ (5)	Probability of surviving to age 2 $(l_f(2))$ (6)	Female survivorship probability from birth $(l_f(n))$ (7)	West level for $l_f(n)$ (8)
0-4	20	0.9970	1975.6	20.7	0.9621	0.9365	20.2
5-9	25	0.9895	1973.3	20.1	0.9551	0.9160	19.6
10-14	30	0.9794	1971.1	19.5	0.9477	0.8955	19.2
15-19	35	0.9658	1968.9	18.9	0.9401	0.8744	18.9
20-24	40	0.9432	1967.0	18.5	0.9348	0.8491	18.6

probabilities, derived from widowhood information classified by duration of marriage. These dates have been copied from table 106 and indicate the periods for which estimates of female child mortality are needed.

Step 3: estimation of probability of surviving to age 2 for females. In chapter III, two detailed examples using data from the Demographic Survey of Panama in 1976 were presented, one based on data classified by age of mother (subsection B.3) and the other based on data classified by marriage duration (subsection C.3). The estimates yielded by each set are not identical, so a choice has to be made between them. The estimates derived from duration data exhibit, in terms of West mortality levels, a fairly linear trend with respect to time (if the estimate associated with duration group 0-4 is disregarded), and they have the relative advantage of being obtained from the same type of data (classified by duration) as the adult survivorship estimates considered here. Hence, any errors present in one set may be expected to be similar to those present in the other and it may therefore be easier to detect them.

Table 121 shows the West mortality levels consistent with the child mortality estimates derived in chapter III, subsection C.3, together with their reference dates. A graph of these levels against time shows that the values from the second to the fifth follow an acceptably linear trend. Hence, the line defined by the second and fifth values can be used to estimate the levels prevalent during the reference dates of the estimates of survivorship derived from widowhood data (shown in column (4) of table 120). The trend line fitted to the estimated child mortality levels has as its slope:

$$(20.0-18.1)/(1973.0-1965.6)=0.257.$$

Hence, the level corresponding to 1971.1, for example, is

$$18.1+0.257(1971.1-1965.6)=19.5.$$

All other mortality levels shown in column (5) of table 120 are obtained in a similar way; and by interpolation between the $I(2)$ values listed in table 236 in annex VIII, one can obtain the $l_f(2)$ estimates required as input for equation (C.5). The full set of $l_f(2)$ estimates is shown in column (6) of table 120.

TABLE 121. WEST MORTALITY LEVELS IMPLIED BY THE ESTIMATES OF CHILD MORTALITY OBTAINED FROM DATA CLASSIFIED BY DURATION OF MARRIAGE, PANAMA, 1976

Duration group (1)	Reference date (2)	West mortality level (3)
0-4	1975.3	21.3
5-9	1973.0	20.0
10-14	1970.6	19.7
15-19	1968.3	18.9
20-24	1965.6	18.1

Step 4: Estimation of survivorship probabilities from birth for females. The estimation equation to be used in the case of male respondents is (C.5) with $SMAM_f=21.95$, the $l_f(2)$ values obtained from column (6) of table 120, the $_nW_m(n-20)$ proportions from column (3) of the same table; and $a(n)$, $b(n)$, $c(n)$ and $d(n)$ from table 119. The use of equation (C.5) is straightforward. Final results are shown in column (7) of table 120. The consistency of the estimates in terms of equivalent mortality levels is impressive, but it should be mentioned that each estimate is, to a large extent, determined by the $l_f(2)$ value used as input, a fact that greatly reduces their range of variation. On the other hand, the influence of $l_f(2)$ is lowest on the mortality estimates for the longer duration groups, for which the mortality estimates are most similar in terms of implied level to those derived from child survival, so the consistency observed cannot be dismissed.

Chapter V

ESTIMATION OF ADULT MORTALITY FROM INFORMATION ON THE DISTRIBUTION OF DEATHS BY AGE

A. BACKGROUND OF METHOD

1. *Use of information on deaths by age and sex*

The straightforward way of calculating mortality rates is by using the information on deaths by age produced by a vital registration system. However, even though many countries possess such a system of registration it is often the case that not all the deaths are registered. As a result, the death rate implied by the reported deaths is usually an underestimate of the true death rate prevalent in the population in question, and some method of adjustment is required to transform the reported death rate into a better estimate of true mortality conditions. Of course, the same statements apply to the calculation of age-specific death rates; if a life table is calculated in the ordinary manner, and if deaths are underreported, the survival function, $l(x)$, will fall too slowly as age increases and estimates of life expectancy will be biased upward.

Over the years, demographers have suggested several methods of adjustment. Two methods were selected for inclusion here: one proposed by Preston and Coale;[1] and the other proposed by Brass.[2] A third method that essentially compares registered deaths between two censuses with the deaths implied by the census age distributions is presented in chapter IX.

The two methods described in this chapter are based on the assumption that the population being studied is stable (see chapter I, section C). They are both fairly robust to violations of the assumption of stability, particularly to recent changes in fertility and to gradual changes in mortality. Nevertheless, the methods hold strictly only for stable populations, and it is therefore helpful to recall the main characteristics of such populations. A stable population is one that has been subject to constant fertility and mortality for a long time. A feature of such a population is that the rate of exponential growth is constant at all ages, from which it follows that the same rate of growth must apply to births, deaths, deaths in any particular age group and the population of any age group.

The methods presented are also based on the assumption that the completeness of reporting of deaths is the same at all ages. That is, it is not assumed that reporting is 100 per cent complete; but if it is, for example, only 80 per cent complete, this figure applies to all age groups. In practice, because childhood and adult deaths are very often underreported to different extents, these methods are used only to estimate adult mortality (robust procedures for estimating childhood mortality are discussed in chapter III). Child mortality estimates obtained from the procedures described in chapter III and age-specific death rates for adults adjusted by the methods described in this chapter may be spliced together in the conventional manner or by using model mortality schedules as described in chapter VI.

2. *Organization of this chapter*

The two methods described in this chapter use information on the age distribution of deaths and that of the population. They both assume that the population is stable and that the degree of completeness of death registration is more or less the same at all ages after childhood (over age 5 or 10). The basic data required may be obtained from a vital registration system (deaths by age) and a census (age distribution of the population), or from surveys, whether retrospective or prospective in nature. In all cases, it is important to make sure that the deaths refer to the population whose age distribution is being used in the analysis. To aid the user in selecting the method best suited for a particular application, brief descriptions of the methods presented are given below (table 122 indicates their data requirements and the parameters they estimate):

Section B. Preston and Coale method. An equation is derived from stable-population theory that relates the population of age x to the deaths over age x expanded by a series of factors incorporating the stable growth rate. The ratios of the estimated population of age x derived from deaths over age x to the reported population of age x, denoted by $\hat{N}(x)/N(x)$, indicate the relative completeness of death registration. Although it is necessary to assume a growth rate, r, in order to calculate $\hat{N}(x)$, a "best" value of r can be selected as being that which produces the most consistent set of $\hat{N}(x)/N(x)$ ratios for different values of x. This method is more robust to departures from stability than the Brass method, but it is more sensitive to certain types of age misreporting;

Section C. Brass growth balance method. In a stable population, the rate of entry into the population aged x and over by reaching age x is equal to the rate of depar-

[1] Samuel Preston, Ansley J. Coale, James Trussell and Maxine Weinstein, "Estimating the completeness of reporting of adult deaths in populations that are approximately stable," *Population Studies*, vol. 46, No. 2 (Summer 1980), pp. 179-202.

[2] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, North Carolina, Carolina Population Center, Laboratories for Population Studies, 1975).

TABLE 122. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER V

Section	Type of input data	Estimated parameters
B. Preston and Coale Method	Deaths in a year classified by five-year age group, and by sex Mid-year population classified by five-year age group, and by sex Provisional estimate of the growth rate	Completeness of death-reporting in relation to population coverage Revised estimate of the growth rate Revised estimate of the death rate over age 10
C. Brass growth balance method	Deaths in a year classified by five-year age group and by sex Mid-year population classified by five-year age group and by sex	Completeness of death-reporting in relation to population coverage An estimate of the growth rate Revised estimate of the death rate over age 10

ture from the same population segment through death plus the stable population growth rate, which is the same for all values of x. This method uses this relation to estimate the stable growth rate and the relative completeness of death registration. It is somewhat less vulnerable to age exaggeration than the Preston and Coale method, but it is more sensitive to the effects of destabilization resulting from a rapid mortality decline.

B. PRESTON AND COALE METHOD

1. *Basis of method and its rationale*

In any population, the number of persons in a particular age group, say 25-29, at a particular time t will be equal to the total number of deaths to those persons from time t, when its members are 25-29, until the last survivor has died. If only 50 per cent of the deaths occurring every year are registered, then the ratio of the total number of deaths reported to the actual population will be 0.5, the value of the completeness of registration. The Preston-Coale procedure is based on this simple idea. Of course, in the example given above, one would have to wait a long time to obtain an estimate of the completeness of registration. However, if the number of deaths that will occur after time t can be estimated from the number of deaths reported for a particular year or calendar period, the comparison of the reported number of persons in a particular age group with the estimated total number of future deaths to the age group should provide an estimate of the completeness with which deaths are registered. In a stable population, there is a precise relationship between the numbers of current deaths and the numbers of persons in the population. The persons now aged x, to whom deaths are currently occurring, are the survivors of births x years ago, which, by the properties of stable populations, must have been smaller in number than current births by a factor of $\exp(-rx)$. Hence, the number of deaths that will occur to the current number of births when they are aged x will be larger than the current number of deaths to persons aged x by a factor of $\exp(rx)$.

It follows that the number of deaths that will be experienced by persons currently aged x (theoretically equal to the number of such persons) can be estimated from the current number of deaths recorded at each age above x. Specifically, if $N(x)$ is the number of persons at age x in a stable population with growth rate r, and $D(x)$ is the number of deaths at age x, then an estimate of $N(x)$, $\hat{N}(x)$ can be expressed as

$$\hat{N}(x) = \sum_{a=x}^{\omega} D(a)\exp(r(a-x)). \qquad \text{(B.1)}$$

If the population is genuinely stable, the rate of growth correctly specified, and deaths and population accurately reported, then $\hat{N}(x)$ will equal $N(x)$. If, however, deaths are underreported by some fixed proportion (say, 20 per cent), $\hat{N}(x)/N(x)$ will be less than 1.0—in this instance, 0.8. Since $\hat{N}(x)$ is an estimate of the population at exact age x, it cannot be compared directly to the reported population, which is normally tabulated by five-year age group. One can estimate $N(x)$ as $({}_5N_{x-5}+{}_5N_x)/10$, where ${}_5N_x$ is the reported population between ages x and $x+5$; or, alternatively, ${}_5N_x$ can be estimated as $2.5(N(x)+N(x+5))$.

Thus, the ratio of $\hat{N}(x)$ to $N(x)$ is an estimate of the completeness of death registration in relation to population enumeration, but it would be unwise to estimate completeness from a single ratio $\hat{N}(x)/N(x)$ or ${}_5\hat{N}_x/{}_5N_x$, since the number of persons reported at a particular age (even if determined as the average of the number over an interval centred on x) is subject to overstatement or understatement because of age-misreporting, or because of differential omission of persons in a particular span of ages. A better estimate of the completeness of death registration in relation to population enumeration can be obtained from erratic $\hat{N}(x)/N(x)$ ratios by considering some type of representative value of individual ratios over an extensive range of ages. For example, the median can be used, or the ratio of the sum of estimates of ${}_5\hat{N}_x$ over an extensive range of ages to ${}_5N_x$ over the same range. The obvious possibility is the ratio

$$C = \sum_{x=\Phi}^{\omega} {}_5\hat{N}_x \Big/ \sum_{x=0}^{\omega} {}_5N_x. \qquad \text{(B.2)}$$

which is a comparison of the total population estimated

from the number of deaths to the total enumerated population. However, this possibility is not necessarily the best because estimates of $\hat{N}(0)$ and ${}_5\hat{N}_0$ are based in part on the reported numbers of infant and child deaths, which usually constitute a large proportion of the total number of deaths and are often subject to a completeness of registration quite different from that of deaths at older ages.

Another problem arising in the estimation of $\hat{N}(x)$ is the determination of the population estimated from reported deaths at the upper ages. At the upper end of the age range, there is always an "open interval", in which the number of deaths and the number of persons are tabulated in an undivided age category, such as 90+, 85+, 80+ or 75+. If the lower boundary of the open interval is denoted by A, the estimation of $\hat{N}(A)$ and of $\hat{N}(A+)$ from $D(A+)$ and the rate of growth requires special procedures because the distribution of deaths within the open interval is not available. It can be proved that the estimation of $\hat{N}(A)$ is less sensitive to uncertainties about the distribution of deaths in the open interval than is the estimation of $\hat{N}(A+)$.

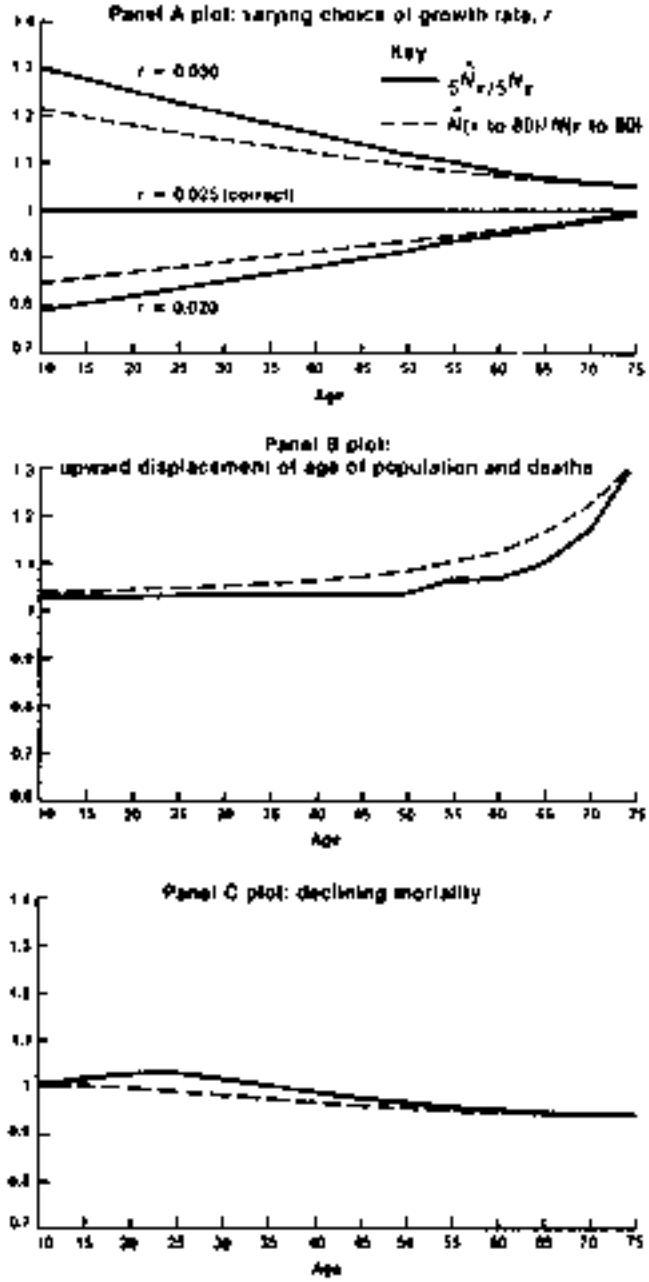
Because of the special difficulties involved in estimating the values of $\hat{N}$ in early childhood and in the open interval, a practical option to obtain the least erratic values of the ratios of estimated to reported population is to divide the sum of the estimated ${}_5N_x$ values over certain ranges of age x (from age 5 up to age $A-5$, five years before the beginning of the open interval) by the sum of the reported ${}_5N_x$ values over the same range.

It is therefore suggested that two sets of estimates be considered in assessing the completeness with which deaths are recorded in relation to the population—the sequence of $10\hat{N}(x)/{}_{10}N_{x-5}$, and the sequence of $\hat{N}(x \text{ to } A)/N(x \text{ to } A)$ ratios for different values of x. In an ideal situation of accurately reported age, a genuinely stable age distribution of known growth rate, and the same completeness of registration of deaths at different ages, both sequences of ratios would be constant with respect to age and both would yield the same fixed estimate of completeness. In actuality (as stated earlier), the first set of estimates is erratic because of misreporting of age in the population or because of differential completeness of enumeration. In addition, it is frequently the case that the estimates both of $\hat{N}(x)$ and of $\hat{N}(x \text{ to } A)$ are affected by violations of the assumptions upon which the method is based. It turns out, however, that the most typical violations of the assumptions produce characteristic deviations from the expected horizontal plot of the $\hat{N}/N$ sequence, deviations that are fairly distinctive and therefore, in certain circumstances, interpretable. In order to examine such effects, the estimation method proposed was applied to data from a true stable population with a growth rate of 0.025, which have been artificially distorted. The conclusions reached from these experiments are summarized below.

(a) *Incorrect choice of growth rate.* In panel A of figure 11, plots of $\hat{N}/N$ are shown for the correct choice of the growth rate, r (0.025), and for values that are too high or too low. It can be seen that if r is too high, the sequence of $\hat{N}/N$ falls with age; the opposite occurs if the value of r is too low.

(b) *Upward displacement of age at death.* Very often the age of older persons is displaced upward, perhaps because social status is enhanced by advanced age. Typically, reported ages at death are displaced upward to a greater extent than the reported ages of the living. A common situation in which age at death is exaggerated in relation to the age of the living produces the rising sequence of $\hat{N}/N$ ratios shown in panel B of figure 11. Overstatement of age at death increases the estimate of $\hat{N}(x)$ in two ways. Recall that $\hat{N}(x)$ is the sum of the products of deaths over age x and exponen-

Figure 11. Illustration of the effects of deviations from the assumptions on the plot of estimated to reported population ratios, $\hat{N}/N$, for model cases



tial terms $\exp(r(a-x))$ (equation (B.1). Overstatement of age at death in the form of a transfer of reported deaths from under to over age x increases $\hat{N}(x)$ by simple exaggeration of the relative number of deaths over age x. Moreover, overstatement of age at death confined within the span of ages over x (that is, with no transfer from under x to over x) also increases $\hat{N}(x)$ for the following reason. When the sum

$$\sum_{a>x}^{\omega} D(a)\exp(r(a-x))$$

that determines $\hat{N}(x)$ is calculated, a transfer of deaths from lower to higher ages within the span over x results in multiplication of the transferred deaths by a larger than warranted exponential factor.

The first or direct effect of an upward transfer of deaths across some age boundary x is substantially greater than the second or indirect effect of age exaggeration above x, resulting in too old an age distribution of deaths over age x, though the indirect effect continues to distort the results for all values of x. The size of the indirect effect declines as x decreases, however, so that by age 10 it will be only about half as large as at age 70, say, if net upward transfers occur only over ages 75, 80 and 85. The overall effect, therefore, of exaggeration of age at death on the plot of $\hat{N}(x)/N(x)$ is likely to be an imperceptible upward trend with x up to the age at which exaggeration begins, followed by a sharp upward kink reflecting the direct effects of exaggeration. The values of the points up to the kink should not be severely distorted, unless exaggeration of age at death begins at fairly young ages.

(c) *Departures from stability.* If fertility has recently declined or if there was a birth deficit at some time in the past that created a small cohort in the population, the age distribution would not in fact be stable. The match between the true number of persons at age x and the number constructed from the number of deaths, even if deaths were completely reported, would be impaired. Specifically, small cohorts caused by low fertility either at the younger ages because of a recent decline, or for another cohort because of the effect of a war, would be reflected in high values of $\hat{N}(x)/N(x)$ for these age intervals. However, if the estimate of completeness of registration of adult deaths is derived by analysing deaths and population only over age 15 or 20, a deviation from stability caused by recent changes in fertility will have no effect on the results. A more frequent situation in contemporary populations in developing countries is a decline in mortality that has occurred over the past 20 or 30 years. A history of declining mortality causes a departure from the stable age distribution, but usually a rather limited departure. The sequence of ratios of $\hat{N}/N$ produced in a typical population in which mortality has been declining is shown in panel C of figure 11. The sequence of $\hat{N}(x)/N(x)$ rises and then falls, while that of $\hat{N}(x \text{ to } A)/N(x \text{ to } A)$ falls slowly at first and then more rapidly. The actual pattern depends upon the particular trend followed by the mortality decline and the time period during which the decline has taken place, but an inverted U pattern is fairly typical.

In any real population, of course, misstatement of reported age by the population will tend to produce a zigzag pattern in the sequence of the $\hat{N}(x)/N(x)$ ratios. Such distortions are lessened when the cumulated version of these ratios, $\hat{N}(x \text{ to } A)/N(x \text{ to } A)$, is considered, that is, when the estimated population whose age ranges from x to the lower limit of the open interval is compared with the number of persons in the enumerated population over the same age interval from x to A. Unfortunately, it is not always easy to distinguish between the effects of the different violations of the underlying assumptions.

Both overstatement of age at death and choice of a growth rate that is too small cause a rising sequence of $\hat{N}(x)/N(x)$. However, too small a value of r causes the sequence to rise linearly at all ages; while overstatement of age at death, concentrated as it often is at the highest ages, causes an upward bend at the first age at which a systematic upward transfer of reported age at death occurs. If the chosen r is too small, one simply tries a larger value. If $\hat{N}(x)/N(x)$ rises monotonically only over age 75, for example, overstatement of age at death is the likely cause. An estimate can be made of the extent to which the number of reported deaths above each age has been increased by age overstatement, and a rough correction can be made by reassigning the excess reported deaths to the next lower age interval. The $\hat{N}(x)/N(x)$ ratios can then be recalculated. Usually this correction only slightly reduces the $\hat{N}(x)/N(x)$ ratios at younger ages. An alternative to redistribution may be the selection of a lower beginning age for the open interval, so that age overstatement is largely confined to that interval.

A more intractable problem is to distinguish a declining set of $\hat{N}(x)/N(x)$ ratios that is associated with a population that has a history of declining mortality from that produced when the growth rate used is too high. Moreover, adjusting the estimates by assigning a smaller value to the rate of growth would lower the estimate of completeness; yet, the estimate derived from the population with a history of declining mortality may already be too low. When faced with a declining sequence of $\hat{N}(x)/N(x)$, one must therefore decide on other grounds whether the population in question has recently experienced extensive declines in mortality. If such a decline has occurred, the median of the ratios of cumulated populations with beginning ages from 10 to 35 or from 10 to 45 can be accepted as an estimate of completeness. On the other hand, if a prolonged and substantial decline in mortality appears unlikely, a lower value of the rate of increase can be employed to produce a more level sequence of $\hat{N}(x)/N(x)$ ratios.

Unfortunately, the generally optimistic tone of this discussion of the diagnostic value of the $\hat{N}(x)/N(x)$ sequence is not invariably justified in practice. Errors are not always as orderly as one would wish, and combinations of different types of errors can give rise to patterns in the $\hat{N}(x)/N(x)$ ratios that cannot be interpreted

according to the simple rules presented above. Hence, it will not always be possible to obtain one single, unambiguous estimate of completeness, though by the judicious use of different growth rates and open intervals, it should at least be possible to obtain some idea of the range within which the true figure lies.

2. *Data required*

The following data are required for this method:

(a) The deaths occurring in a specific time period, normally in a given year (but the average over a two-year or a five-year period may be used as well), classified by age. Five-year age groups are adequate. It is useful, though not essential, to classify by sex;

(b) The population by age group (and sex) corresponding to the mid-period for which deaths are given;

(c) An estimate of the growth rate during the period. One need not have a precise estimate, since an incorrect choice of r should be evident in a diagnostic plot of $\hat{N}(x)/N(x)$. Indeed, the final choice of r may be determined by that value which produces the most level trend over the central age groups.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: estimation of growth rate. Since the method is applied only to adults, the provisional estimate of the growth rate could be obtained as the intercensal rate of growth of the population over age 10 or 15. However, a better estimate can usually be found by taking the median of the growth rates of the population over ages 10, 15, 20, ..., 60. If censuses are taken at times t_1 and t_2, and the population in question at time t_1 is N_1 and at time t_2 is N_2, then the growth rate is calculated as $r = \ln(N_1/N_2)/(t_1 - t_2)$. If it is not possible to calculate intercensal growth rates, then an iterative procedure that would begin with a reasonable value of r and modify it until the sequence of $\hat{N}(x)/N(x)$ is as close to being horizontal as possible would be necessary.

Step 2: adjustment of reported population to mid-point of period. Strictly speaking, when the data on deaths are for a given year, the age distribution of the mid-year population should be used. If a census was taken on or near the middle of the year, its data can be used without adjustment. If the population data refer to the beginning or the end of the year, one possible course of action is to use the average of the data on deaths for the two adjacent years. Another, more general solution is to adjust the population figures so that they conform in time to the death data. The simplest way to accomplish this task is to assume that the population is growing at the growth rate estimated in step 1. Thus, if t_m is the date corresponding to the middle of the period (or year) to which the death data refer and t_c is the reference date of the census, the adjusted population values are computed as

$$ {}_5N_x^{adj} = {}_5N_x \exp(r(t_m - t_c)). $$

Note that the values of ${}_5N_x$ and $N(x+)$ would all be multiplied by the same adjustment factor $\exp(r(t_m - t_c))$. Thus, in practice, it is much simpler to apply this method without adjusting the data at this stage, but making sure to adjust the estimated completeness of death registration, C, by the factor $\exp(-r(t_m - t_c))$. Furthermore, if one is interested in adjusted age-specific mortality rates and not in the level of completeness *per se*, this adjustment is not necessary. When no adjustment is performed, C will be an estimate of completeness of death registration in relation to the size of the unadjusted population, so multiplying the reported deaths by $1.0/C$ and dividing by the unadjusted population will yield the same estimated mortality rates as those obtained when the population is adjusted as explained above.

Another common situation that requires adjustment of the ${}_5N_x$ values arises when the deaths correspond to an intercensal period. In this case, the population figures should be the average of those recorded by the two censuses and the death figures should be the average annual number of deaths.

Step 3: estimation of population from registered deaths. It can be shown that equation (B.1), from which the estimated population at exact age x, $\hat{N}(x)$, is calculated, can be expressed in recursive fashion as $\hat{N}(x) = \hat{N}(x+5)\exp(5r) + {}_5D_x \exp(2.5r)$. Hence, the calculation of $\hat{N}(x)$ is straightforward, except for the open-ended age interval. In general, since the estimation for this interval requires a certain amount of approximation, it is advisable to use as narrow an open interval as the data permit, taking into consideration the possible effects of age exaggeration.

For each of the five-year age intervals from x to $x+4$, it is assumed that the length of time from x to the mid-point of the interval provides an adequate estimate of the average length of time over which deaths need to be inflated by the stable growth rate. In the case of the open interval (deaths at age A and over), there is no mid-point, so some alternative procedure is required to include these heavily weighted deaths in the analysis. There are two different problems involved, the first being to decide what age A to select as the lower boundary of the open interval, and the second being to estimate the age point at which the deaths in the interval should be assumed to be concentrated in order to apply the correct growth rate adjustment to them.

The first problem is sometimes resolved by the form in which the data are available. If the tabulations of deaths by age stop at age groups 55-59, 60-64 or 65-69, leaving open-ended intervals beginning at values of A of 60, 65 or 70, respectively, then nothing better can be done than to use the value of A as given. If more age detail is available, the question arises whether to use the A of the basic data or to use a lower value. The advantage of using the highest possible value according to the data tabulations is that the higher the value of A, the smaller the approximation made in estimating the age point at which deaths in the open interval are assumed to be acting. The advantage of using a lower value of A

is that the age errors in death reporting, which probably increase with age, will be reduced. In general, a value of A of 75 may be adopted except in cases where age reporting is extremely good, when values of 80 or 85 can be used. Age 75 is selected because it offers the most advantageous balance between gains obtained by reducing the effects of age errors at very high ages and losses related to the approximation involved in estimating the weight of the deaths in the open interval.

The second problem, that of estimating the weight for the deaths in the open interval, can be solved by using models. If A is the lower boundary of the open interval, the number of people aged A is given by

$$\hat{N}(A) = \sum_{x=A}^{\omega} D(x)\exp(r(x-A)),$$

an expression that cannot be evaluated from the data because the values of $D(x)$ are not available. However, there exists a length of time, $z(A)$, such that

$$D(A+)\exp(rz(A)) = \sum_{x=A}^{\omega} D(x)\exp(r(x-A))$$

so that the number of people aged A can be calculated from the number of deaths over age A, $D(A+)$, as

$$\hat{N}(A) = D(A+)\exp(rz(A)). \qquad \text{(B.3)}$$

Values of $z(A)$ have been calculated for a range of model cases having different growth rates, mortality levels, mortality patterns and values of A. Least-squares regression was used to relate the values of $z(A)$ to two parameters available in any application, namely, the growth rate r and the exponential value of the ratio of deaths over age 45 to deaths over age 10. The coefficients shown in table 123 make it possible to estimate $z(A)$ for values of A ranging from 45 to 85 for each family of the Coale-Demeny model life tables by using the following equation:

$$z(A) = a(A) + b(A)r + c(A)\exp\left[\frac{D(45+)}{D(10+)}\right]. \qquad \text{(B.4)}$$

In any particular application, therefore, the value of $z(A)$ may be estimated from the growth rate, r, as estimated in step 1, and the ratio of deaths at age 45 and over to deaths at age 10 and over, a ratio that can be readily calculated from the distribution of reported deaths by age group. The population aged A can then be estimated using equation (B.3). If no indications exist as to which family of life tables should be used, the West family is a satisfactory default.

Once the value of $\hat{N}(A)$ has been calculated from deaths in the open age interval, the remaining calculations are straightforward. One begins with the open interval and continues downward, using the following recursive equations to calculate $\hat{N}(A-5)$ from $\hat{N}(A)$, $\hat{N}(A-10)$ from $\hat{N}(A-5)$ and so on:

$$\hat{N}(x) = \hat{N}(x+5)\exp(5r) + {}_5D_x\exp(2.5r), \qquad \text{(B.5)}$$

$${}_5\hat{N}_x = 2.5(\hat{N}(x) + \hat{N}(x+5)) \qquad \text{(B.6)}$$

where ${}_5D_x$ is the number of reported deaths in the interval from x to $x+4$.

TABLE 123 COEFFICIENTS[a] FOR ESTIMATION OF THE AGE FACTOR FOR THE OPEN INTERVAL, $z(A)$, FROM THE RATIO OF DEATHS OVER AGE 45 TO DEATHS OVER AGE 10 AND THE POPULATION GROWTH RATE

Regional family (1)	Age A (2)	Coefficient $a(A)$ (3)	Coefficient $b(A)$ (4)	Coefficient $c(A)$ (5)
North	45	−11.42	185.2	17.02
	50	−10.63	167.2	14.99
	55	−9.78	147.8	12.96
	60	−8.57	126.1	10.85
	65	−6.83	101.6	8.62
	70	−4.53	74.6	6.28
	75	−1.91	47.1	3.98
	80	0.46	22.7	2.00
	85	1.82	6.4	0.67
South	45	−15.26	183.4	18.23
	50	−14.91	168.4	16.36
	55	−14.22	151.2	14.38
	60	−12.89	130.8	12.22
	65	−10.67	106.4	9.80
	70	−7.53	78.4	7.15
	75	−3.84	48.8	4.47
	80	−0.47	22.6	2.14
	85	1.47	5.6	0.63
East	45	−15.87	174.3	18.06
	50	−15.14	158.5	16.06
	55	−13.97	140.4	13.93
	60	−12.10	118.8	11.60
	65	−9.43	93.9	9.05
	70	−6.07	66.5	6.38
	75	−2.52	39.3	3.81
	80	0.37	16.8	1.73
	85	1.79	3.5	0.48
West	45	−13.43	181.4	17.57
	50	−12.69	163.6	15.49
	55	−11.24	143.7	13.24
	60	−9.50	121.2	11.07
	65	−7.21	96.1	8.67
	70	−4.48	69.2	6.23
	75	−1.64	42.9	3.91
	80	0.72	20.5	1.98
	85	2.03	5.9	0.70

Note: $z(A)$ is the age that satisfies the relationship

$$D(A+)\exp(rz(A)) = \sum_{x=A}^{\omega} D(x)\exp(r(x-A))$$

and it is estimated from the equation

$$z(A) = a(A) + b(A)r + c(A)\exp\left[\frac{D(45+)}{D(10+)}\right]$$

[a] Based on 11 levels of mortality, with e_0 ranging from about 40 years to about 75 years.

Step 4: estimation of completeness of death registration. The completeness of death registration, C, may be taken as the average level of the sequence of $\hat{N}(x)/N(x)$ values, or of the sequence $\hat{N}(x \text{ to } A)/N(x \text{ to } A)$. Use of the median or the mean of the values over ages where the latter sequence is approximately flat, say, from age 10 to age 45, is suggested. If the plot of the $\hat{N}(x)/N(x)$

ratios shows a generally rising trend with x, suggesting that the growth rate used was too low, or a falling trend with x, suggesting that the growth rate used was too high, the growth rate should be modified accordingly and the calculations repeated. If some test is required to decide which set of points provides the closest approximation to a horizontal line, the absolute deviations of the points, excluding obvious outliers, from their grand mean (excluding again the obvious outliers) could be summed; and that growth rate giving the minimum sum could be accepted. Another procedure would be to divide the points, again excluding outliers, into two groups of equal size (for instance, points for ages from 10 to 35 and from 40 to 65); to calculate the mean value of $\hat{N}(x)/N(x)$ for each group, and to accept the growth rate that minimizes the difference between the two means. If the slope of $\hat{N}(x)/N(x)$ indicates that the growth rate has been misspecified, the following rule of thumb can be used to estimate an adjusted value of r. The effect of an estimate of r that incorporates an error Δr on $\hat{N}(x)$ is to increase $\hat{N}(x)$ by a multiplier $1+(\Delta r)e_x$, where e_x is the expectation of life at age x. Given that e_x does not vary rapidly with the overall level of mortality for advanced values of x, the estimation of e_x for particular applications can be avoided while still obtaining a reasonable first approximation to Δr. If the values of $\hat{N}(x)/N(x)$ for values of x from 20 to 40 and from 40 to 60 are averaged, and the difference between them (a rough indicator of slope) is obtained by subtracting the 20-40 average from the 40-60 average, a first approximation to Δr can be found by dividing the result by 14.5. This value of 14.5 is suitable for an e_{10} of about 55 years; if the true value of e_{10} were 45 years, the value of the denominator should be about 12.7, whereas for 65 it should be 18.4.

Step 5: adjustment of reported death rates for underregistration and calculation of a life table for adults. If the age intervals are, for example, 10-14, ..., 75-79, 80+, then adjusted death rates are calculated in a straightforward manner as follows:

$$ {}_5m_x^{adj} = \frac{{}_5D_x}{C} \cdot \frac{1.0}{{}_5N_x} \quad \text{for } x = 10, \ldots, 75 \qquad (B.7) $$

and

$$ {}_{\infty-80}m_{80}^{adj} = \frac{D(80+)}{C} \cdot \frac{1.0}{N(80+)} \quad \text{for } x = 80. \qquad (B.8) $$

Lastly, the ${}_5m_x^{adj}$ values are converted into life-table ${}_5q_x$ values in the usual manner for the age groups from 10-14 to 75-79:

$$ {}_5q_x = \frac{(5.0)_5m_x^{adj}}{1.0+(2.5)_5m_x^{adj}} \qquad (B.9) $$

Then, using the relation $l(x+5) = l(x)(1.0 - {}_5q_x)$, values of $l(x)$ for ages from 15 to 80 can be determined with a radix $l(10)$ of 1.0 at age 10. If an estimate of $l(10)$ is available from the procedures described in chapter III, it can be employed directly in calculating the full life table. However, some irregularities generally will remain in the sequence of adjusted ${}_5m_x$ values, because of age-misreporting or other errors; and further adjustment and smoothing, for example, by comparison with mortality models, will be desirable. For further details on these types of adjustments, see chapter VI.

4. *First detailed example*

The case of the female population of El Salvador in 1961 is considered first. The deaths registered in that year, classified by age, are shown in table 124, along with the population enumerated at the time of the 1961 census, with a reference date of 5 May 1961.

TABLE 124. FEMALE POPULATION AND DEATHS BY AGE, EL SALVADOR, 1961

Age (1)	Registered female deaths ${}_5D_x^a$ (2)	Reported female population ${}_5N_x$ (3)	Female population aged [years x to 75 (4) to 75] (4)
0	6 909	204 089	1 258 060
5	610	190 234	1 043 971
10	214	149 538	853 737
15	266	125 040	704 199
20	291	113 490	579 159
25	271	91 663	465 669
30	315	77 711	374 006
35	349	72 936	296 295
40	338	56 942	223 359
45	357	46 205	166 417
50	385	38 616	120 212
55	387	26 154	81 596
60	647	29 273	55 442
65	449	14 984	26 169
70	504	11 205	11 205
75	1 360[a]	16 193[a]	-
TOTAL	13 652	1 274 253	

[a] For the open interval 75+.

TABLE 125. INTERCENSAL GROWTH RATES FOR FEMALES, EL SALVADOR, 1950-1971

Age range (1)	Annual growth rate 1950-1961 (2)	1961-1971 (3)	Average (4)
10+	0.0236	0.0330	0.0283
15+	0.0224	0.0307	0.0266
20+	0.0232	0.0289	0.0260
25+	0.0246	0.0287	0.0266
30+	0.0258	0.0290	0.0274
35+	0.0253	0.0298	0.0276
40+	0.0266	0.0307	0.0287
45+	0.0284	0.0309	0.0296
50+	0.0293	0.0311	0.0302
55+	0.0351	0.0326	0.0338
60+	0.0366	0.0327	0.0346

Step 1: estimation of growth rate. Censuses were taken in El Salvador in 1950, 1961 and 1971. From the cumulated population figures, $N(x+)$, growth rates for the population over ages 10, 15, 20,..., 60 were calculated as $r = \ln[N_2(x+)/N_1(x+)]/(t_2 - t_1)$ for each of the two intercensal periods. Results are shown in table 125, along with the averages of the two. If the population

were really stable and all data were accurately recorded, then the growth rates would all be the same. Examination of table 125 reveals that they are not, so some choice must be made. The median of the average values (which roughly correspond to the growth rates in 1961) for ages ranging from 20 to 60, 0.0287, was selected. If this choice is a poor one, the diagnostic plot of $\hat{N}/N$ should reveal the error and an improved choice can be made.

Step 2: adjustment of reported population to mid-point of period. The population figures correspond to the census reference date of 5 May, while the deaths are centred on 1 July. Thus, the ${}_5N_x$ and $N(x$ to $A)$ values are too small by a tiny fraction. The difference in time between the mid-year and 5 May is 56 days, or 0.1534 of a year. Thus, the adjustment factor that should be applied to the population data is $\exp[(0.1534)(0.0287)] = 1.0044$. However, as stated before, the population data need not be adjusted at this stage, since it is simpler to adjust the resulting completeness of death registration (see step 4 below).

Step 3: estimation of population from registered deaths. In this case, the open interval is the age group 75 and over, so A is taken equal to 75. Thus, in order to begin the recursive calculation of $\hat{N}$, an estimate of $z(75)$ is needed.

The rate of growth has been estimated as 0.0287, and the mortality pattern regarded as suitable is West, so all that is required to estimate $z(75)$ is the ratio of deaths at ages 45 and over to deaths at ages 10 and over. Cumulating from the bottom of column (2) of table 124,

$$D(45+) = {}_{\infty}{}_{-15}D_{75} + {}_5D_{70} + \cdots + {}_5D_{45}$$

$$= 1{,}360 + 504 + \cdots + 357 = 4{,}089.$$

The value of $D(10+)$ can be obtained most readily by subtracting deaths under age 10 from all deaths:

$$D(10+) = D(0+) - {}_5D_0 - {}_5D_5$$

$$= 13{,}652 - 6{,}909 - 610 = 6{,}133.$$

The ratio is then calculated:

$$D(45+)/D(10+) = 4{,}089/6{,}133 = 0.6667:$$

and its exponential value found:

$$\exp[D(45+)/D(10+)] = \exp(0.6667) = 1.948.$$

Using the coefficients from table 123 in equation (B.4),

$$z(75) = a(75) + b(75)(0.0287) + c(75)(1.948)$$

$$= -1.64 + (42.9)(0.0287) + (3.91)(1.948) = 7.21.$$

$\hat{N}(75)$ is then obtained by applying equation (B.3):

$$\hat{N}(75) = D(75+)\exp[rz(75)] =$$

$$(1{,}360)\exp[(0.0287)(7.21)]$$

$$= (1{,}360)(1.2299) = 1{,}672.9$$

Then $\hat{N}(70)$ is computed using the recursive equation (B.5):

$$\hat{N}(70) = \hat{N}(75)\exp(5r) + {}_5D_{70}\exp(2.5r)$$

$$= (1{,}672.9)\exp[(5.0)(0.0287)]$$

$$+ (504)\exp[(2.5)(0.0287)]$$

$$= 1{,}931.0 + 541.5 = 2{,}472.5.$$

TABLE 126 VALUES OF REPORTED DEATHS, ESTIMATED POPULATION FOR DIFFERENT AGES AND RATIOS OF ESTIMATED TO REPORTED POPULATION, FEMALES, EL SALVADOR, 1961

Age x (1)	Reported deaths ${}_5D_x$ (2)	Estimated population $\hat{N}(x)$ (3)	Estimated population ${}_5\hat{N}_x$ (4)	Estimated population $\hat{N}(x$ to $75)$ (5)	Ratio of estimated to reported population ${}_5\hat{N}_x/{}_5N_x$ (6)	Ratio of estimated to reported population $\hat{N}(x$ to $75)/N(x$ to $75)$ (7)	Ratio of estimated to reported population $\hat{N}(x$ to $60)/N(x$ to $60)$ (8)
75	1 360[a]	1 672.9	-	-	-	-	-
70	504	2 472.5	10 363.5	10 364	0.925	0.925	-
65	449	3 338.4	14 522.3	24 886	0.970	0.951	-
60	647	4 546.4	19 707.0	44 593	0.873	0.804	-
55	387	5 663.7	25 525.3	70 118	0.976	0.900	0.992
50	385	6 951.3	31 537.5	101 656	0.817	0.847	0.895
45	357	8 407.5	38 397.0	140 053	0.831	0.842	0.873
40	336	10 068.0	46 188.8	186 241	0.811	0.832	0.856
35	349	11 996.5	55 161.3	241 403	0.756	0.815	0.829
30	315	14 186.1	63 456.5	306 860	0.842	0.820	0.835
25	271	16 686.3	77 131.0	383 991	0.841	0.825	0.839
20	291	19 530.7	90 542.5	479 533	0.798	0.819	0.832
15	266	22 853.3	106 010.0	580 543	0.848	0.824	0.837
10	214	26 609.6	123 657.3	704 200	0.827	0.825	0.837
5	610	31 371.0	144 951.5	849 152	0.762	0.813	0.824

[a] For the open-ended age group 75 and over

Estimates of $\hat{N}(x)$ for all the required values of x are shown in column (3) of table 126. Next, ${}_5\hat{N}_{70}$ is calculated as

$${}_5\hat{N}_{70} = 2.5(\hat{N}(75) + \hat{N}(70)) = 2.5(1,672.9 + 2,472.5) = 10,363.5$$

and $\hat{N}(x \text{ to } 75)$ is calculated by cumulation. The complete sets of estimated ${}_5\hat{N}_x$ and $\hat{N}(x \text{ to } 75)$ values are shown in columns (4) and (5) of table 126.

Step 4: estimation of completeness of death registration. Values of ${}_5\hat{N}_x/{}_5N_x$ and $\hat{N}(x \text{ to } 75)/N(x \text{ to } 75)$ are shown in columns (6) and (7) of table 126 (the denominators of the ratios are found in columns (3) and (4) of table 124); both sets of values are displayed in figure 12. To illustrate the stability of the ratios $\hat{N}(x \text{ to } A)/N(x \text{ to } A)$ with respect to A, their values using an open interval of 60+ are also shown in table 126. Although they are slightly higher than those obtained using as open interval 75+, the differences observed are small. Because, as figure 12 shows, the sequence of $\hat{N}(x \text{ to } 75)/N(x \text{ to } 75)$ ratios is approximately flat over the age range from 5 to 60, their median, 0.825, is selected as an unadjusted estimate of C, the completeness of death registration. Adjustment of C is necessary to make allowance for the difference existing between the date of enumeration of the population and the mid-year (see step 2). Thus, the adjusted completeness of death registration, $\hat{C}$, is

$$\hat{C} = 0.825/1.0044 = 0.821$$

Because in this case the difference in the dates mentioned above is relatively small, the difference between the adjusted and unadjusted C is minimal, and for all practical purposes, adjustment can by bypassed. However, it is important to remember its theoretical necessity in cases where the date difference is large.

Step 5: adjustment of reported death rates. The age-specific mortality rates can be adjusted in the straightforward manner by using equation (B.7) with the unadjusted estimate of C. For example, the adjusted death rate for age group 50-54 is given by

$${}_5m_{50}^{adj} = \frac{{}_5D_{50}}{\hat{C}({}_5N_{50})} = \frac{385}{(0.825)(38,616)} = 0.0121.$$

5. *Second detailed example*

The second example, using data from Andra Pradesh State in India, illustrates how sets of $\hat{N}/N$ ratios can be used to estimate the completeness of death registration when the sequence of ratios is more irregular than in the case of El Salvador. The details of the calculations are not presented, however, since they are similar to those of the first example.

Step 1: estimation of growth rate. In Andra Pradesh, the male population aged 10 and over increased from 13,137,000 on 1 March 1961 to 15,806,000 on 1 April 1971. The intercensal period was thus 10 years and 31 days, or 10.085 years, so the intercensal growth rate of the population aged 10 and over, $r(10+)$, is given by

$$r(10+) = [\ln(15.806) - \ln(13.137)]/10.085 = 0.0183.$$

Step 2: adjustments to basic data. The basic data (for males) on deaths by age and population by age for 1970 and 1971 come from the Indian Sample Registration

Figure 12. Plot of ratios of estimated to reported female population, El Salvador, 1961

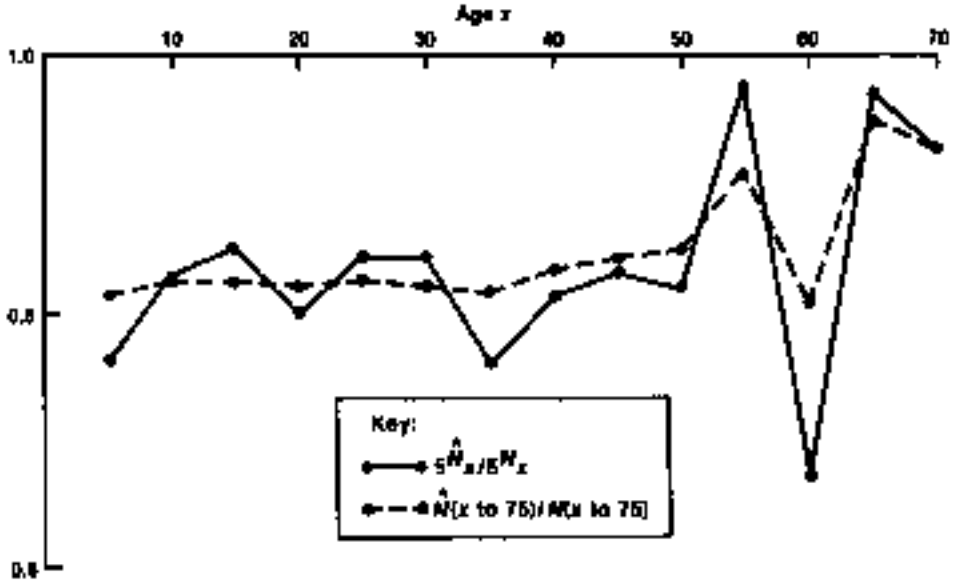


TABLE 127. AGE DISTRIBUTION OF MALE POPULATION, CENTRAL DEATH RATES AND DERIVED NUMBERS OF DEATHS, BY AGE GROUP, ANDHRA PRADESH STATE, INDIA, 1970 AND 1971

Age group (1)	1970			1971			Total number of deaths (8)	Total population (9)
	Male population (2)	Central death rate (3)	Number of deaths (4)	Male population (5)	Central death rate (6)	Number of deaths (7)		
0-4	1 534	0.0462	70.87	1 355	0.0447	60.57	131.44	2 889
5-9	1 388	0.0049	6 80	1 416	0.0050	7.08	13 88	2 804
10-14	1 211	0.0020	2.42	1 230	0.0023	2.83	5.25	2 441
15-19	871	0.0027	2.35	1 008	0.0021	2.12	4.47	1 879
20-24	735	0.0043	3.16	728	0.0035	2.55	5.71	1 463
25-29	696	0.0060	4.18	708	0.0069	4.89	9.07	1 404
30-34	682	0.0044	3.00	673	0.0041	2.76	5.76	1 355
35-39	620	0.0070	4.34	627	0.0064	4.01	8.35	1 247
40-44	608	0.0105	6.38	591	0.0100	5.91	12.29	1 199
45-49	458	0.0125	5.73	467	0.0133	6.21	11.94	925
50-54	425	0.0201	8.54	423	0.0266	11.25	19.79	848
55-59	245	0.0283	6.98	240	0.0302	7.25	14.23	485
60-64	258	0.0993	12.72	247	0.0403	9.95	22.67	505
65-69	111	0.0607	6.74	143	0.0646	9.24	15.98	254
70+	158	0.1221	19.29	144	0.1279	18.42	37.71	302
TOTAL	10 000			10 000			318.52	20 000
Deaths at age 45+							122.32	
Deaths at age 10+							173.22	

System and are published in the form of the age distribution of the sampled population per 10,000 population of all ages and of central death rates, ${}_5m_x$. Data by five-year age group are available up to age group 65-69, the open interval being 70 and over. No adjustments are required for the population data, which are intended to represent person-years of exposure, but deaths by age group must be obtained by multiplying the central death rates, ${}_5m_x$, by the population exposed to the risk of dying, ${}_5N_x$. The calculations are carried out separately for 1970 and 1971 in table 127. Total deaths and total population for the period 1970-1971 are then obtained by summing the figures for individual years, the results being shown in columns (8) and (9) of table 127.

Step 3: estimation of population from reported deaths. The ratio of $D(45+)/D(10+)$ is calculated from table 127 as (122.32)/(173.22), or 0.7062. The value of its exponential is therefore exp (0.7062) = 2.0263. Assuming that mortality in India conforms to a South pattern, $x(70)$ is estimated as

$$x(70) = -7.53 + (78.4)(0.0183) + (7.15)(2.0263)$$

$$= 8.393.$$

The value of $\hat{N}(70)$ is then obtained as

$$\hat{N}(70) = D(70+)\exp[(8.393)(0.0183)]$$

$$= (37.71)(1.1660) = 43.97.$$

Full results are shown in table 128. The ${}_5\hat{N}_x/{}_5N_x$ and $\hat{N}(x \text{ to } 70)/N(x \text{ to } 70)$ ratios are plotted in figure 13.

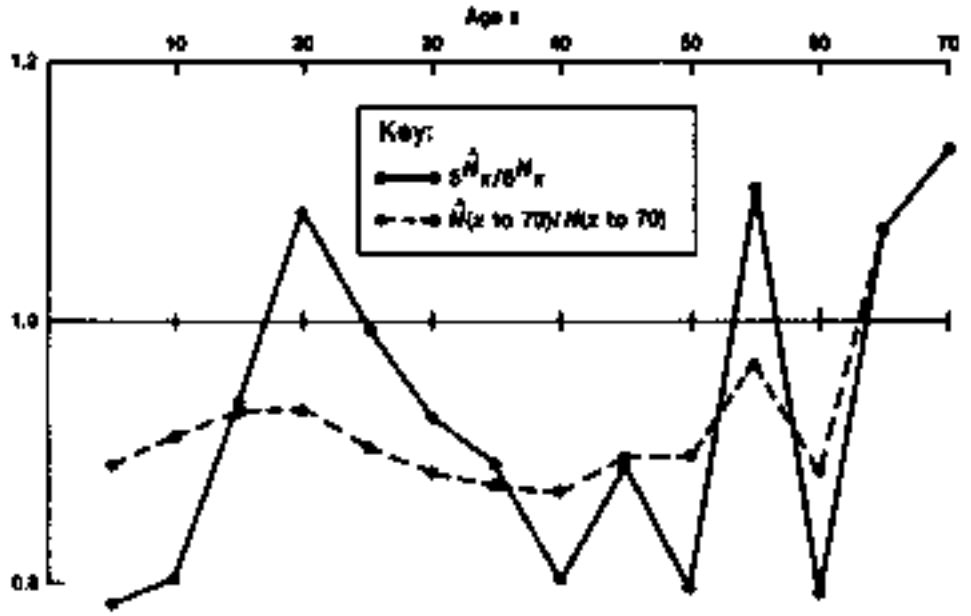
The plot of ${}_5\hat{N}_x/{}_5N_x$ rises from a value below 0.8 at

TABLE 128. VALUES OF REPORTED DEATHS, ESTIMATED POPULATION FOR DIFFERENT AGES AND RATIOS OF ESTIMATED TO REPORTED POPULATION, MALES, ANDHRA PRADESH STATE, INDIA, 1970-1971

Age x (1)	Reported deaths ${}_5D_x$ (2)	Reported population		Estimated population			Ratio of estimated to reported population	
		${}_5N_x$ (3)	$N(x \text{ to } 70)$ (4)	$\hat{N}(x)$ (5)	${}_5\hat{N}_x$ (6)	$\hat{N}(x \text{ to } 70)$ (7)	${}_5\hat{N}_x/{}_5N_x$ (8)	$\hat{N}(x \text{ to } 70)/N(x \text{ to } 70)$ (9)
70	37.71	302	-	43.97	-	-	-	-
65	15.98	254	254	64.91	272.20	272.2	1.072	1.072
60	22.67	505	759	94.86	399.42	671.6	0.791	0.885
55	14.23	485	1 244	118.85	534.28	1 205.9	1.102	0.969
50	19.79	848	2 092	150.95	674 50	1 880.4	0.795	0.899
45	11.94	925	3 017	177.91	822.15	2 702.6	0.889	0.896
40	12 29	1 199	4 216	207.82	984 33	3 666.9	0.804	0.870
35	8 35	1 247	5 463	236.47	1 110.73	4 777.6	0.891	0.875
30	5.76	1 355	6 818	265.16	1 254.08	6 031.7	0.926	0.885
25	9.07	1 404	8 222	300.06	1 413.05	7 444.7	0.994	0.905
20	5.71	1 463	9 685	334 79	1 587.13	9 031.9	1.085	0.933
15	4.47	1 879	11 564	371.55	1 765.85	10 797.7	0.940	0.934
10	5.25	2 441	14 005	412.65	1 960.50	12 758.2	0.803	0.911
5	13.88	2 804	16 809	466.72	2 198.43	14 956.6	0.784	0.890
0	131.44	2 889	19 698	649.03	(2 789.38)	(17 746.0)	(0.966)	(0.901)

age 5 to a value over 1.0 at age 20, then falls to 0.80 at age 40, remaining between 0.79 and 0.89 from ages 35 to 60, except for age 55. The probable explanation of this up-and-down movement is the deviation of the reported age distribution from a stable form, which, although attributable to a true lack of stability is more likely to be the result of differential omission, or of the tendency to misstate age. It should be borne in mind that the construction of the estimated population ${}_5\hat{N}_x$ from the recorded distribution of deaths and the growth rate, being cumulative in form, tends to follow a stable age distribution quite closely. Thus, if there are segments of the age distribution that are undercounted in the census or survey, the value of ${}_5\hat{N}_x/{}_5N_x$ will reach a peak at these points; and where, through age-misreporting, the population is over-enumerated, ${}_5\hat{N}_x/{}_5N_x$ will show a minimum. The high point of ${}_5\hat{N}_x/{}_5N_x$ at age 20 is almost certainly the result of an undercount of the cohort aged 20-24. The cumulated values, $\hat{N}(x \text{ to } 70)/N(x \text{ to } 70)$, show no general trend with age; and the median value of 0.896 can be accepted as a sensible estimate of completeness of death registration.

Figure 13. Plot of ratios of estimated to reported male population, Andra Pradesh State, India, 1970-1971



It is likely that the age-misreporting that appears to have distorted the reported age distribution of the population also produced an inexact reported age distribution of deaths. Thus, the life table obtained from the central death rates, ${}_5m_x$, adjusted for underreporting of deaths by a factor of about 1.0/0.9, might represent the overall level of mortality moderately well, but the detailed structure of the resultant schedule of mortality rates and the accompanying life table would probably be distorted. It is therefore preferable to calculate the expectation of life at age 10 in this life table and to utilize a model life table with the same life expectancy as perhaps a more trustworthy representation of mortality.

C. Brass growth balance method

1. *Basis of method and its rationale*

In 1975, Brass[3] proposed a method to estimate the completeness of registration of adult deaths. This method is based on the equation:

$$N(x)/N(x+) = r + D^*(x+)/N(x+) \qquad \text{(C.1)}$$

where $N(x)$ is the number of persons of exact age x; $N(x+)$ is the total number of persons aged x and over; $D^*(x+)$ is the total number of deaths occurring to persons aged x and over; and r is the growth rate.

Brass proved that equation (C.1) is exact for stable, closed populations. Its validity can be explained in common-sense terms by the following argument: since $N(x)$ may be thought of as being the number of persons in a year entering the group of those aged x and over, the ratio $N(x)/N(x+)$ can be interpreted as a "birth rate" for the population aged x and over. $D^*(x+)/N(x+)$ is the death rate corresponding to the same population; and, if one denotes by $r(x+)$ the growth rate for the population aged x and over, the equation

$$N(x)/N(x+) = r(x+) + D^*(x+)/N(x+) \qquad \text{(C.2)}$$

[3] *Ibid.*

simply states the familiar truism that in a closed population the birth rate is equal to the sum of the growth rate and the death rate. Now invoking the assumption of stability, it is the case that in a stable population the growth rate $r(x+)$ is, by definition, the same for every x; and, therefore, $r(x+)$ can be replaced by r, and equation (C.2) becomes equation (C.1), that proposed by Brass.

Once the validity of equation (C.1) is established by assuming stability, a second assumption is made. Suppose that instead of observing $D^*(x+)$ (the total number of deaths over age x), only a proportion of them was recorded, say $D(x+)$, where

$$D(x+) = C(x)D^*(x+), \qquad \text{(C.3)}$$

$C(x)$ being a factor representing the completeness of registration of deaths at age x and over.

If it is then assumed that the completeness of death registration does not vary with age, at least over age 5 or age 10, $C(x)$ can be replaced by a constant C that does not change with age. Letting $K = 1/C$ and substituting equation (C.3) in equation (C.1), the following relationship is obtained.

$$N(x)/N(x+) = r + K(D(x+)/N(x+)). \qquad \text{(C.4)}$$

For a closed, stable population, where the completeness of death registration is the same at every age and where age-reporting is accurate, equation (C.4) provides a method by which to estimate the completeness of death registration. According to equation (C.4), the relationship between $D(x+)/N(x+)$ and $N(x)/N(x+)$ is linear; and the slope of the line defined by the points $[D(x+)/N(x+),\ N(x)/N(x+)]$ is the value of the adjustment factor K. Hence, to estimate K, one needs only to find the slope of the line defined by the points $[D(x+)/N(x+),\ N(x)/N(x+)]$. Note, however, that no explicit allowance is being made for age-misreporting, whether of age at death or of age of the living, though the use of cumulation is likely to smooth out some of the effects of age errors. It is fair to say, therefore, that whenever the age structure of deaths or of the population is distorted by poor reporting, the estimates yielded by this method may be biased, although the use of cumulation and judicious elimination of points from the fitting procedure will reduce the effects of such errors.

In practice, the points $[D(x+)/N(x+),\ N(x+)/N(x+)]$ seldom fall exactly on a straight line, and K is obtained by selecting the line that best fits the observed points. In some cases, however, the deviations of these points from a linear trend are so marked that the use of this method of estimation is unwarranted. Large deviations from linearity may be due to several causes. The most common cause is the inaccuracy of the data (usually the existence of age-misreporting), but lack of linearity may also be due to differential completeness of death registration by age or to the lack of stability of the population considered (in this case, $r(x+)$ would not be constant for every x). So, in general, a plot of the observed values $D(x+)/N(x+)$ against $N(x)/N(x+)$ provides a good diagnostic tool to assess the validity of using this method to estimate K. Unfortunately, when the points are markedly non-linear it is not generally possible to identify a unique cause for this lack of linearity, nor to adjust the data in such a way as to make the use of this method of estimation possible. If, however, the deviation from approximate linearity is confined to points for the elderly, a straight line can be fitted excluding the deviant points and therefore avoiding serious distortion.

It is important to notice that the relationship expressed by equation (C.4) allows the estimation both of K, the adjustment factor for registered deaths, and of r, the growth rate of the population. This estimate of r is not robust to deviations from the hypotheses under which equation (C.4) was derived, and the user should not regard it as necessarily sound. However, the estimate of r may be compared with estimates obtained from other sources, and reasonable agreement may be interpreted as some confirmation that the assumptions being made are justified.

When a solid, independent estimate of r is available for a stable population where age-misreporting is not prevalent, equation (C.4) may be modified in the following way:

$$N(x)/N(x+) = r + K(x)D(x+)/N(x+) \qquad \text{(C.5)}$$

where the adjustment factor K is no longer a constant, but is allowed to change with age. The assumption of equal underenumeration of deaths at all ages is, therefore, being dropped; and, since r is assumed to be known, equation (C.5) can be rewritten in the following form to allow the estimation of an adjustment factor, $K(x)$, for each open-ended age group:

$$K(x) = (N(x) - rN(x+))/D(x+). \qquad \text{(C.6)}$$

Sadly, equation (C.6) can seldom be used in practice because it is uncommon to find cases of stable populations where age-reporting is fairly accurate and where $K(x)$ would therefore measure differential underregistration of deaths by age rather than differential age-misreporting.

To conclude this discussion, some comments need to be made on the validity of estimates obtained by this method when the population being studied is not stable. A stable population is established when fertility and mortality have both remained constant for a fairly long period of time. In today's world, few, if any, populations are truly stable. In most cases, destabilization has been brought about by changes in mortality. Most human populations have seen their chances of survival improve during the past 40 years and cannot, therefore, be stable. However, simulation has shown that when stable populations are destabilized by prolonged mortality changes that occur slowly, the bias introduced in the estimation of K by this lack of stability is relatively

small.[4] Only when abrupt changes occur does the bias become large.[5] Of course, these simulations have been carried out using data that, apart from representing a non-stable population, are otherwise perfect. It is possible, therefore, that in reality even the biases introduced by rapid mortality changes may be small compared with those caused by the poor quality of the available data. However, it is important to remember that declining mortality usually biases the value of K upward, thus producing an underestimate of the completeness of death registration. Therefore, when this method is used to estimate completeness in a population where mortality has been declining and where age is reported accurately, the death-rate estimate obtained may safely be interpreted as an upper bound for the true rate.

The effect that changes in fertility have on the estimation of K has not been as extensively studied as the effects of changing mortality. The reason is that fertility changes have only recently become common in countries with poor data; and because these changes affect mainly the youngest age groups, they have little impact on the performance of this method of estimation.

2. *Data required*

The data required for this method are described below:

(a) The number of deaths occurring during a period (normally one year) classified by age and sex (though classification by sex is not necessary). Five-year age groups are adequate. The last age group must be open-ended; that is, it must include all the deaths at or over some age A;

(b) The number of persons in each age group at the mid-point of the period being considered, classified by sex if the deaths are so classified. The age groups used must correspond to those used for deaths. The last age group must once again be open, including all the population aged A and over.

3. *Computational procedure*

Two variants of the Brass method have been proposed, the difference lying essentially in the way in which the left-hand side of equation (C.4) is evaluated. One procedure, and that originally proposed, is to attempt to estimate $N(x)$, the number of persons reaching age x in a year, from the number of persons in the five-year age groups adjacent to x; the second is to estimate $N(x)$ for the mid-point of a five-year age group. The latter procedure is theoretically preferable, because the non-linearity of the age distribution is less marked over a five-year age span than over a 10-year span. However, for a population in which there is more than a trivial amount of age-heaping, the theoretical advantage of the second procedure is outweighed by the practical advantage of the first, that is, that the effects of heaping are reduced when $N(x)$ is estimated from two adjacent five-year age groups, one of which is likely to be inflated (the one including an age ending in zero) and one of which is likely to be deflated. Numerous applications have shown that the first procedure yields results that are more easily interpreted. Hence, only this procedure to estimate $N(x)$ is described here. The steps of the complete computational procedure are given below.

Step 1: the person-years lived by the population subject to the risk of dying. This step, which consists of adjusting the reported population to the mid-point of the period for which the death data are available, is exactly the same as step 2 of the Preston-Coale procedure. It need not, therefore, be repeated, especially since it can generally be omitted, in accordance with the discussion under step 2 in subsection B.3.

Step 2: calculation of population at an exact age. By definition, $N(x)$ is the number of persons who reach age x during the course of the year under consideration. When the total population for the *mid-year* is classified by single year of age, $N(x)$ can be estimated from the number of persons in the two contiguous age groups, ${}_1N_{x-1}$ and ${}_1N_x$. Thus, if one lets ${}_1N_x$ be the number of persons of age x enumerated by the census or survey,

$$N(x) = ({}_1N_{x-1} + {}_1N_x)/2.0. \qquad \text{(C.7)}$$

If the classification by age is made in five-year age groups and ${}_5N_x$ is the number of persons in the age group from x to $x+4$ at the time of the census or survey, $N(x)$ can be estimated as

$$N(x) = ({}_5N_{x-5} + {}_5N_x)/10.0. \qquad \text{(C.8)}$$

In general, equation (C.8) is preferable because it contributes to the reduction of the effects of age-heaping.

Step 3: calculation of population over an exact age. The total number of persons aged x and over is denoted by $N(x+)$. Thus, when the data are available for five-year age groups,

$$N(x+) = \sum_{i=x}^{A-5} {}_5N_i + N(A+) \qquad \text{(C.9)}$$

where $N(A+)$ represents the number of persons in the last, open-ended age interval. If the data are available only by single year of age, then

$$N(x+) = \sum_{y=x}^{A-1} {}_1N_y + N(A+). \qquad \text{(C.10)}$$

Note that there is no advantage in calculating $N(x+)$ from single-year rather than grouped data, unless the entire analysis is to be carried out by single years of age, a procedure that is likely to be excessively tedious and more unreliable when any degree of age-heaping is present.

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[4] Hoda M. Rashad, "The estimation of adult mortality from defective registration", unpublished doctoral dissertation, University of London, 1978.

[5] Linda G. Martin, "A modification for use in destabilized populations of Brass's technique for estimating completeness of birth registration", *Population Studies*, vol. XXXIV, No. 2 (July 1980), pp. 381-395.

Step 4: calculation of number of deaths after an exact age. The calculation of the cumulated number of deaths from age x onward is very similar to that of $N(x+)$. $D(x+)$ is just the total number of deaths recorded as having occurred to persons aged x and over. Thus, if the deaths are classified by five-year age group,

$$N(x+) = \sum_{j=x}^{A-5} {}_5D_j + D(A+) \qquad \text{(C.11)}$$

where $D(A+)$ denotes the deaths in the open-ended age interval A and over. If the data are only available by single year, or it is wished for some reason to carry out the analysis by single years of age, then

$$D(x+) = \sum_{y=x}^{A-1} {}_1D_y + D(A+). \qquad \text{(C.12)}$$

Step 5: points defined by partial death and birth rates. Using the quantities calculated in the previous steps, the calculation of the values $D(x+)/N(x+)$ and $N(x)/N(x+)$ is straightforward. Once calculated, they should be displayed graphically, plotting the values of $D(x+)/N(x+)$ on the x-axis and those of $N(x)/N(x+)$ on the y-axis. The same scale should be used on both axes. The plotted points should ideally follow a linear trend. If it is clear that the points do lie roughly on a straight line, the next step can be performed with some confidence. Otherwise, the use of this estimation method may have to be abandoned.

Step 6: selection of a best fitting line. There are several ways of fitting a straight line to a series of points. Probably the most widely used method is that known by the name of "least-squares". The least-squares line is that which minimizes the sum of the squared distances between its points and the points to be approximated. In the sense that it approximates all these points as closely as possible, it provides a best fit. If the data points derived in step 5 were affected only by random errors, this line would certainly be that chosen to estimate the adjustment factor K. However, given the inaccuracies typically present in the data sets considered here, the use of other methods of fitting appears to be preferable. Two of the simplest are suggested below:

(a) *The "mean" line.* This line is defined by the means of the abscissae (horizontal axis values) and ordinates (vertical axis values) of the derived points when those points are divided, according to age, into two groups of approximately equal size. For example, if 15 age groups are used, the values of $D(x+)/N(x+)$ for the first eight values of x are averaged, as are the values for the last eight x values. (The middle point is included in both averages.) Similarly, mean values are obtained for the first eight and last eight values of $N(x)/N(x+)$. The desired line is that which passes through the two points defined by the two pairs of mean co-ordinates. This line will not be as near as possible to all the points derived in step 5, but it will closely approximate their general trend when this trend is linear. One possible disadvantage of this fitting procedure is that it gives equal weight to all the derived points. In practice, it is often the case that one or two of these points (usually at the two extremes of the age range) depart markedly from the linear trend followed by the rest. In such cases, it may be better to ignore them altogether than to take them into account in selecting a representative straight line;

(b) *The "robust" line.* To deal with such cases as that described above, where linearity is apparent in the middle of the age range but where distortions are present at the extremes, it is recommended that trimmed means be used in fitting the mean line, instead of using the usual means. The line fitted by using trimmed means is called a "robust" line because trimming makes the fit less sensitive to large deviations from linearity at the extremes of the age range. Trimmed means are a generalization of the usual mean or average, which is calculated by giving equal weight to each entry. When trimming is performed, the weights applied to different observations are not constant for all observations; the sum of weights times observations is standardized by dividing it by the sum of the weights used. The procedure for computing trimmed means is relatively simple and is fully described in the detailed examples.

Step 7: adjustment of death rate. Once the straight line that best fits the points derived in step 5 is selected, the values of the adjustment factor, K, and of the growth rate, r, are given by its slope and y-intercept, respectively. The value of the completeness of death registration, C, is just the reciprocal of K, that is, $C = 1.0/K$, provided all the necessary data adjustments were carried out in step 1. If the value of r is plausible, one can attach greater confidence to C, the estimate of completeness. In such a case, an adjusted death rate can be obtained by multiplying the reported death rate by K. Note, however, that the trend of the points $[N(x)/N(x+), D(x+)/N(x+)]$ is determined mainly by those corresponding to fairly advanced ages, so that effectively the level of completeness estimated refers only to adult deaths. Therefore, the value of K should be regarded as an adjustment factor only for the death rate over age 5 or 10, and may not be applicable to the death rate of the population as a whole.

The value of K may also be used to adjust the derived age-specific death rates, ${}_5m_x$. This procedure is valid only if the assumption of constant completeness of death registration by age holds true. The adjusted ${}_5m_x$ values can then be used, together with independent estimates of child mortality (see chapter III), to construct a life table for the population considered. In practice, this life table is seldom constructed directly from the adjusted ${}_5m_x$ values since age-misreporting makes them too unreliable; however, they can be used as a basis for fitting a mortality model from which the final life table will be derived. For further details of this procedure, see chapter VI.

4. *First detailed example*

In this example, the case of the female population of El Salvador in 1961 is again analysed. The numbers of

female deaths and women classified by age have already been presented in table 124 in the previous detailed example. Step 1 is omitted here, as adjustment for the census date is not necessary at this point.

Step 2: calculation of population at an exact age. The value of $N(x)$, the number of persons of exact age x, is obtained by adding the number of persons in two contiguous age groups and dividing by the number of years spanned by those age groups (usually 10). In this case, $N(10)$, for example, is obtained as follows:

$$N(10) = (190{,}234 + 149{,}538)/10 = 33{,}977.2.$$

Column (2) of table 129 shows the values obtained for all values of x up to 70; no value can be computed for 75, because the open age interval begins at that age.

TABLE 129. ELEMENTS NEEDED TO ESTIMATE COMPLETENESS OF DEATH REGISTRATION AMONG ADULT FEMALES, EL SALVADOR, 1961

Age x (1)	Population at exact age x $N(x)$ (2)	Population aged x and over $N(x+)$ (3)	Reported deaths (exact age) $D(x)$ (4)
5	40 432.3	1 060 164	6 743
10	33 977.2	869 930	6 133
15	27 457.8	720 392	5 919
20	23 853.0	595 352	5 653
25	20 515.3	481 862	5 362
30	16 937.4	390 199	5 091
35	15 064.7	312 488	4 776
40	12 987.8	239 552	4 427
45	10 314.7	182 610	4 089
50	8 482.1	136 405	3 732
55	6 477.0	97 789	3 347
60	5 542.7	71 635	2 960
65	4 423.7	42 362	2 313
70	2 616.9	27 398	1 864

Step 3: calculation of population over an exact age. The number of persons aged x and over, denoted by $N(x+)$, is computed by cumulating the numbers reported in each age group (shown in table 124) from that beginning with age x upward. For example, $N(50+)$ is

$$N(50+) = (38{,}616 + 26{,}154 + 29{,}273 + 14{,}964 + 11{,}205 + 16{,}193) = 136{,}405.$$

All values of $N(x+)$ are given in column (3) of table 129. Note that in practice one calculates $N(x+)$ by cumulating downward, each time adding the population of one more five-year age group to $N((x+5)+)$, so $N(50+) = N(55+) + {}_5N_{50}$, for example.

Step 4: calculation of number of deaths after an exact age. The calculation of $D(x+)$ is exactly analogous to that of $N(x+)$. Thus, for example, $D(50+)$ is

$$D(50+) = (385 + 387 + 647 + 449 + 504 + 1{,}360) = 3{,}732$$

where the deaths reported in each age group have been taken from table 124. Also, it is important to point out that to calculate both $N(x+)$ and $D(x+)$, the number of persons whose age was recorded as unknown is ignored. Since these numbers are generally small, serious biases are not likely to be introduced by disregarding them.

Step 5: points defined by partial birth and death rates. Using the values obtained in the previous steps, the ratios $D(x+)/N(x+)$ and $N(x)/N(x+)$ are calculated. Their values are shown in table 130. The calculation of the partial birth rate, $N(x)/N(x+)$, involves simply dividing the entries listed in column (2) of table 129 by the corresponding entries in column (3) of the same table. For example,

$$N(40)/N(40+) = 12{,}987.8/239.552 = 0.0542.$$

Similarly, the calculation of the partial death rate, $D(x+)/N(x+)$, involves the division of the entries in column (4) by those in column (3) of table 129. Thus, for example,

$$D(40+)/N(40+) = 4{,}427/239.552 = 0.0185.$$

Table 130 gives results for other ages and figure 14 shows a plot of the points. Note that the values of $D(x+)/N(x+)$ are plotted on the x-axis (abscissa), while those of $N(x)/N(x+)$ appear on the y-axis (ordinate). The same scale has been used to plot both values. The use of equal scales for both axes aids in the assessment of linearity of the plotted points.

TABLE 130. PARTIAL DEATH AND BIRTH RATES FOR FEMALES, EL SALVADOR, 1961

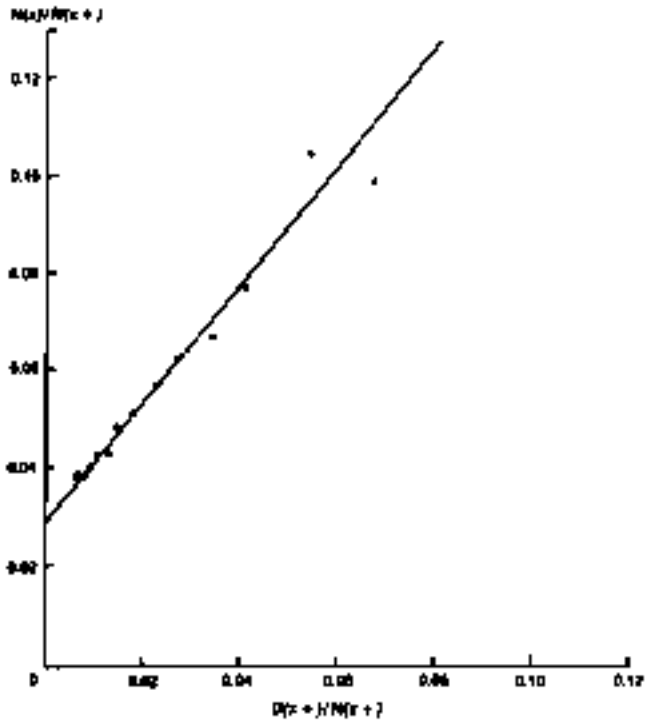
Age x (1)	Partial death rate $D(x+)/N(x+)$ (2)	Partial birth rate $N(x)/N(x+)$ (3)
5	0.0064	0.0381
10	0.0070	0.0391
15	0.0082	0.0381
20	0.0095	0.0401
25	0.0111	0.0426
30	0.0130	0.0434
35	0.0153	0.0482
40	0.0185	0.0542
45	0.0228	0.0565
50	0.0274	0.0622
55	0.0342	0.0662
60	0.0415	0.0774
65	0.0546	0.1044
70	0.0680	0.0955

It is important at this point to examine figure 14 with some care. Ideally, as explained in the general introduction to this method, the points should lie approximately on a straight line. Deviations from linearity are, however, inevitably present in empirical data. The possible causes of such deviations are varied and the investigator should consider all the relevant ones. In the case of females in El Salvador in 1961, the points show a definite linear trend, though the last point (for age 70) deviates markedly from the linear trend suggested by the others. As for the strange pattern displayed by the first three points, it is frequently encountered in empirical

data and is associated with distortions of the values of $N(x)$ brought about by age-misreporting.

Examination reveals that the data at hand are far from perfect. Aside from the possible lack of stability of the population being analysed, errors in age-reporting are undoubtedly present. Yet, the relative linearity of the central segment of the plotted points still warrants an attempt to obtain a rough estimate of the completeness of death registration. This parameter is calculated in the next step.

Figure 14. Plot of partial birth rates, $N(x)/N(x+)$, against partial death rates, $D(x+)/N(x+)$, for females, El Salvador, 1961



Step 6: selection of a best fitting line. An examination of the points shown in figure 14 suggests that all of them, except the last two (those for age 65 and over and age 70 and over), lie around an approximate straight line. A straight line is therefore fitted to the remaining 12 points, from ages 5 to 60. The points are divided into two equally sized groups, one comprising points for ages ranging from 5 to 30, the other from 35 to 60. The mean abscissa and ordinate values for each group are calculated by summing the observations for each group and dividing the sum by the number of observations in each group. Thus, the mean of the first group of partial death rates $D(x+)/N(x+)$, denoted by X_1, is

$$X_1 = (0.0064 + 0.0070 + 0.0082 + 0.0095 +$$

$$0.0111 + 0.0130)/6.0 = 0.0092.$$

The groups and their means are shown in table 131.

TABLE 131. FITTING OF A STRAIGHT LINE BY GROUP MEANS. FEMALES, EL SALVADOR, 1961

Age x (1)	Partial death rate $D(x+)/N(x+)$ (2)	Partial birth rate $N(x)/N(x+)$ (3)
	(a) *Group 1*	
5	0.0064	0.0381
10	0.0070	0.0391
15	0.0082	0.0381
20	0.0095	0.0401
25	0.0111	0.0426
30	0.0130	0.0434
TOTAL	0.0552	0.2414
MEAN	$X_1 = 0.0092$	$Y_1 = 0.0402$
	(b) *Group 2*	
35	0.0153	0.0482
40	0.0185	0.0542
45	0.0224	0.0565
50	0.0274	0.0622
55	0.0342	0.0662
60	0.0413	0.0774
TOTAL	0.1591	0.3647
MEAN	$X_2 = 0.0265$	$Y_2 = 0.0608$

Once the means of each group have been computed, the slope of the fitted line is calculated according to the equation

$$K = (Y_2 - Y_1)/(X_2 - X_1) = (0.0608 - 0.0402)/$$

$$(0.0265 - 0.0092) = 1.191.$$

The value of K implies that the completeness of death registration is $C = 1.0/K = 0.840$, or 84 per cent, and taking into account the slight adjustment necessary for the difference between the census date and the mid-year (see step 2 of subsection B.4), the final estimate of completeness is $\hat{C} = 0.840/1.0044 = 0.836$. Once more, in this case the necessary adjustment is so small that for all practical purposes it can be ignored.

From the estimate of K, the corresponding value of the growth rate r can be obtained as follows:

$$r' = Y_1 - KX_1 = 0.0402 - 1.191(0.0092) = 0.0292.$$

Note that this rate of growth is almost identical to that of 0.0287 obtained in subsection B.4 from census data.

Even though the estimate of K obtained above is probably the best possible, as an example another is computed by fitting a robust line. This exercise will serve to show that the use of trimmed means reduces the influence of extreme points on the estimated value of K.

Table 132 illustrates the way in which trimmed means are computed. All the observations have been used, but three have been "trimmed" at each end of the age range. The weights associated with each observation are listed in column (2). Note that the data set has been divided into groups of equal size and that the weights are symmetrical with respect to the centre. The line labelled "Weighted total" shows under column (2) the sum of the weights used, and under columns (3) and (4), the sum of the weighted observations. Thus, for example,

$$0.0602 = (0.25)(0.0064) + (0.50)(0.0070) + (0.75)(0.0082)$$

$$0.0095 + 0.0111 + 0.0130 + 0.0153,$$

and

$$0.2320 = (0.25)(0.0381) + (0.50)(0.0391) + (0.75)(0.0381)$$

$$+0.0401 + 0.0426 + 0.0434 + 0.0482.$$

TABLE 132. ESTIMATION OF AN ADJUSTMENT FACTOR, K, BY USE OF TRIMMED MEANS, FEMALES, EL SALVADOR, 1961

Age x (x)	Weights (2)	Partial death rate $D(x+)/N(x+)$ (3)	Partial birth rate $N(x)/N(x+)$ (4)
	(a) *Group 1*		
5	0.25	0.0064	0.0381
10	0.50	0.0070	0.0391
15	0.75	0.0082	0.0381
20	1.00	0.0095	0.0401
25	1.00	0.0111	0.0426
30	1.00	0.0130	0.0434
35	1.00	0.0153	0.0482
Weighted total	5.50	0.0602	0.2320
Weighted mean		$X_1^* = 0.0109$	$Y_1^* = 0.0422$
	(b) *Group 2*		
40	1.00	0.0185	0.0542
45	1.00	0.0224	0.0565
50	1.00	0.0274	0.0622
55	1.00	0.0342	0.0662
60	0.75	0.0413	0.0774
65	0.50	0.0546	0.1044
70	0.25	0.0650	0.0955
Weighted total	5.50	0.1778	0.3732
Weighted mean		$X_2^* = 0.0323$	$Y_2^* = 0.0679$

The "totals" for the second group are obtained in exactly the same way. Once these totals are computed, the means are obtained by dividing the sum of the weighted observations by the sum of the weights, that is, the totals under columns (3) and (4) by the total under column (2). For example,

$$X_1^* = 0.0602/5.5 = 0.0109.$$

The X^* and Y^* values are the trimmed means of the grouped observations. From them, the calculation of estimates of K, C and r is straightforward:

$$K^* = (Y_2^* - Y_1^*)/(X_2^* - X_1^*)$$

$$= (0.0679 - 0.0422)/(0.0323 - 0.0109) = 1.201;$$

$$C^* = 1.0/K^* = 0.0833;$$

$$r^* = Y_1^* - K^* X_1^* = 0.0422 - 1.201(0.0109) = 0.0291.$$

Note that the estimates obtained for each of these three parameters by using trimmed means are very similar to those obtained earlier using simple means. The rounded value of C^* still implies that adult female death registration in El Salvador was only 83 per cent complete in 1961 (disregarding for the moment the adjustment for census date). The value of r^* is virtually identical to r'. It is also interesting to recall that the use of the Preston-Coale procedure described in subsection B.3 led to an estimate of coverage of 0.825 using a 75+ open-ended interval and a growth rate estimate of 0.0287, again disregarding the adjustment for census date (see step 4 of subsection B.4). The similarity of these estimates is reassuring.

Step 7: adjustment of death rate. To obtain an adjusted death rate or adjusted age-specific mortality rates, one simply multiplies the observed rates by K' or by K^*. Because the trimmed-mean estimate is somewhat more robust, it has been preferred. For example, for the death rate over age 5,

$$d(5+) = K^*[D(5+)/N(5+)] = 1.201(0.0064) = 0.0077.$$

TABLE 133. VALUES OF POPULATION AT DIFFERENT AGES, REPORTED DEATHS AND PARTIAL BIRTH AND DEATH RATES FOR MALES, ANDRA PRADESH STATE, INDIA, 1970-1971

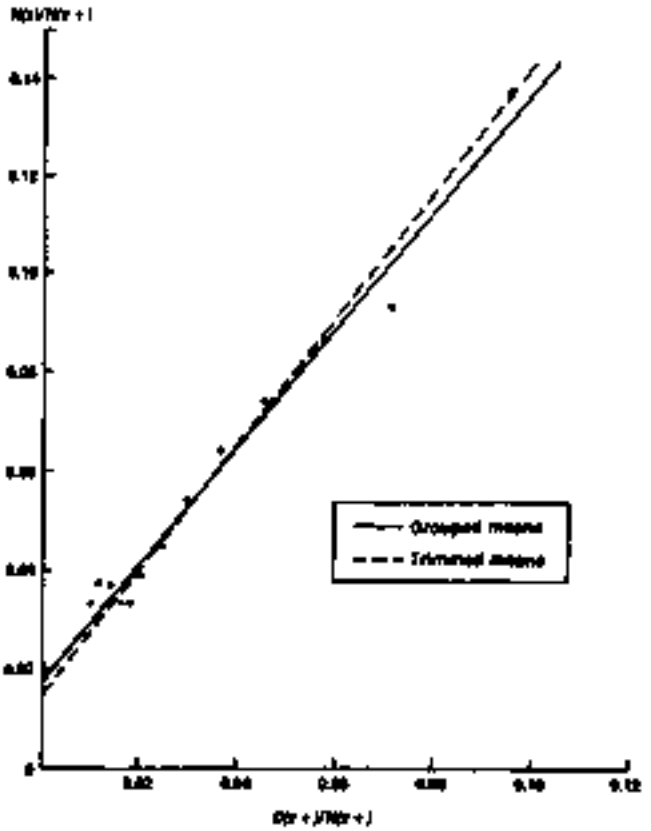
Age x (x)	Population at exact age x $N(x)$ (1)	Population aged x and over $N(x+)$ (2)	Reported deaths over age x $D(x+)$ (3)	Partial birth rate $N(x)/N(x+)$ (4)	Partial death rate $D(x+)/N(x+)$ (5)
5	569.3	17 111	187.10	0.0333	0.0109
10	524.5	14 307	173.22	0.0367	0.0121
15	432.0	11 866	167.92	0.0364	0.0142
20	334.2	9 987	163.50	0.0335	0.0164
25	286.7	8 524	157.79	0.0336	0.0185
30	275.9	7 120	148.72	0.0388	0.0209
35	260.2	5 765	142.96	0.0451	0.0248
40	244.6	4 518	134.61	0.0541	0.0298
45	212.4	3 319	122.32	0.0640	0.0369
50	177.3	2 394	110.38	0.0741	0.0461
55	133.3	1 546	90.59	0.0862	0.0586
60	99.0	1 061	76.36	0.0933	0.0720
65	75.9	556	53.69	0.1365	0.0966

5. *Second detailed example*

Andra Pradesh provides an interesting example of what happens to the application of the Brass growth balance method when age is poorly reported. The basic data have already been presented in table 127. The values of $N(x)$, $N(x+)$, $D(x+)$, $N(x)/N(x+)$ and $D(x+)/N(x+)$ are shown in table 133. The values of $N(x)/N(x+)$ and $D(x+)/N(x+)$ are plotted in figure 15.

The first five points are a trendless jumble, but the points from age 30 upward follow something like a straight line, although if the last point were excluded, the points for 30-60 would look as if they were following a curve. A straight line can be fitted to the points from 30 on; the line, fitted by group means, has a slope of 1.169 and an intercept of 0.0177, indicating that adult deaths were underreported by nearly 14 per cent. However, such a line is strongly affected by the last point and also by the exclusion of the first five points; it might, therefore, represent a case where trimming would be very effective. Following the previous example, using all available points, but weighting the first and last by 0.25, the second and second from last by 0.50, and the third and third from last by 0.75, gives a rather different straight line with a slope of 1.257 and an intercept of 0.0144; thus, the robust line indicates a rather lower growth rate and a level of omission of some 20 per cent.

Figure 15. Plot of partial birth rates, $N(x)/N(x+)$, against partial death rates, $D(x+)/N(x+)$, for males, Andra Pradesh State, India, 1970-1971



Of the two, the robust line should probably be preferred, but not a great deal of confidence can be attached to the results of this method when the derived points do not approximate a straight line. One feature that is not prominent in figure 15 is evidence of exaggeration of age at death; such exaggeration should lead to a tendency for the plotted points to curve off to the right, because the partial death rates over each age would increase more rapidly than the partial birth rates. The points from 30 to 60 do show something akin to such a pattern, but the point for age 65 and over fails to confirm the continuation of the pattern to higher ages. In general, exaggeration of age at death is likely to result in overestimates of completeness, since the points for higher ages would tend to pull the slope down.

In conclusion, it is difficult to interpret the results of applying the Brass method in this case. It may be mentioned in passing that in this application, the results of the Brass method are rather different from the ones obtained by applying the Preston-Coale method, which in subsection B.5 gave an estimate of coverage of nearly 90 per cent and a growth rate of 0.0183. Thus, this example should be taken as a warning against the mechanical application of any method. In applying the Brass procedure, careful examination of the behaviour of the plotted points is essential.

Chapter VI

DERIVATION OF A SMOOTH LIFE TABLE FROM A SET OF SURVIVORSHIP PROBABILITIES

A. Background of methods

1. *Necessity of smoothing and completing sets of survivorship probabilities*

There are several situations in which one can obtain estimates of life-table probabilities of survivorship, $l(x)$, but in which one would still want to smooth[1] these estimates with reference to a model life table. For example, one could calculate a life table directly by converting observed ${}_nm_x$ values into ${}_nq_x$ and hence into $l(x)$ values. In other instances, the $l(x)$ values available may not define a complete life table because some may apply to childhood and some to advanced adult ages (as those obtained from maternal orphanhood data). In the most common situation, one would estimate childhood survivorship probabilities from information on children ever born and children surviving (see chapter III) and estimate adult survivorship probabilities from information on spouse survival or on parental survival (see chapter IV), and the age ranges covered by these sets of estimates will not exhaust the 0-85 life-table range. In such cases, both an element of smoothing and some interpolation or extrapolation to obtain missing $l(x)$ values may be necessary.

In yet other situations, one may wish to derive complete sets of life-table probabilities of survivorship, $l(x)$, from childhood mortality estimates and conditional survivorship probabilities for adults (such as the probability of surviving from age A to age B, $l(B)/l(A)$). Conditional probabilities are usually obtained from information on widowhood or orphanhood (see chapter IV) or by the use of some death-distribution method (see chapter V). The problem one faces in this circumstance is to link the two independent estimates. Again, this linkage is usually accomplished by reference to a model life table.

2. *Organization of this chapter*

This chapter focuses on methods to derive complete life tables from partial information on survivorship probabilities. Two types of methods are distinguished: those which allow the smoothing and interpolation of $l(x)$ values (called the "SI methods"); and those which, in addition to incorporating some element of smoothing and allowing interpolation, permit the linkage of direct or unconditional probabilities of survivorship, $l(x)$, with conditional probabilities, i.e., the ratios of $l(x)$ values (called the "L methods"). The sections presenting each type of method are described below:

TABLE IV.1. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER VI

Section	Subsection and method	Type of input data	Estimated parameters
B. Smoothing and interpolation of an incomplete set of survivorship probabilities		A set of $l(x)$ values. It may be the complete $l(x)$ function for $x = 1, 5, 10, \ldots, 80$, or some subset of those values A reliable, complete set of $l(x)$ values to be used as standard	Parameters α and β defining the fitted $l(x)$ function in the logit system generated by the selected standard A complete life table
C. Linkage of child survivorship probabilities with conditional adult survivorship probabilities	C.2. Using the logit system	A set of $l(x)$ values, usually for childhood ages A set of conditional survivorship probabilities, of the form $l(x)/l(y)$ A reliable, complete set of $l(x)$ values to be used as standard	Parameters α and β defining the fitted $l(x)$ function in the logit system generated by the selected standard A complete life table
	C.3. Using the Coale-Demeny models	A set of $l(x)$ values, usually for childhood ages A set of conditional survivorship probabilities, of the form $l(x)/l(y)$	A complete life table

Section B. Smoothing and interpolation of an incomplete set of survivorship probabilities. This section presents a smoothing and interpolation method based on the use of the logit life-table system. It requires as input a set of survivorship probability estimates, $l(x)$.

Section C. Linkage of child survivorship probabilities with conditional adult survivorship probabilities. This section presents two linkage methods based, respectively,

[1] The term "to smooth" is used in this *Manual* in its most general sense to mean the elimination or minimization of irregularities often present in reported data or in preliminary estimates obtained from them. In this sense, the set of possible "smoothing techniques" encompasses a wide variety of procedures, ranging from the fitting of models to simple averaging. The traditional smoothing techniques applied to age distributions and to observed age-specific mortality rates are part of this set, but they do not exhaust it. The somewhat rougher procedures described in this *Manual* are necessary because the basic data available are both deficient and incomplete.

on the use of the logit life-table system and the Coale-Demeny life tables. Both require as input a set of child survivorship probabilities, $l(x)$, and a set of conditional survivorship probabilities of the form $l(x)/l(y)$. See table 134 for more details.

B. Smoothing and interpolation of an incomplete set of survivorship probabilities

1. *Basis of method and its rationale*

Perhaps the simplest technique for smoothing and interpolating between a sequence of $l(x)$ values is provided by the logit system described in chapter I, subsection B.4. Because all life-table $l(x)$ functions belonging to the same logit system are related linearly on the logit scale, one way of smoothing the observed $l(x)$ values would be to plot their logit transformations against those of some standard life table. If the estimated life table does conform to the logit system generated by this standard, then the plotted points should form a fairly straight line with slope β and intercept α; any method of line-fitting can then be used to estimate the actual values of these parameters. However, when the plotted points depart from a straight line, the problem of selecting the best fit is less tractable. If the observed deviations from linearity are systematic, as, for example, when the deviations become larger as age increases or as age decreases, or when the plot is decidedly curvilinear, the use of a different standard should be considered. If, on the other hand, the deviations from a linear trend seem to be random in nature, the exclusion of some of the points may be necessary before a line is fitted to the remaining points. In such a case, the use of fitting techniques (such as regression) that assume some homogeneity in the errors affecting the data is not warranted, because the errors involved are likely to have different variances at different points and these variances cannot be estimated from the data usually available. Hence, in cases of rough overall linearity, where distortions are mostly due to errors, rather crude fitting techniques (such as those described in chapter V, subsection C.3), coupled with the judicious selection of the most reliable points, is probably the most acceptable procedure to follow.

In the application of this smoothing procedure, the selection of an appropriate standard is of great importance, since only when the mortality pattern of the standard resembles that of the observed $l(x)$ function will the linear relationship on the logit scale be evident. The most common sources of standards are sets of model life tables. The general standard proposed by Brass[2] is always one possibility to be considered (see chapter I, table 2), as are the four families of the Coale-Demeny[3] life tables (see annex XI) and the United Nations model patterns for developing countries[4] (see chapter I, subsection B.5). In using these models, it is helpful to remember that the general standard has a pattern very similar to that of the West model in the Coale-Demeny set, and that the logit transformations of life tables at different levels within the same family of Coale-Demeny models have an approximately linear relationship to one another, but that linearity is not maintained when the logit transformations of life tables from different families are compared.

2. *Data required*

The data listed below are required for this method:

(*a*) A set of $l(x)$ values, estimated either directly from observed data or by using any of the procedures described in this *Manual*. Values at five-year intervals (5, 10, 15, 20 and so on) are sufficient, but the complete range from 0 to 80 or 85 need not be covered;

(*b*) A standard life table, which may be selected from the Coale-Demeny models, the United Nations models for developing countries, the general standard or any reliable life table thought to approximate the pattern of mortality in the population being studied.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of logit transformation of the estimated survivorship probabilities. If one denotes by $\lambda(x)$ the logit transformation of the survivorship probabilities, $l(x)$, then

$$\lambda(x) = \text{logit}(1.0 - l(x)) = 0.5 \ln((1.0 - l(x))/l(x)). \qquad \text{(B.1)}$$

Step 2: plot of logit transformation of the estimated life table against logit transformation of the standard. The logit transformations of the $l(x)$ values for the life table of the population under study should be plotted against those of the standard. If the relationship is linear (or approximately so, with no systematic deviations), then the parameter values α and β can be estimated. If systematic distortions are evident, a different standard should be employed, though departures from linearity may arise from errors in the observed $l(x)$ values rather than from the use of an unsuitable standard.

Step 3: estimation of parameter values. If the plot in step 2 reveals a nearly linear relationship, then estimates of the parameter values, α and β, the constant term and the slope of the line representing this relationship, may be obtained by almost any line-fitting procedure: either by least squares; or by the procedures described in chapter V, subsection C.3. If certain groups of $l(x)$ values are thought to be more reliable than others, then only those values should be used in fitting a line, as is illustrated in the detailed example given below.

---

[2] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, North Carolina, Carolina Population Center, Laboratories for Population Studies, 1975).

[3] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966).

[4] *Model Life Tables for Developing Countries* (United Nations publication, Sales No. E.81.XIII.7).

Step 4: computation of smoothed life-table values. Once estimates of the parameters α and β are available, the smoothed life-table values, $l^*(x)$, are obtained as

$$l^*(x) = (1.0 + \exp(2\alpha + 2\beta\lambda_s(x)))^{-1} \qquad (B.2)$$

where $\lambda_s(x)$ denotes the logit transformation of the standard life table at age x. Note that $l(0)$ and $l(\omega)$, where ω is the highest age that may be attained, cannot be calculated by using equation (B.2); instead, the radix of the life table, $l(0)$, is set equal to one, and $l(\omega)$ is set equal to zero.

TABLE 135 CHILDHOOD SURVIVORSHIP PROBABILITIES FOR FEMALES, ESTIMATED USING TRUSSELL COEFFICIENTS WITH DATA CLASSIFIED BY AGE AND BY MARRIAGE DURATION OF MOTHER, PANAMA, 1976

Age x (1)	Duration: Childhood survivorship probability $l(x)$ (2)	Duration: West level (3)	Duration: Reference date (4)	Age: Childhood survivorship probability $l(x)$ (5)	Age: West level (6)	Age: Reference date (7)
2	0.9688	21.3	1975.3	0.9560	20.2	1974.3
3	0.9505	20.0	1973.0	0.9405	19.2	1972.5
5	0.9428	19.7	1970.6	0.9324	19.0	1970.5
10	0.9238	18.9	1968.3	0.9091	18.1	1967.9
15	0.9026	18.1	1965.6	0.9035	18.3	1965.3
20	...	...	...	0.8691	17.0	1962.4

4. *A detailed example*

The case of female mortality in Panama is considered in this example. In chapters III and IV, the data gathered by the Demographic Survey of Panama in 1976 were used to estimate female child and adult survivorship. Table 135 shows estimates of female $l(x)$ values for childhood ages obtained both from data classified by age of mother (see chapter III, subsection B.3) and from those classified according to the duration of mother's first marriage (see chapter III, subsection C.3). Each $l(x)$ value presented in table 135 is accompanied by the mortality level it implies in the West family of Coale-Demeny life tables and by its reference period, both estimated as described in chapter III.

Table 136 shows a set of $l(x)$ values for females estimated from data on the widowhood status of male respondents classified by duration of marriage. Again, each of the estimates presented is accompanied by the mortality level it implies in the West models and by its time reference period (see chapter IV, subsection C.3 (c)(iii)). As these tables indicate, the two distinct sets of estimates, those referring to childhood and those referring to adult ages, cover only the age range from 2 to 40. Furthermore, since different estimates refer to different time periods, it is clear that for a given period, say from 1970 to 1975, no complete set of $l(x)$ estimates is available covering that age range. Hence, it is necessary to employ the procedure described in this section to derive from this partial information on survivorship probabilities a complete life table referring to the recent past.

The steps of the computational procedure are described below.

Step 1: calculation of logit transformation of the estimated survivorship probabilities. Equation (B.1) is used to calculate the logit transformation of each survivorship probability, $l(x)$, given in tables 135 and 136. The resulting $\lambda(x)$ values are shown in table 137. As an example, the logit transformation of $l(30)$ is shown below:

$$\lambda(30) = \text{logit}(1.0 - 0.8955) = 0.5 \ln((1.0 - 0.8955)/0.8955)$$
$$= 0.5 \ln(0.1167) = -1.0741.$$

Column (2) of table 137 gives the logit transformations of the $l_s(x)$ probabilities corresponding to the standard used. In this case, the general standard has been selected as an adequate representation of the pattern of mortality prevalent in the population under study; therefore, the $\lambda_s(x)$ values listed in table 137 have been copied from table 2 (see chapter I).

TABLE 136. FEMALE SURVIVORSHIP PROBABILITIES ESTIMATED FROM DATA ON THE WIDOWHOOD STATUS OF MALE RESPONDENTS CLASSIFIED BY DURATION OF MARRIAGE, PANAMA, 1976

Age x (1)	Survivorship probability $l(x)$ (2)	West mortality level (3)	Reference date (4)
20	0.9365	20.2	1975.6
25	0.9160	19.6	1973.3
30	0.8955	19.2	1971.1
35	0.8744	18.9	1968.9
40	0.8491	18.6	1967.0

TABLE 137. LOGIT TRANSFORMATION OF THE ESTIMATED AND STANDARD SURVIVORSHIP PROBABILITIES, PANAMA, 1976

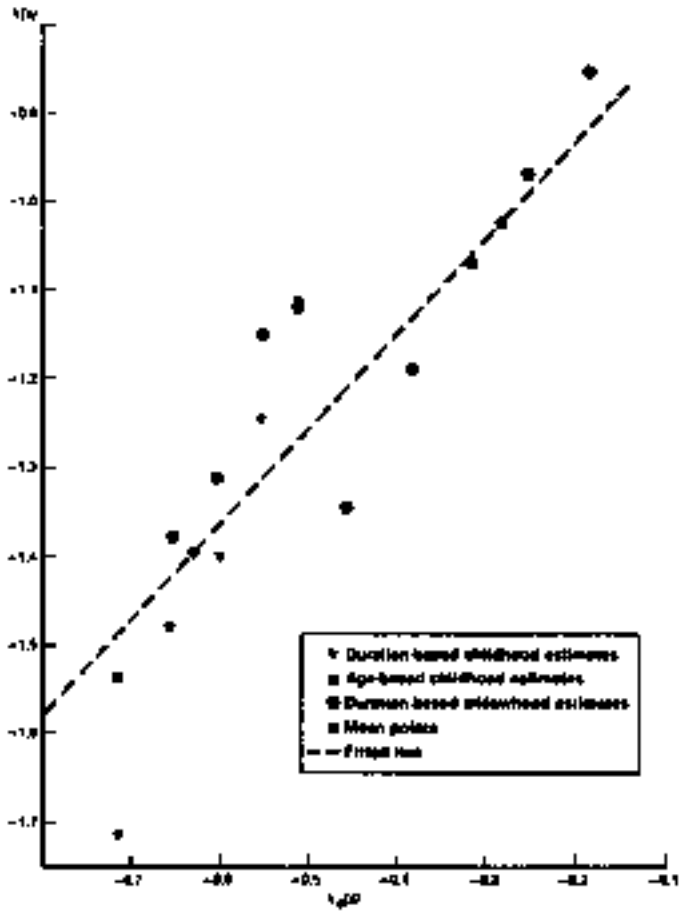
Age x (1)	General standard $\lambda_s(x)$ (2)	Duration-based estimates $\lambda(x)$ (3)	Age-based estimates $\lambda(x)$ (4)
2	−0.7132	−1.7178	−1.5393*
3	−0.6552	−1.4775*	−1.3002*
5	−0.6015	−1.4012*	−1.3121*
10	−0.5498	−1.2476*	−1.1513
15	−0.5131	−1.1132	−1.1186
20	−0.4553	−1.3456	−0.9465
25	−0.3829	−1.1946*	..
30	−0.3150	−1.0741*	...
35	−0.2496	−0.9702*	..
40	−0.1816	−0.8638*	..

* Values used in fitting procedure.

Step 2: plot of logit transformation of the estimated life table against logit transformation of the standard. The points defined by each pair of values $[\lambda_s(x), \lambda(x)]$ are plotted in figure 16. Note that because of the nature of the data used in deriving the different $l(x)$ estimates, at least three different sets of points have been identified in the plot. The points corresponding to the two sets of child mortality estimates cluster, in general, to the bottom left of the graph. They are similar, but the desirable degree of coincidence between one set and the other is not achieved. Notably deviant points are those associated with the $l(2)$ estimate obtained from data classified by duration and with the $l(x)$ estimates for ages 15 and 20 (even the point associated with $l(10)$ of the age-based estimates is somewhat out of line with the rest). The estimates derived from widowhood data, on the other hand, cluster near the upper right-hand side of the graph. They display a fairly linear trend, but the point associated with $l(20)$, and even perhaps that associated with $l(25)$, would have to be disregarded in order for the widowhood-based estimates to be considered a continuation of a straight line defined by the points derived from child survivorship probabilities.

A rapid appraisal of the West mortality levels associated with each estimate and of their respective reference dates suggests that most of the deviant points among the childhood estimates are associated with periods preceding 1970 or following 1975, and that they usually correspond to mortality levels (in the Coale-Demeny set) substantially lower or higher than the rest. Deviant points in the case of the widowhood-based estimates are associated with the most recent dates (after 1975) and correspond to fairly high mortality levels. On the basis of these observations, the points indicated by footnote a in table 137 are chosen as the most reliable indicators of child and adult mortality during, roughly, the period 1969-1975.

Figure 16. Plot of the logit transformation of the estimated female survivorship probabilities, $l(x)$, against those of the general standard, Pakistan, 1976



Step 3: estimation of parameter values. Having selected the most reliable set of points representing survivorship probabilities for childhood and adult ages, a line is fitted to them by using group means (see chapter V, subsection C.3). Means must be obtained both for the logit transformations of the estimated survivorship probabilities and for those of the standard. If one denotes the means of the former values by $\Theta(2, 3, 3, 5, 5, 10)$ and $\Theta(25, 30, 35, 40)$, and those of the latter by $\Theta_s(2, 3, 3, 5, 5, 10)$ and $\Theta_s(25, 30, 35, 40)$, then the line passing through the points defined by these mean values satisfies the following two equations:

$$\Theta(2, 3, 3, 5, 5, 10) = \alpha + \beta\Theta_s(2, 3, 3, 5, 5, 10), \quad (B.3)$$

and

$$\Theta(25, 30, 35, 40) = \alpha + \beta\Theta_s(25, 30, 35, 40). \quad (B.4)$$

Hence, an estimate of the parameter β is given by

$$\beta^* = \frac{\Theta(2, 3, 3, 5, 5, 10) - \Theta(25, 30, 35, 40)}{\Theta_s(2, 3, 3, 5, 5, 10) - \Theta_s(25, 30, 35, 40)}$$

$$= \frac{-1.3930 + 1.0257}{-0.6297 + 0.2823} = \frac{-0.3673}{-0.3474} = 1.0573, \quad (B.5)$$

and parameter α is then found by substituting the estimated value of β into either equation (B.3) or equation (B.4) as follows:

$$\alpha^* = \Theta(2, 3, 3, 5, 5, 10) - \beta^*\Theta_s(2, 3, 3, 5, 5, 10)$$

$$= -1.3930 - [(1.0573)(-0.6297)] = -0.7272,$$

or

$$\alpha^* = \Theta(25, 30, 35, 40) - \beta^*\Theta_s(25, 30, 35, 40)$$

$$= -1.0257 - [(1.0573)(-0.2823)] = -0.7272.$$

Step 4: computation of smoothed life-table values. Once best estimates of α and β have been calculated, the full set of estimated smoothed life-table survivorship probabilities, $l^*(x)$, shown in column (3) of table 138, can be obtained by using equation (B.2). For example, $l^*(20)$ is calculated as

$$l^*(20) = \frac{1.0}{1.0 + \exp(2(-0.7272) + 2(1.0573)(-0.4551))}$$

$$= 0.918.$$

It should be stated that this procedure may not be robust to different choices of groups in fitting a line to the logit points. For example, if instead of choosing as the point representative of childhood mortality the mean of those associated with the age-based $l(2)$, $l(3)$ and $l(5)$, and with the duration-based $l(3)$, $l(5)$ and $l(10)$, just the mean of the three duration-based estimates had been used, β^* would have been 1.0934 instead of its previous value of 1.0573. Similarly, α^* would have been slightly higher, with an estimated value of −0.7170. Hence, unless one is fairly certain that a particular group as a whole is correct, other combinations should be tried in order to assess the sensitivity of the final life table to alternative choices. In this case, however, most of the reasonable choices produce fairly similar life tables.

TABLE 138. ESTIMATED SMOOTHED LIFE-TABLE SURVIVORSHIP PROBABILITIES FOR FEMALES OBTAINED BY USE OF DIFFERENT SMOOTHING AND LINKAGE METHODS, PANAMA, 1976

Age x	General standard $\lambda_x(t)$ (2)	Estimated smoothed survivorship probabilities, $l(x)$: Smoothing and interpolation method (3)	Linkage methods: Logit system (4)	Coale-Demeny (5)
0	-	1.000	1.000	1.000
1	−0.7152	0.951	0.948	0.948
5	−0.6015	0.939	0.940	0.940
10	−0.5498	0.932	0.935	0.934
20	−0.4551	0.918	0.927	0.923
30	−0.3150	0.893	0.913	0.911
40	−0.1816	0.863	0.897	0.893
50	−0.0212	0.817	0.875	0.859
60	0.2100	0.733	0.835	0.789
70	0.5818	0.556	0.752	0.636
80	1.2375	0.238	0.551	0.342

C. LINKAGE OF CHILD SURVIVORSHIP PROBABILITIES WITH CONDITIONAL ADULT SURVIVORSHIP PROBABILITIES

1. *Basis of method and its rationale*

As stated in chapter IV, orphanhood or widowhood data properly reflect only adult mortality experience. Hence, one should be able to obtain better estimates of conditional survivorship probabilities (for example, $l(35)/l(20)$) than for unconditional $l(x)$ values. Nevertheless, one is most often interested in the unconditional probabilities, since they define a conventional life table. It is useful, therefore, to have some procedure for linking conditional probabilities of adult survivorship, of the form $l(x)/l(y)$, with other information about survivorship, particularly that in childhood. Not only would such a procedure make possible the derivation of a complete life table, but it would also incorporate some element of smoothing over the range of survivorship estimates available.

If a number of survivorship probabilities of the type $l(x)/l(y)$ are available for different values of x, but for a fixed value of y, all that is needed in order to obtain estimates of $l(x)$ is an estimate of $l(y)$. Information about child survivorship—say, an estimate of the probability of surviving from birth to age 2, $l(2)$—can then be coupled with a system of model life tables to estimate a value of $l(y)$, which can in turn be multiplied by the adult survivorship probabilities to obtain $l(x)$. The logit system provides a convenient basis for such a procedure, and an iterative process that also introduces a powerful smoothing component is described below. An alternative procedure using the Coale-Demeny model life tables is also presented below (in subsection C.3).

2. *Linkage method using a logit life-table system*

(a) *Data required*

The data required for use of the first linkage method, that utilizing a logit life-table system, are listed below:

(*a*) Conditional probabilities of adult survivorship, usually in the form of $l(x)/l(y)$ ratios estimated by using the widowhood or orphanhood methods described in chapter IV;

(*b*) An estimate of child survivorship. Survivorship probabilities $l(x)$ for childhood ages can be estimated by using the methods described in chapter III;

(*c*) A standard model life table. The general standard, one of the Coale-Demeny models or of the United Nations new models, or any other reliable life table thought to approximate the mortality experience of the population in question may be used as standard.

(b) *Computational procedure*

In the two-parameter logit life-table system, any pair of survivorship probabilities, one from birth and another conditional on attaining a certain age, uniquely determine values of the parameters α and β defining a life table in the system. However, because one of the probabilities is conditional, the values of these parameters have to be estimated iteratively, as there is no way of solving for them algebraically. The procedure described here provides a way of finding satisfactory values of α and β when a number of conditional survivorship probabilities, all referring to the same population, are available, without having to find parameter values for each one, a process that would be tedious without a computer. The steps of the computational procedure are described below.

Step 1: initial estimate of parameter α. Given an estimate of $l(x)$ for children, such as $l(2)$ or the average of a group of estimates, the initial estimate of parameter α is obtained as

$$\alpha_1 = \lambda(x) - \lambda_s(x)$$

under the assumption that $\beta_1 = 1.0$.

Step 2: initial estimate of survivorship probability appearing as denominator. Given the value of α_1 estimated in step 1 and continuing to assume that $\beta_1 = 1.0$, a first estimate of this survivorship probability, $l(y)$, denoted by $l_1(y)$, is obtained from equation (B.2):

$$l_1(y) = (1.0 + \exp(2\alpha_1 + 2\lambda_s(y)))^{-1}$$

Step 3: initial estimates of survivorship probabilities from birth. The initial approximation to $l(y)$, $l_1(y)$, is now used to calculate initial estimates of the survivorship

probabilities from birth, $l(x)$, for each value of x from the observed ratios $l(x)/l(y)$:

$$l_1(x) = \frac{l(x)}{l(y)} l_1(y). \qquad \text{(C.1)}$$

Step 4: modified estimate of parameter β. If one denotes by $\lambda_1(x)$ the logit transformation of $l_1(x)$, each pair of points $[\lambda_s(x), \lambda(x)]$ and $[\lambda_s(x), \lambda_1(x)]$ determines parameters α and β with respect to the standard being used. The main interest at present is in parameter β, which can be found as

$$\beta_2(x) = \frac{(\lambda(x) - \lambda(x))}{(\lambda_s(x) - \lambda_s(x))}. \qquad \text{(C.2)}$$

A single estimate of β_2 can then be obtained by averaging the $\beta_2(x)$ values. It is often the case that the values of $\beta_2(x)$ vary sharply with x, and it may be decided that a best estimate of β_2 can be obtained by averaging the $\beta_2(x)$ values after excluding obvious outliers, such as the highest and lowest values.

Step 5: a second estimate of parameter α. A second estimate of α, denoted by α_2, is obtained by repeating step 1, but now using the estimate of β_2 obtained in step 4 instead of the first assumed value of $\beta_1 = 1.0$. Thus,

$$\alpha_2 = \lambda(x) - \beta_2 \lambda_s(x). \qquad \text{(C.3)}$$

Step 6: second estimate of survivorship probability used as denominator. The new value of α, denoted by α_2, and the second approximation to β, denoted by β_2, are now used to obtain a revised value of $l(y)$, denoted by $l_2(y)$, as follows:

$$l_2(y) = (1.0 + \exp(2\alpha_2 + 2\beta_2 \lambda_s(y)))^{-1}. \qquad \text{(C.4)}$$

Step 7 and onward: further iteration. The iterative procedure continues, by repeating step 3 to obtain a second set of estimates of $l(x)$, then repeating step 4 to find a revised estimate of β, then re-estimating α in step 5 and $l(y)$ in step 6 with the new α and β values, and so on, until the first two or three decimal places of the estimate of β no longer change with continued iteration. This unchanging value of β, denoted by β^*, and the value of α^* it implies (calculated by using equation (C.3)) are then accepted as best estimates of the parameters defining a life table consistent with the available survivorship probabilities in the logit system being used.

With practice, the iterative process does not take long; four or five iterations are usually sufficient for the β estimate to converge. A slight short-cut can also be taken, since β^*, the unchanging estimate of β, is always further away, generally about half as far again, from the first assumed value of 1.0 than is the first estimate obtained. Thus, as an approximation,

$$\beta_3 = 1.5\beta_2 - 0.5. \qquad \text{(C.5)}$$

Then, instead of substituting the value of β_2 in step 5, a more rapid convergence can be achieved by substituting β_3 instead. Thereafter, however, no short-cuts can be taken.

(c) *A detailed example*

Table 139 shows conditional survivorship probabilities for the female population of Panama derived from data on the widowhood status of male respondents classified by age (see chapter IV, subsection B.3 (c) (iii)). Given the age-based estimates of childhood mortality shown in table 135, the way in which a full life table may be constructed is illustrated below.

TABLE 139. ESTIMATED CONDITIONAL FEMALE SURVIVORSHIP PROBABILITIES, $l(x)/l(20)$, AND CORRESPONDING MORTALITY LEVELS IN THE WEST FAMILY OF MODEL LIFE TABLES. PANAMA, 1976

Age x (1)	Survivorship probabilities $l(x)/l(20)$ (2)	West mortality level (3)
25	0.9962	22.2
30	0.9878	21.3
35	0.9775	21.0
40	0.9641	20.8
45	0.9417	20.4

Step 1: initial estimate of parameter α. The life table used as standard is, once more, the general standard presented in table 2 in chapter I. In the first iteration, an estimate of α_1 is obtained by assuming that $\beta_1 = 1.0$. If one assumes that the estimates of $l(2)$, $l(3)$ and $l(5)$ are reasonably accurate, then α_1 can be estimated as

$$\alpha_1 = \Theta(2, 3, 5) - \Theta_s(2, 3, 5)$$

$$= -1.4105 + 0.6573 = -0.7532$$

where $\Theta(2, 3, 5)$ denotes, as usual, the mean of the logit transformations of $l(2)$, $l(3)$ and $l(5)$, that is, the mean of $\lambda(2)$, $\lambda(3)$ and $\lambda(5)$ in this case.

Step 2: initial estimate of survivorship probability appearing as denominator. In this case, the survivorship ratios being used have as denominator the value of $l(20)$, so $y = 20$. A first estimate of $l(20)$ is obtained by setting $\beta_1 = 1.0$, and using the α_1 estimate from step 1:

$$l_1(20) = (1.0 + \exp(2(-0.7532) + 2(-0.4551)))^{-1}$$

$$= 0.9181.$$

Step 3: initial estimates of survivorship probabilities from birth. Each value of $l(x)/l(20)$ is multiplied by $l_1(20)$ in order to obtain first estimates of $l(x)$. The results are shown in table 140. The value for $x = 40$ is obtained as

$$l_1(40) = \frac{l(40)}{l(20)} l_1(20) = (0.9641)(0.9181) = 0.8851.$$

Step 4: modified estimate of parameter β. Each estimate of $l_1(x)$, in combination with the pooled estimates of $l(2)$, $l(3)$ and $l(5)$, implies a value of β which is equal to the ratio of the difference in the observed logit transfor-

TABLE 140. ITERATION PROCESS TO ESTIMATE THE α AND β PARAMETERS DEFINING A LIFE TABLE FOR FEMALES IN THE LOGIT SYSTEM GENERATED BY THE GENERAL STANDARD, PANAMA, 1976

Age x	Estimated unadjusted survivorship probability $l_s(x)/l_s(20)$	First iteration: $\alpha_1 = -0.7552$, $\beta_1 = 1.0000$, $l_1(20) = 0.9181$			Second iteration: $\alpha_2 = -0.8695$, $\beta_2 = 0.8230$, $l_2(20) = 0.9220$			Third iteration: $\alpha_3 = -0.8195$, $\beta_3 = 0.7676$, $l_3(20) = 0.9255$		
		$l_1(x)$	$\lambda_1(x)$	$\beta_2(x)$	$l_2(x)$	$\lambda_2(x)$	$\beta_2(x)$	$l_3(x)$	$\lambda_3(x)$	$\beta_3(x)$
25	0.9962	0.9146	−1.1856	0.820	0.9198	−1.2198	0.695	0.9220	−1.2349	0.640
30	0.9878	0.9069	−1.1382	0.796	0.9120	−1.1692	0.705	0.9142	−1.1830	0.665
35	0.9775	0.8974	−1.0843	0.800	0.9025	−1.1127	0.730	0.9047	−1.1253	0.700
40	0.9641	0.8851	−1.0208	0.819	0.8901	−1.0459	0.766	0.8923	−1.0572	0.743
45	0.9417	0.8646	−0.9270	0.879	0.8695	−0.9483	0.840	0.8715	−0.9571	0.824
AVERAGE				0.823			0.747			0.714

Age x	Estimated unadjusted survivorship probability $l_s(x)/l_s(20)$	Fourth iteration: $\alpha_4 = -0.8442$, $\beta_4 = 0.7169$, $l_4(20) = 0.9284$			Fifth iteration: $\alpha_5 = -0.8497$, $\beta_5 = 0.7010$, $l_5(20) = 0.9267$			Sixth iteration: $\alpha_6 = -0.8538$, $\beta_6 = 0.6960$, $l_6(20) = 0.9269$		
		$l_4(x)$	$\lambda_4(x)$	$\beta_5(x)$	$l_5(x)$	$\lambda_5(x)$	$\beta_5(x)$	$l_6(x)$	$\lambda_6(x)$	$\beta_6(x)$
25	0.9962	0.9229	−1.2412	0.617	0.9232	−1.2433	0.609	0.9234	−1.2447	0.604
30	0.9878	0.9151	−1.1888	0.648	0.9154	−1.1907	0.642	0.9156	−1.1920	0.638
35	0.9775	0.9056	−1.1305	0.687	0.9058	−1.1317	0.684	0.9060	−1.1329	0.681
40	0.9641	0.8931	−1.0614	0.734	0.8934	−1.0630	0.731	0.8936	−1.0640	0.728
45	0.9417	0.8724	−0.9612	0.817	0.8727	−0.9625	0.815	0.8729	−0.9634	0.813
AVERAGE				0.701			0.696			0.693

mations of the $l(x)$ estimates and the difference in those of the standard (see equation (C.2)). The logit transformations of the $l_1(x)$ values and the values of $\beta_2(x)$ implied by each are shown in table 140. As an example, for $x = 40$,

$$\lambda_1(40) = \text{logit}(1.0 - l_1(40)) = 0.5 \ln((1.0 - 0.8851)/0.8851)$$

$$= -1.0208;$$

and since $\lambda_s(40)$ is -0.1816, $\Theta(2, 3, 5)$ is -1.4105, and the equivalent value for the standard, $\Theta_s(2, 3, 5)$, is -0.6573, then

$$\beta_2(40) = \frac{-1.0208 - (-1.4105)}{-0.1816 - (-0.6573)} = 0.819.$$

The values of $\beta_2(x)$ are fairly consistent, except for $\beta_2(45)$. However, it is not sufficiently deviant to warrant its exclusion. Therefore, β_2 is estimated as the average of all $\beta_2(x)$ values, that is, 0.823.

Step 5: a second estimate of parameter α. Step 1 is now repeated, but using the estimate of $\beta_1 = 0.823$ instead of 1.0. Thus,

$$\alpha_2 = \Theta(2, 3, 5) - \beta_2 \Theta_s(2, 3, 5)$$

$$= -1.4105 - (0.823)(-0.6573)$$

$$= -0.8695.$$

Step 6: second estimate of survivorship probability to age 20. Step 2 is now repeated, but using the new values of $\alpha_2(-0.8695)$ and $\beta_2(0.823)$ obtained after the first complete iteration. Thus,

$$l_2(20) = (1.0 + \exp(2(-0.8695) + 2(0.823)(-0.4551)))^{-1}$$

$$= 0.9233.$$

Step 7 and onward: further iteration. Table 140 shows the intermediate results obtained during the first six iterations. The value of β is clearly converging towards 0.69, so this value can be adopted as an estimate of β^*. The value of α corresponding to β^* is then calculated using equation (C.3), which in this case yields a value of -0.957 for α^*.

Final step: calculation of fitted life table. Once final estimates of α and β have been arrived at, the estimated $l^*(x)$ function of the fitted life table can be calculated using equation (B.2); and, as usual, the other functions of the life table can then be derived from it. For example, for age 40,

$$l^*(40) = (1.0 + \exp(2(-0.957) + 2(0.69)(-0.2816)))^{-1}$$

$$= 0.8970.$$

The resulting $l^*(x)$ values are shown in column (4) of table 138.

(d) *Comment on the detailed example*

There are several points to note in this detailed example. First, the final α^* and β^* estimates are very far from the neutral values of 0.0 and 1.0, respectively, for the standard being used, suggesting that another standard might have been a better model of the estimates of survivorship being linked in this example. Secondly, the consistency of the $\beta_j(x)$ estimates with age becomes less and less satisfactory as the iteration proceeds, so that the initial range (from 0.796 to 0.879) widens greatly by iteration six (from 0.604 to 0.813). This is a further indication that the standard used may not be suitable. In this regard, it must be noted that no allowance has been made for the fact that different survivorship estimates refer to different periods. It is possible that the pattern of the standard used may not represent adequately the

various mortality levels that have prevailed in the past. Thirdly, it will be seen that in this case six iterations were needed for β to converge; use of the proposed short-cut would have reduced the number somewhat. It may be pointed out in passing that the selection of $\beta_1 = 1.0$ as the beginning value of β has no impact on the final estimates to which the procedure converges.

The life table shown in column (4) of table 138 and implied by the values of α^* and β^* just estimated is moderately similar at earlier ages to the life table given in column (3) of the same table (generated by the smoothing and interpolation method presented in subsection B.3), but the two tables become increasingly different as age advances. The survivorship probabilities shown in column (4), generated by this linkage method, are always higher than those in column (3), generated by the smoothing and interpolation method, and become increasingly higher at older ages. Although it may not be immediately evident, the observed differences in the final life-table estimates are due in part to differences in the original survivorship estimates themselves and do not arise entirely from the smoothing and linking techniques used. It should also be pointed out that the $l(x)$ values given in column (4) of table 138 cannot be believed. It cannot be accepted that 55 per cent of women in Panama will actually reach age 80, and the mortality rates for advanced ages implied by the $l(x)$ function are unacceptably low. In this case, the errors in the data are such that a less flexible smoothing procedure than that based on the logit system is required. This example should serve as a warning that alternative estimation methods, even if they employ the same data (in this case the proportions of male respondents with surviving first wives), do not always yield the same results. Because general rules about the reliability of the results yielded by these methods cannot be given, it is recommended that as many techniques as possible be applied to a particular data set, so that decisions about the validity of the results can be based on a thorough appraisal.

3. *Use of Coale-Demeny model life tables*

In a one-parameter model life-table system such as that proposed by Coale and Demeny,[5] any survivorship probability, whether from birth or conditional on having reached a certain age, uniquely determines a life table within the system, once a family of models has been selected. Thus, each child survivorship probability from birth, $l(x)$, implies a life table, as does each conditional survivorship probability of the type $l(x)/l(y)$ (see chapters III and IV for descriptions of the procedure followed in determining these life tables). The information for the two distinct age ranges can be linked together by adopting the life table implied by the average level of the child mortality estimates up to age y (which can be regarded as the pivotal age) and then completing the life table over age y by applying the conditional probabilities of survivorship from age y consistent with the average level implied by the adult survivorship estimates.

The procedure can be illustrated by using again the data referring to Panamanian females. Table 135 shows female probabilities of surviving, $l(x)$, from birth to ages 2, 3 and 5, obtained from information on female children ever born and surviving classified by age of mother. Each $l(x)$ value implies a mortality level in a family of Coale-Demeny model life tables; column (6) shows the levels implied in the West family. The average female child mortality level is 19.5. The conditional female adult survivorship probabilities estimated from data on the widowhood status of male respondents classified by duration also imply mortality levels, as is shown in table 139; the average mortality level is 21.1. The data therefore suggest that, for females, and ignoring timing effects, child mortality in Panama is approximated by level 19.5 in the West family of model life tables for females, whereas adult mortality is approximated by level 21.1. A composite life table is then constructed by adopting a level 19.5 life table for females up to age 20, calculated by averaging the $l(x)$ functions of the life tables of levels 19 and 20 shown in table 236 (see annex VIII), from which age onward the life table is completed by applying survivorship probabilities from age 20 taken from a level 21.1 life table for females. The latter value can be calculated up to age 60 by weighting the values of $l(x)/l(20)$ for levels 21 and 22 in table 222 (see annex VII), those for level 22 being weighted by 0.1 and those for level 21 by 0.9 (that is, linear interpolation is used). The ${}_nq_x$ values of the resultant life table would show some irregularity around the pivotal age, but the quantitative importance of any discontinuity would be negligible. The final $l^*(x)$ values of the life table estimated in this manner are shown in column (5) of table 138.

In comparing these $l^*(x)$ estimates with those shown in column (4), it is remarkable that the latter values are quite different, especially at older ages. Since these two $l(x)$ functions have been fitted to exactly the same survivorship probabilities, the differences observed can only be ascribed to differences in the procedures used and in the models underlying them. Because the mortality pattern of the general standard is fairly similar to that of the West family of Coale-Demeny models, the choice of standard is unlikely to be the cause of the differences observed. It seems more likely that these divergences have arisen because the procedure based on the Coale-Demeny models introduces a more powerful smoothing component than does the one based on the logit system. Indeed, if one considers the West mortality levels associated with each of the conditional probabilities of survival presented in table 139, it is immediately evident that they cover a fairly wide range of levels. It is not surprising therefore that the shape of the general standard has to be twisted considerably in order to approximate them all at the same time (thus, the estimated β is substantially lower than one). In fitting Coale-Demeny life tables no such twisting is involved, since a single level (21.1 in this case) is used to represent

[5] *Op. cit.*

adult mortality in general. In this instance, the smoothed life table obtained by using the Coale-Demeny system is much more plausible than that obtained by using the logit system, which, being more flexible, incorporates data distortions as well as the actual underlying mortality pattern. This example, therefore, helps to underscore the fact that these methods of fitting cannot be used mechanically. The user must be well aware of the nature and significance of the data at hand in order to establish which elements are truly compatible and merit being used as input for the fitting procedures described here.

Chapter VII

FERTILITY AND MORTALITY ESTIMATION USING MODEL STABLE AGE DISTRIBUTIONS

A. BACKGROUND OF METHODS

1. *General principles underlying the use of model stable populations for estimation purposes*

Lotka[1] proved that the age distribution of any population that is subject, for a sufficiently long time, to constant fertility and mortality becomes fixed. He called this end-product of constant demographic conditions a "stable population", characterized both by an unchanging age distribution and by a fixed annual rate of natural increase.

The age distribution of a stable population is jointly determined by the mortality schedule (that is, by the life table) to which the population has been subject and by its annual rate of growth. In chapter I, section C, the equation defining the density function determining the age distribution of a stable population was presented. It has the form

$$c(x) = b \exp(-rx)\, l(x) \qquad \text{(A.1)}$$

where $c(x)$ is the infinitesimal proportion of the stable population at exact age x; b is the constant birth rate; r is the constant rate of natural increase; and $l(x)$ is the probability of surviving from birth to age x (that is, it is the usual life table function with an $l(0)$ radix of 1.0).

The usefulness of model stable populations in estimating the parameters determining the growth and structure of actual populations derives from three considerations. First, it has been found that for a population in which fertility has been approximately constant and in which mortality has recently undergone a steady reduction, the age distribution closely resembles that of a stable population generated by equation (A.1), with $l(x)$ being the probability of survival to age x in the current life table and r being the current rate of increase. Secondly, these conditions (the recent course of fertility approximately constant and mortality either approximately constant or recently declining) are, or have until recently been, characteristic of the population of many developing countries. Thirdly, the availability of fairly flexible sets of model stable populations, such as those generated on the basis of the Coale-Demeny[2] life tables with a range of growth rates, makes it possible to identify with relative ease a stable age distribution approximating a reported age distribution in an actual application. The underlying characteristics of the stable population can then be adopted as estimates of the demographic parameters of the population being studied.

Useful estimates can be made by selecting a model stable population matching two characteristics, presumed to have been accurately recorded or estimated, of the reported population. In the Coale-Demeny system, a model stable population within each family is fully determined by the level of mortality and the growth rate; thus, two parameters are sufficient to identify a stable population within a given family. For example, the reported proportion under any given age, in conjunction with any one of the following parameters—an estimated $l(x)$ for any age x, the rate of increase or the population death rate—uniquely determines a model stable population, provided that a specific pattern of mortality has been deemed an appropriate representation of that experienced in reality.

When a model stable population has been identified, selected characteristics of the stable population serve as estimates of the corresponding characteristics of the population in question. For example, the birth rate in a given population can be estimated by identifying a model stable population in which the probability of surviving from birth to age 5, $l(5)$, and the proportion under age 15 are the same as those in the given population. The birth rate in the stable population is then taken as an estimate of the actual birth rate.

The value of estimates based on fitting model stable populations is limited by a number of practical considerations. First, no actual population is genuinely stable. Fertility may vary as a result of recent trends or because of special past episodes, such as wars or epidemics. Age-selective and sex-selective migration can affect both the rate of growth and the age distribution of the population, and the recent mortality declines observed in many countries produce age distributions that do not conform exactly to those predicted by the equation defining a stable population. Secondly, the characteristics of the population being studied that are available to identify a model stable population are often imprecisely recorded (the age distribution is distorted by differential omission by age, or the intercensal rate of increase calculated from census counts at two points in time is biased because of differential completeness of coverage between the two censuses). Lastly, the esti-

[1] Alfred J. Lotka and F. R. Sharpe, "A problem in age-distribution", *Philosophical Magazine*, vol. 21, No. 124 (April 1911), pp. 435-438.

[2] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966).

mates of a certain parameter (the birth rate, for example) obtained by using different sets of model stable populations (such as those generated by the different mortality families of the Coale-Demeny set) may be quite different. In most cases, there is uncertainty about which mortality pattern approximates most closely that experienced by a given population; and since the number of model families is limited, there is no guarantee that they cover all possible experience, so the accuracy of the estimates obtained may often be questionable. In this respect, it is appropriate to mention that the availability of the United Nations model life tables for developing countries,[3] in conjunction with stable populations generated by them,[4] will substantially enhance the estimation capabilities by expanding the spectrum of models from which one can choose.

Ideally, the estimates derived by fitting a model stable population to a reported age distribution should be insensitive to the practical difficulties mentioned above. Therefore, they should not be much affected by the deviations from stability most commonly encountered in practice. To achieve this end, the fitted stable population should be identified on the basis of parameters for which estimates are least likely to be biased by the usual data flaws (such as age-misreporting) and which should assume much the same values whichever set of model stable populations is employed to derive them. (In this chapter, only stable populations based on the four Coale-Demeny families are used, so this last desirable feature will at most be satisfied with respect to variations between these four models.)

Experience has shown that the most efficient use of stable populations requires the identification not of one single stable population from which all parameters are derived, but rather of a number of stable populations identified by using different pairs of reported values and applied to estimate different sets of underlying parameters. For example, it is often the case that the best estimate of fertility in the population under review (measured as the birth rate or as total fertility) may be obtained by selecting the model stable population that matches the value of $l(5)$ and the proportion of the population of both sexes under age 15; while in order to estimate mortality levels over age 5, a model stable population should be fitted on the basis of the estimated growth rate and the proportion of the population under age 15. Surprisingly enough, some of the estimates derived from a model stable population by using the first of these strategies are quite accurate even when the population in question is very far from being stable. As examples, table 141 shows the estimated birth rates obtained by fitting a stable population, on the basis of the reported proportion under age 15 for both sexes combined, $l(5)$ and the growth rate, to populations that, as in the case of Sweden, cannot be considered even remotely stable since they have experienced substantial declines both in fertility and in mortality. For comparison, table 141 also displays estimates of the birth rate derived from registered births (in all the countries considered, birth registration is virtually complete). The coincidence between the two sets of estimates is reassuring.

2. *Organization of this chapter*

Essentially, only two methods of estimation using

[3] *Model Life Tables for Developing Countries* (United Nations publication, Sales No. E.81.XIII.7).

[4] *Stable Populations Corresponding to the New United Nations Model Life Tables for Developing Countries* (ST/ESA/SER.R/44).

TABLE 141. ESTIMATION OF AVERAGE BIRTH RATE OVER THE 15-YEAR PERIOD PRECEDING ENUMERATION FROM THE PROPORTION OF THE POPULATION UNDER AGE 15, $C(15)$, THE PROBABILITY OF SURVIVING TO AGE 5, $l(5)$, AND THE CONSTANT RATE OF NATURAL INCREASE, r, FOR SELECTED NON-STABLE POPULATIONS

Country (1)	Intercensal period used to estimate growth rate (2)	Date of enumeration (3)	Average birth rate during the 15 years preceding enumeration: From registered births (4)	From proportion under age 15, $C(15)$, survivorship probability, $l(5)$, and growth rate, r (5)
(a) *Survivorship probability estimated from data on children ever born and surviving for women aged 30-34*				
Belgium	1961-1970	Dec. 1970	0.0163	0.0167
Bulgaria	1956-1965	Dec. 1965	0.0184	0.0180
Costa Rica	1963-1973	Mar. 1973	0.0406	0.0417
Poland	1960-1970	Dec. 1970	0.0202	0.0199
Yugoslavia	1961-1971	Mar. 1971	0.0215	0.0213
(b) *Survivorship probability obtained from published life tables corresponding to about 7.5 years before enumeration*				
Hong Kong	1960-1971	Mar. 1971	0.0302	0.0295
Japan	1950-1965	Oct. 1965	0.0190	0.0196
Netherlands	1918-1922-1933-1937	1933-1937	0.0235	0.0238
Sweden	1798-1802-1833-1837	1813-1817	0.0319	0.0313
Sweden	1918-1922-1933-1937	1933-1937	0.0167	0.0167
Sweden	1948-1965	1965	0.0148	0.0150

model stable populations are presented in this chapter. Experience has shown that these variants of the more general procedures described in a United Nations manual[5] are more likely to yield acceptable parameter estimates in populations that are not truly stable (because of recent changes in mortality) or whose basic data are distorted by reporting errors. Model stable populations can also be used to assess the extent of such errors and to identify typical distortion patterns in reported age distributions; and because it is important to explore the quality of the available data before any estimation method is applied, one of the sections that follows is devoted to the description of a simple procedure that allows the assessment of age distributions. To guide the reader in the use of this chapter, the contents of the sections are briefly described below (table 142 presents the data requirements of, and the parameters estimated by, the methods suggested):

TABLE 142. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER VII

Section	Type of input data	Estimated parameters
B. Evaluation of age distributions	Population classified by five-year age group and by sex	Assessment of data quality, with particular emphasis on age-reporting
C. Estimation of fertility from the proportion of the population under age 15 and the probability of surviving to age 5	Population classified by five-year age group (and by sex, if available) An estimate of $l(5)$, the probability of surviving from birth to age 5, for both sexes and referring to the middle of the 15-year period preceding enumeration	Average birth rate over the 15-year period preceding enumeration Average total fertility during the same period Average gross reproduction rate during the same period
	Population classified by five-year age group An estimate of the growth rate for the 15-year period preceding enumeration An estimate of $l(5)$ for both sexes, referring roughly to the mid-point of the 15-year period preceding enumeration	Adjusted estimate of the birth rate
D. Estimation of the expectation of life at age 5 and of the death rate over age 5 from the proportion under age 15 and the rate of increase	Population classified by five-year age group and by sex An estimate of the rate of natural increase for the period preceding enumeration (usually obtained from census counts at two points in time)	Expectation of life over age 5 Death rate over age 5 A life table over age 5

Section B. Evaluation of age distributions. This section presents a simple procedure for assessing the quality of a reported age distribution and its similarity to that of a truly stable population;

Section C. Estimation of fertility from the proportion of the population under age 15 and the probability of surviving to age 5. This section presents a method that allows the estimation of the birth rate and of total fertility by fitting a stable population on the basis of the reported proportion under age 15 for both sexes, denoted by $C(15)$, and an estimate of $l(5)$, the probability of surviving from birth to age 5, also for both sexes;

Section D. Estimation of the expectation of life at age 5 and of the death rate over age 5 from the proportion under age 15 and the rate of increase. This section presents a method that allows the estimation of mortality over age 5 by fitting a stable population on the basis of the reported proportion under age 15 for both sexes, $C(15)$, and an estimate of the rate of natural increase, r, of the entire population.

B EVALUATION OF AGE DISTRIBUTIONS

1. *Basis of method and its rationale*

Knowledge of the population classified by age and sex, that is, the population age distribution, is generally a starting-point in identifying an appropriate model stable population. Because of major errors that may be present in the recorded age distribution, it is not always easy to select a model stable population that best fits the underlying true age distribution, even when that distribution is very close to being genuinely stable.

[5] *Manual IV: Methods of Estimating Basic Demographic Measures from Incomplete Data* (United Nations publication, Sales No. E.67.XIII.2)

There are two main causes of error in age distributions: the selective omission of persons of a given age; and the misreporting of age of those counted. Very often, it is impossible to distinguish the magnitude of each of these two types of error, since their effects are similar. For example, a deficit of persons in a certain age group may be caused by their total omission or by their transference to other age groups, or by both factors combined. Age-misreporting is generally the most prevalent error in age distributions, but differential coverage by age and sex is also a likely cause of some distortions for particular age ranges.

Misclassification of the enumerated population according to age is often due to defective age-reporting by its members (or to defective estimation of age by interviewers). A very common type of age-misreporting is known as "age-heaping", which consists of the tendency on the part of respondents or interviewers to "round" the reported age to a slightly different age ending in some preferred digit. In practice, ages ending with zero or five are usually preferred over other endings (one or nine, for example), so age distributions by single years of age often exhibit peaks at ages 20, 25, 30 and so on, while troughs are associated with ages 29, 31, 39, 41, etc. If, for cultural reasons, some other ages are important to the population in question, they may also attract more respondents than would be expected. Thus, it is frequently found that age 12 is more attractive than either age 10 or age 15.

Age-heaping is reduced when age can be ascertained by means of a question about date of birth rather than about age itself. Respondents are less likely to round year of birth than age, although interviewers, when assigning ages in cases of uncertainty, may mix the rounding of the year of birth with the translation of rounded ages into years of birth. In addition, some heaping on years of birth ending in preferred digits may be expected; and the analyst must bear this possibility in mind as its age pattern may not resemble that of conventional age-heaping. Furthermore, this method of investigating age (through date of birth) only performs better than a direct question on age if most members of the population know their date of birth (an age distribution based on dates of birth may exhibit less heaping than one based on information about age but be no more accurate on an overall basis). In countries where a sizeable proportion of the population is illiterate and where the reckoning of age is traditionally not important, the use of a question on age may be equally appropriate, since often the respondents will not—or cannot—supply a year of birth or even an age, which must therefore be estimated by the interviewer.

When an age distribution by single year of age is available, the incidence of age-heaping can be easily ascertained by plotting the numbers observed at each age. If age-reporting were perfect and there were no omission, the age distribution would approximate a smooth curve, disturbed only by genuinely large or small cohorts resulting from wars or other episodes causing sharp fluctuations in the birth rate or in infant mortality. However, such genuine fluctuations will not recur at the same digits in the successive age decades.

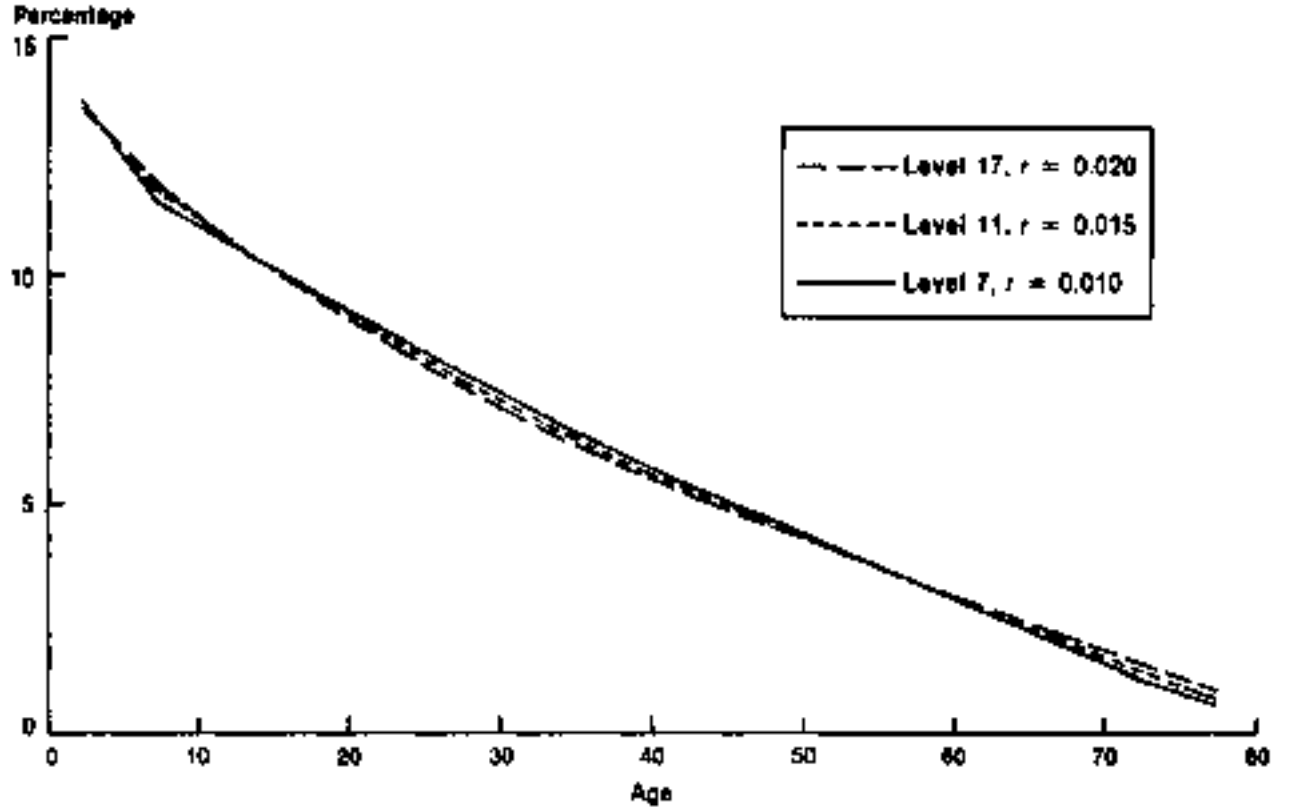
Because most age distributions are more or less affected by heaping, the use of data classified by single year of age is not recommended for most demographic analysis. Rather, cumulated data should be used, since cumulation reduces the effects of heaping and other forms of age-misreporting. The most common form of cumulation is a five-year group, which, though less sensitive to heaping than a single-year group, is still quite affected by transfers into and out of each age group. For most countries, a graph of the numbers observed within each five-year age group, though less ragged than the plot of the age distribution by single years, still exhibits marked irregularities.

A better way of reducing the effects of heaping, and of age-misreporting in general, is to cumulate from age zero to age x (usually a multiple of five); or, equivalently, from age x to the upper age-limit of the population, ω. Age groups of this type are affected only by transfers across their only boundary (age x) and therefore introduce a strong smoothing element. Because of their robustness to age-misreporting, this type of age group is used when identifying a best fitting stable population.

A simple procedure employing the Coale-Demeny model stable populations makes it possible to gain a visual impression of how closely the reported age distribution resembles a stable age distribution; or, conversely, how much it is distorted by misreporting, selective omission or genuine differences. The method is based on a remarkable feature of stable populations that are within the same family (North, South, East or West) of the Coale-Demeny model stable populations. Each stable age distribution generated by a particular combination of a model life table and rate of increase is matched very closely by other stable distributions formed by life tables with the same overall pattern but with quite different levels of mortality, combined with different rates of increase. For example, as shown in figure 17, the following model stable populations in the family of West model stable populations for females are very similar: that with an expectation of life at birth of 35 years and a growth rate of 0.010; that with an expectation of life at birth of 45 years and a growth rate of 0.015; and that with an expectation of life at birth of 60 years and a growth rate of 0.020. In other words, a genuinely stable population can be rather closely matched (in terms of its proportions in successive five-year age groups, and especially in terms of its cumulative proportions under successive ages five years apart) by an appropriately chosen model stable population at an arbitrary level of mortality. It follows that, in order to judge the congruence of a reported age distribution with that of a stable population, it suffices to compare the recorded age distribution with the model stable populations generated by a particular mortality level (arbitrarily chosen) and different rates of increase.

Specifically, the procedure recommended is the determination of the sequence of birth rates, $b(x)$, for $x = 5$,

Figure 17. Comparison of age distributions of different stable populations, West model



10, . . . , 70, 75, 80, calculated by finding, through interpolation, the model stable populations (at an arbitrarily chosen level of mortality) that have the same proportions under these ages x as the population in question.

The diagnostic value of the $b(x)$ sequence arises from the remarkable feature of stable populations mentioned earlier. If the $b(x)$ sequence is calculated for a truly stable population, the value of $b(x)$ for different ages x is very nearly constant. For example, a model stable population with an expectation of life of 40 years will generate a sequence of $b(x)$ values that is virtually constant with x when using model stable populations based on a life table with an e_0 of 50 years. Ups and downs in an estimated $b(x)$ sequence indicate the extent to which the given age distribution conforms to or differs from a stable population. When the age distribution is distorted by age-misreporting, or age-selective omission, $b(x)$ may be far from constant. A value of $b(x)$ that is too high indicates that too high a fraction of the population is reported as under age x, either because of a downward transfer of persons across age x by age-misreporting or because of relatively greater omission of persons above age x. Conversely, a value of $b(x)$ that is too low indicates either an upward transfer of persons from below age x to above age x by age-misreporting or relatively greater omission of persons under age x. ("Too low" and "too high" are in relation to the hypothetical constant value of the $b(x)$ estimates that would have been produced with a genuinely stable population having an accurately recorded age distribution.)

2. *Data required*

The only information required is the age distribution of a population by sex. This distribution can be calculated from the enumerated population classified by five-year age group and by sex. Availability of the population counts by single year of age is useful but not necessary.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of cumulated age distribution. Throughout this chapter, $C(x)$ is used to denote the proportion of the population under exact age x. This proportion, for a given age x (assumed to be a multiple of five), is calculated by adding the population of each five-year age group from 0 to 4 to $x-5$ to $x-1$ and dividing the resulting sum by the total enumerated population (excluding those of unknown age). When the basic data are classified by sex, the proportions $C(x)$ are calculated for each sex separately. In general, if ${}_5N_x$ is used to denote the population aged from x to $x+4$, and N is the total population, then

$$C(x) = [\sum_{y=0}^{x-5} {}_5N_y]/N.$$

Step 2: estimation of a set of representative birth rates at mortality level 13 of the Coale-Demeny model stable populations. This step constitutes the computational core of this procedure; and it requires that, within the set of model stable populations subject to mortality level 13 of the Coale-Demeny life-table system, those whose proportions under ages x match the reported $C(x)$ values be identified by means of a birth rate. To indicate that a given birth-rate estimate determines the stable population consistent with the observed $C(x)$ value at mortality level 13, such an estimate is denoted by $b(x)$. Note should be taken that this birth rate still refers to the whole of the stable population. The index x does not mean that only a portion of the stable population is considered.

The set of $b(x)$ estimates is obtained by interpolating linearly (see annex IV) between the printed $C(x)$ values of the model stable populations. For a given, reported value of $C(x)$, denoted by $C_o(x)$, the two stable populations whose $C(x)$ values bracket this value (in the sense that $C_1(x) \leqslant C_o(x) \leqslant C_2(x)$) are identified; and $b(x)$ is obtained by interpolating between their corresponding birth rates, denoted here by $b_1(x)$ and $b_2(x)$, respectively. The detailed example that follows will help to clarify this calculation process.

Step 3: plot of estimated birth rates against age and interpretation of results. The birth-rate values, $b(x)$, obtained in the previous step are plotted against age, and any deviations they display from a horizontal line may be taken as indication of deviations from true stability or of the existence of distortions caused by age-reporting errors. See subsection B.1 for a discussion of the possible significance of certain types of deviations.

4. *A detailed example*

The age distribution from the 1960 census of Brazil is used to illustrate the application of the evaluation procedure (see table 143).

The steps of the computational procedure are given below.

Step 1: calculation of cumulated age distribution.

TABLE 143. AGE DISTRIBUTION AND FITTED BIRTH-RATE ESTIMATES FOR MALES, BRAZIL, 1960

Age group (1)	Proportion in each age group (2)	Age x (3)	Cumulated age distribution $C(x)$ (4)	Estimated birth-rate $b(x)$ (5)
0-4	0.1625	5	0.1625	0.04114
5-9	0.1477	10	0.3102	0.04316
10-14	0.1227	15	0.4329	0.04337
15-19	0.0986	20	0.5315	0.04253
20-24	0.0855	25	0.6170	0.04202
25-29	0.0727	30	0.6897	0.04151
30-34	0.0644	35	0.7541	0.04137
35-39	0.0563	40	0.8104	0.04147
40-44	0.0474	45	0.8578	0.04159
45-49	0.0398	50	0.8976	0.04186
50-54	0.0317	55	0.9293	0.04203
55-59	0.0235	60	0.9528	0.04163
60-64	0.0206	65	0.9734	0.04203
65+	0.0266	ω	1.0000	-

Column (2) of table 143 presents the recorded five-year age distribution of the male population of Brazil in 1960. Note that the raw census counts are not presented and that the proportions listed in column (2) refer only to five-year age groups. Therefore, although it is no longer necessary to divide by the total male population, the desired $C(x)$ must still be obtained by adding the proportions in column (2). For example, $C(25)$ is obtained as

$$C(25) = 0.1625 + 0.1477 + 0.1227 +$$

$$0.0986 + 0.0855 = 0.6170.$$

The full set of $C(x)$ values is shown in column (4) of table 143.

Step 2: estimation of a set of representative birth rates at mortality level 13 of the Coale-Demeny model stable populations. Calculation of the values of representative birth rates, $b(x)$, identifying the West model stable populations at level 13 having the same $C(x)$ values as those of Brazilian males is carried out by interpolating linearly (see annex IV) between the printed model stable populations (in this case, those referring to males are used). To illustrate the process followed, $b(25)$ is calculated here, that is, the birth rate in the male stable population at level 13 having a $C(25)$ identical to the reported value. According to the printed West model stable populations for males at level 13,[6] a stable population with a birth rate of 0.03756 has a $C_1(25) = 0.5813$, while that with a birth rate of 0.04230 has a $C_2(25) = 0.6193$ (note that the printed stable population values have to be divided by 100 to conform with the conventions followed in this *Manual*, while the printed birth rates need to be divided by 1,000 for the same reason). Therefore, to find the value of $b(x)$ corresponding to the observed $C(25)$ of 0.6170, the interpolation factor θ is

$$\theta = (0.6170 - 0.5813)/(0.6193 - 0.5813) = 0.94.$$

Hence

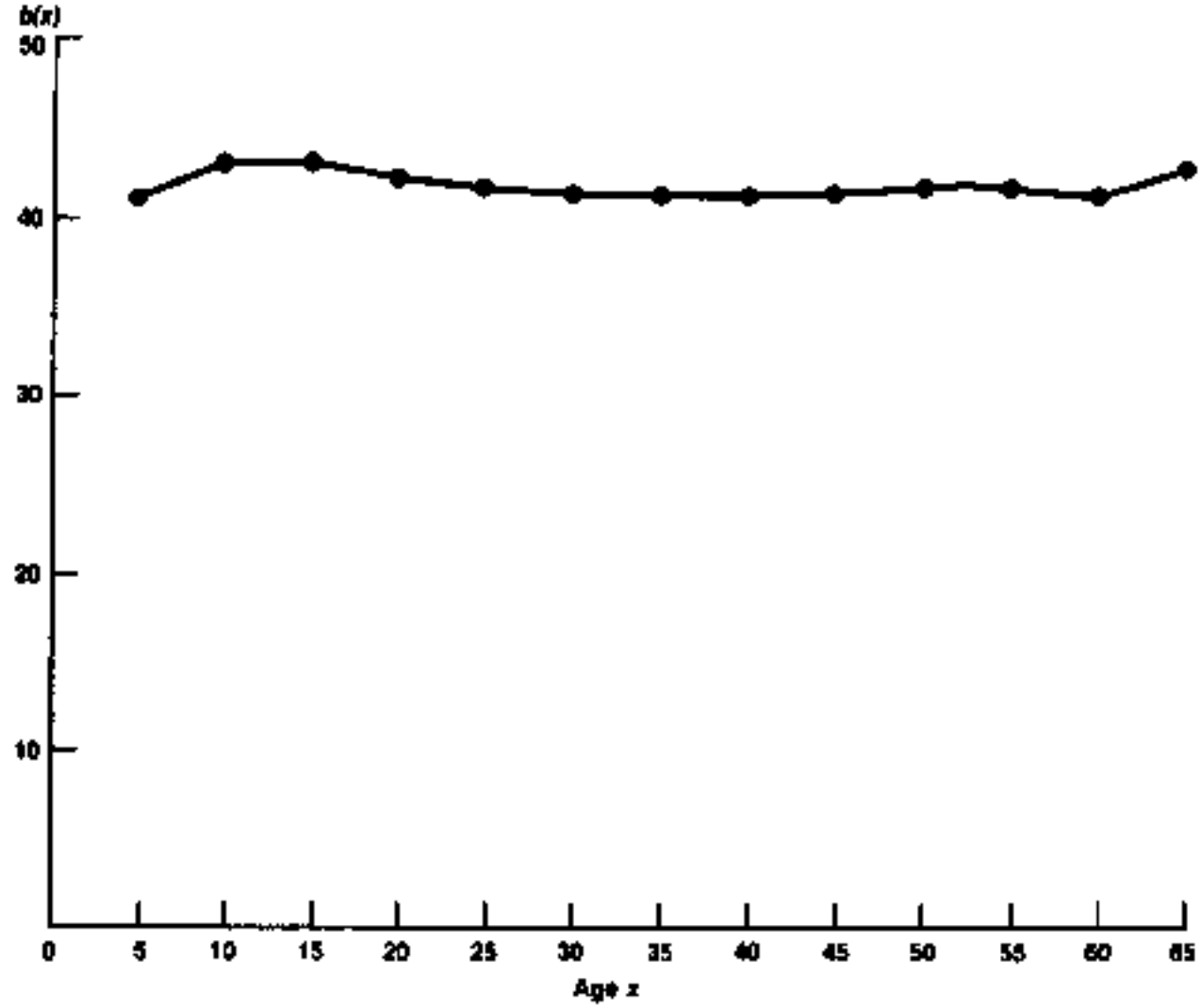
$$b(25) = 0.06(0.03756) + 0.94(0.04230) = 0.04202.$$

The full set of $b(x)$ values estimated in this manner is shown in column (5) of table 143.

Step 3: plot of estimated birth rates against age and interpretation of results. Figure 18 shows a plot of the birth-rate estimates, $b(x)$, against age. It appears that, according to these estimates, the age distribution of the male population of Brazil in 1960 is fairly consistent with that of a stable population (the plotted points do not deviate markedly from a horizontal line). However, the recorded age distribution is not entirely free from distortions. For example, the relatively high $b(x)$ values observed at ages 10 and 15 imply that males older than those ages have been reported as younger, and some distortions are also evident at older ages. Yet, as the exam-

[6] Coale and Demeny, *op. cit.*

Figure 18. Plot of the estimated birth-rate, $b(x)$, sequence based on the model stable male population, West family, level 13; Brazil, 1960



ples that appear in the next section prove, the observed distortions are minimal; and, in and by themselves, they would not be sufficient to preclude the use of estimation methods based on model stable populations in this case.

5. *Typical patterns of the birth-rate estimates*

The evaluation procedure described in this section was applied to a series of reported and model stable populations in order to ascertain whether typical patterns of the birth-rate estimates, $b(x)$, with respect to age could be identified. The seven panels of figure 19 show plots of the $b(x)$ estimates obtained in each case. They were all calculated by interpolating between the female model stable populations generated by a West model life table at level 13. Panel A of figure 19 shows the values of $b(x)$ for populations that are genuinely stable, or nearly so, and that are not much distorted by age-misreporting. Two of the sets of $b(x)$ values in this panel are derived from model stable populations. The stable populations selected have values of life expectancy at birth of 40 and 60 years, respectively, instead of the expectation of life of 50 years of the reference stable population of level 13. These examples illustrate the type of deviations from a horizontal line that may be expected solely on account of selecting an inappropriate mortality level as a basis for comparisons. As stated above in subsection B.1, such deviations are minor and follow a systematic pattern with respect to age.

It is appropriate to add at this stage that, in the discussion that follows, age-misreporting is implicitly assumed to be the main cause of deviations of the $b(x)$ sequence from the expected horizontal line. In certain populations, however, these deviations may, in fact, be mainly due to lack of stability or, as exemplified in panel A, to true discrepancies between the population's mortality pattern and that embodied by the model stable populations being used. Because it is difficult, if not impossible, to disentangle in practice the effects due to each of these phenomena (age-misreporting, lack of stability

Figure 19. Birth-rate estimates, $b(x)$, obtained from the proportions under age x, $C(x)$, in different populations
Panel A

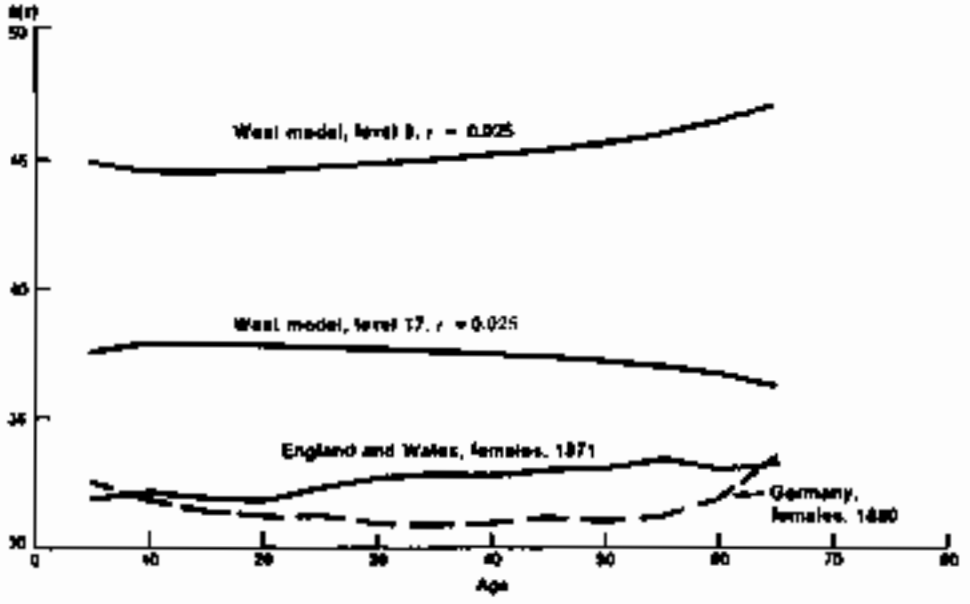
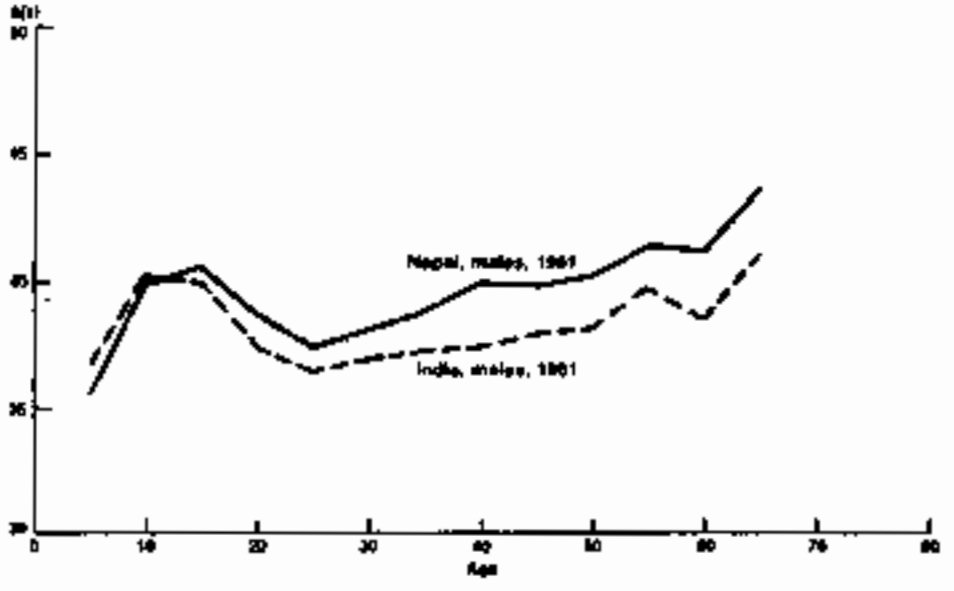


Figure 19 (continued)
Panel B



and inadequacy of the selected model), casting the discussion in terms of the most likely cause is both expeditious and acceptable, especially because, irrespective of their main cause, large deviations of the $b(x)$ sequence from a constant value suggest that the basic data may not be trustworthy and that "stable" estimates derived from them must be carefully assessed before being accepted.

Proceeding with the discussion of the curves displayed in panel A, note that two for populations that are very nearly stable and whose data are reliable, namely, Germany, and England and Wales, both in the late nineteenth century, have been included. These sets of $b(x)$ values, although not perfectly horizontal, vary within a narrow range. They serve to illustrate the case in which, even when the data are reliable, perfect agree-

Figure 19 (continued)
Panel C

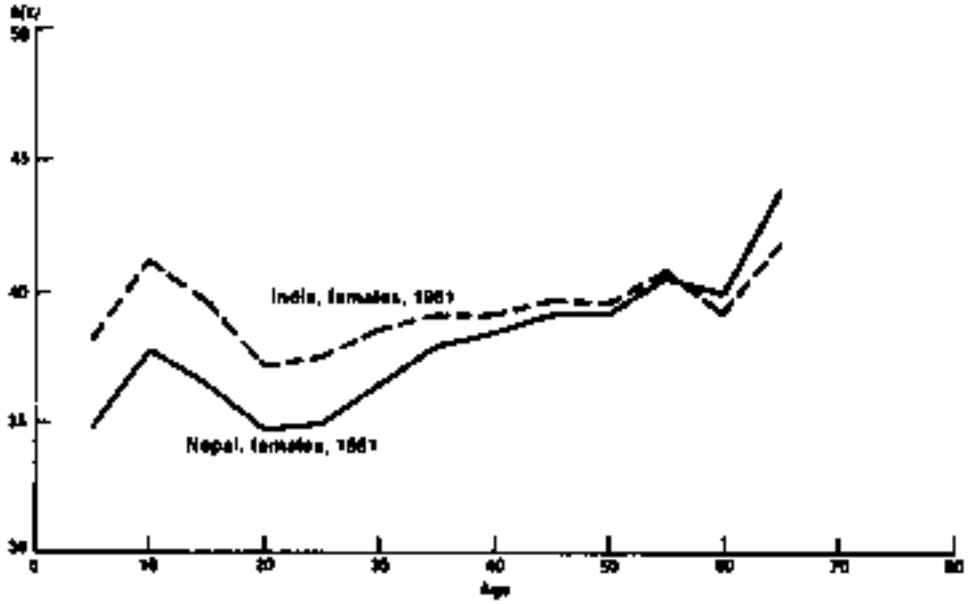
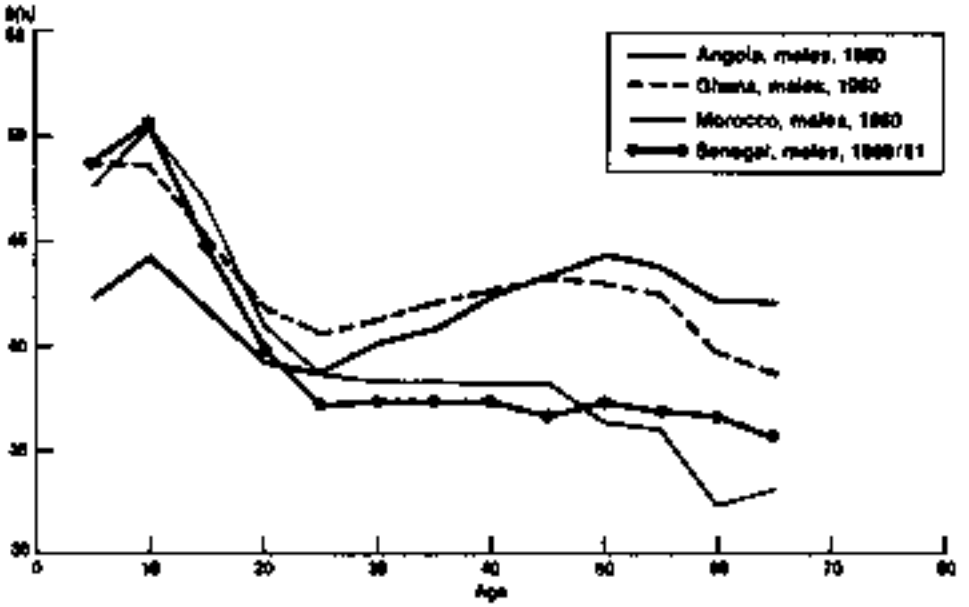


Figure 19 (continued)
Panel D



ment between the models and the observed data is not achieved.

In other panels, the $b(x)$ values derived from the age distributions of males and females are plotted separately. Age distributions for several countries are considered: India and Nepal; four populations in Africa; three populations in Latin America; and the Philippines. The $b(x)$ sequences for India and Nepal show deviations of similar magnitude and some similarities of pattern, particularly for females, to the sequences for Africa (Angola, Ghana, Morocco and Senegal). The values of $b(x)$ for females in these populations have typically a local maximum at age 10, followed by a steep descent to a low point at 20, and then a gradual increase. For the males in these populations, the descent of $b(x)$ from 10 to 15 is less marked (or non-existent); and the lowest point is reached at age 25 instead of at age 20. A descending segment of $b(x)$ from x to $x+5$ implies that this five-year age group comprises a lower proportion of the reported population than of the stable population

Figure 19 (continued)
Panel E

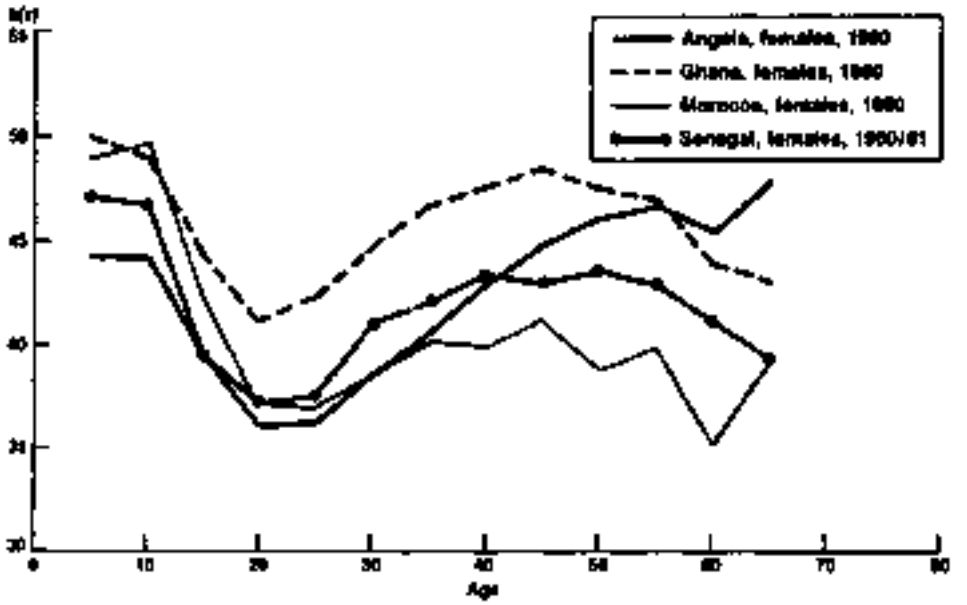
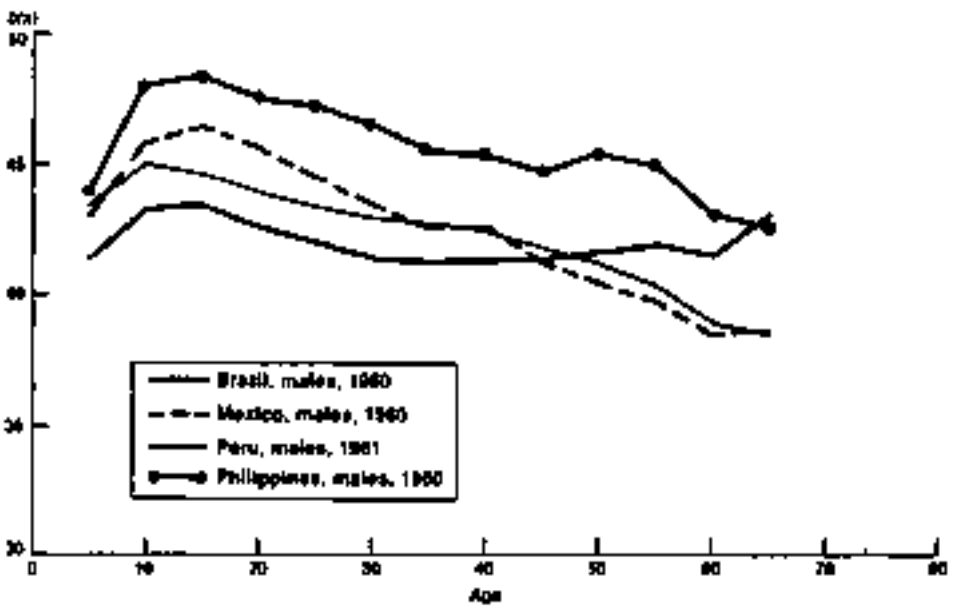


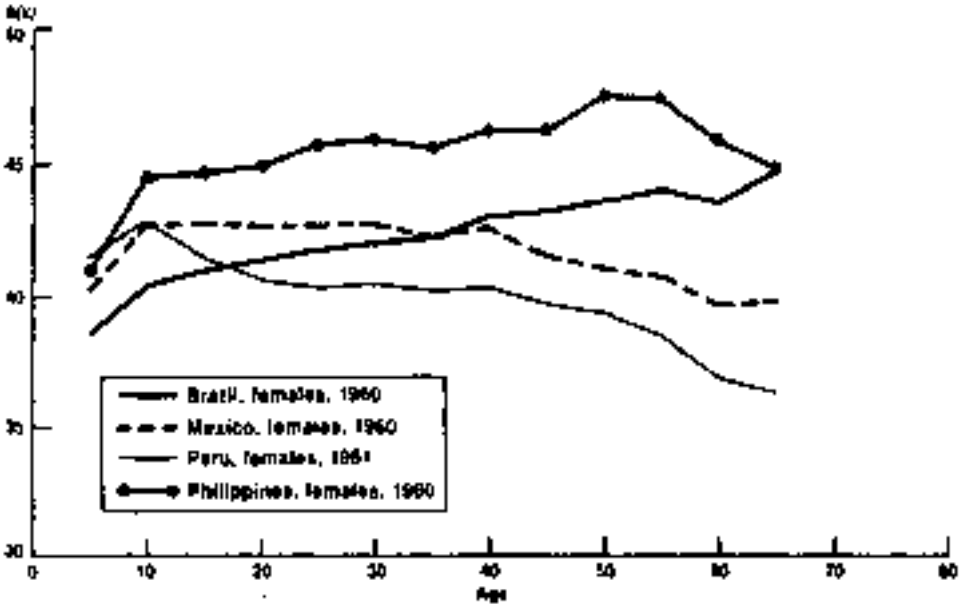
Figure 19 (continued)
Panel F



fined to $C(x)$, and a rising segment implies that there is a higher proportion of the reported population in this five-year age interval than in the stable population with the same $C(x)$. Thus, the rise in the $b(x)$ sequences for India and Nepal from age 5 to age 10 implies a reported proportion in age group 5-9 higher than that in the stable population, and the universal decline from 10 to 15 and from 15 to 20 for females implies that these two age groups (10-14 and 15-19) comprise a lower proportion of the reported than of the stable population. The low values at 20 and 25 imply a substantial upward transfer (caused by age-misreporting) across these two age boundaries.

The $b(x)$ sequences for selected male and female populations from Latin America and the Philippines show less extreme ups and downs than do the sequences for the populations of Africa and other countries of Asia. Moreover, their overall pattern is wholly different. The value of $b(15)$ for males is usually a local maximum, and the sequence of values above age 15 for males usually follows a downward trend. If this downward trend is the effect of age misstatement, it implies

Figure 19 (continued)
Panel G



that there is a progressively greater tendency for males to overstate age. The values of $b(x)$ for females in Brazil and the Philippines follow an upward trend from the earliest point to age 50 or higher. This upward trend may be caused either by a progressively greater understatement of age as age increases or by a higher omission rate among older females. The fairly regular pattern of deviations with respect to age also suggests that other phenomena, such as a discrepancy between the model and the true patterns of mortality, may be responsible for their existence. However, in spite of these distortions, it is clear that the age distributions reported by populations in Latin America are less irregular than those reported in Africa or in India.

The examples displayed in this section show that, in the seriously distorted age distributions reported for populations of Africa or South Asia, there is typically an understated proportion of females in the age interval from 10 to 14, as evidenced by the steep negative slope of $b(x)$ from 10 to 15 in the female populations shown in panels C and E of figure 19. It also appears that there is often an upward transfer across age 15 because of age-misreporting in these female populations. It is probable that such misstatement arises because the interviewer has to estimate the age of young women in cases where the woman interviewed has passed puberty, and especially when she is already married, her age is likely to be estimated as 15 or over.

In contrast, the value of $b(x)$ for males in the Asian populations shown in panel B does not decline much if at all, from 10 to 15; and for the populations of Africa (panel D), the decline is less sharp than for females.

This discussion suggests that for South Asian populations, $C(15)$ for both sexes combined may be a good choice as the age-distribution parameter to which a stable population is fitted, since it appears that the tendency for males to be transferred downward across age 15 is balanced, to some extent, by a tendency for females to be transferred in the other direction. Furthermore, $C(15)$ for both sexes may be an appropriate choice for the pattern for Africa, since although there is no evidence in panels D and E that male and female errors around age 15 might balance each other, in both cases the point for 15 does seem to approximate the average of all the $b(x)$ values. However, there are exceptions: for the male population of Morocco, for example, $b(15)$ is clearly well above the average of all the $b(x)$ values. Thus, though $C(15)$ may be a good starting-point for selecting a stable population, there is no guarantee that it will be the optimal choice; and a number of selections based on different $C(x)$ values may be required. In the sections that follow, however, the presentation of the estimation methods suggested is in terms of $C(15)$ for both sexes combined, but the principles involved may be applied to other $C(x)$ values if a detailed examination of the age distribution suggests that such an approach may be necessary.

C. Estimation of fertility from the proportion of the population under age 15 and the probability of surviving to age 5

1. *Basis of method and its rationale*

The selection of a model stable population on the basis of the proportion of the population under age 15 (both sexes) and the probability of surviving to age 5, $l(5)$, in order to estimate the birth rate, has several of the desirable properties identified in subsection A.1. The proportion under age 15 for both sexes combined is often less affected by age-misreporting than other points on the cumulated age distribution, although it must be conceded that in some circumstances, when there are

high omission rates of young children or when certain problems arise at the time of interview (as, for instance, when heaping concentrates at age 14 as a result of women trying to avoid being interviewed individually in surveys where the target population is that aged from 15 to 49), $C(15)$ can be either too low or too high. Therefore, although fairly robust to the effects of typical age-misreporting patterns, the estimation methods based on $C(15)$ may still yield biased estimates in particular situations. Hence, a careful evaluation of the accuracy of $C(15)$ is recommended before these methods are applied.

The use of $I(5)$ as an indicator of mortality in childhood also enhances the overall robustness of this method. As described in chapter III, $I(5)$ can be estimated from the proportion of children surviving among those ever borne by women aged from 30 to 34 (or duration of marriage of 10-14 years). This estimate of mortality is usually fairly reliable; and when mortality has been changing, it refers to a period located some six or seven years before the time of the interview, about the appropriate time reference for estimating the average birth rate during the 15 years preceding enumeration.

Thirdly, model stable populations identified on the basis of values of $C(15)$ and $I(5)$ from the four families of Coale-Demeny models have nearly the same birth rate and the same total fertility. Such consistency does not extend to stable populations from the different models having the same $C(15)$ and rate of increase. For example, in model stable populations with a $C(15)$ value of 0.4195 and an $I(5)$ of 0.8185, the birth rate ranges only from 0.0408 to 0.0414; and total fertility varies only from 5.70 to 5.89 (to calculate these total fertility rates, the mean age of the fertility schedule was assumed to be 29 years and the sex ratio at birth used was 105 male births per 100 female births). On the other hand, in model stable populations with the same value of $C(15)$ (0.4195) and a rate of increase of 0.025, the birth rate ranges from 0.0408 to 0.0440 and total fertility varies from 5.70 to 6.22.

Lastly, and most surprisingly, the selection of a model stable population on the basis of $C(15)$ and $I(5)$ provides an estimate of the birth rate that closely matches the average birth rate during the 15 years preceding enumeration, even when the population in question is far from stable.

The remarkable accuracy of this approximation in the case of non-stable populations can be explained in heuristic terms by viewing the estimation of a stable birth rate from $C(15)$ and $I(5)$ as a form of reverse projection yielding an estimate of the average birth rate during the 15 years preceding the time of enumeration. In a stable population, the average birth rate during this period is, of course, the same as the current birth rate, but it could be estimated from the stable age distribution by the conventional procedure of reverse projection. The number of births in the period, estimated by reverse projection to birth of the population in each age interval under 15, divided by the number of person-years lived during the period by the total population (estimated by reverse projection of the current population using the stable rate of natural increase), would yield the desired birth-rate estimate. For a non-stable population, the equivalent calculations would involve the same reverse projection to birth of the persons in each five-year age group in the 0-14 range, and the division of the resulting total number of births by the total population reverse-projected according to the actual rate of increase.

A stable population selected so that its parameters match the observed $C(15)$ and $I(5)$ values must incorporate a life table with $l(x)$ values up to age 15 that are fairly similar to those characterizing mortality in the given population over the preceding 15 years or so. The reported population under 15 may be allocated to the three five-year age groups in the 0-14 range in a different way than that of the fitted stable population, but because of the low mortality rates that usually characterize these ages (at least from age 3 to age 14), the differences in the age distribution under 15 between the reported and the stable populations should have only a slight effect on the number of births calculated by reverse projection. Thus, the fitted stable population may be expected to provide an adequate representation of the numerator of the birth rate, that is, the number of births in the preceding 15 years.

A more important difference is likely to affect the denominator; and it arises from differences between the observed and the stable rates of increase, especially if there has been a recent change in fertility. In such an instance, the stable population fitted to the observed $C(15)$ and $I(5)$ values is unlikely to have the same growth rate as the reported population had over the preceding 15 years; in general, the stable population has a lower growth rate. Continuing with the analogy to reverse projection, the birth rate of the stable population could be improved, as an estimate of the true average birth rate over the preceding 15 years, if an adjustment were made for the difference between the true and the stable growth rates in order to estimate more accurately the denominator of the birth rate. If the stable growth rate is lower than the true growth rate, according to the reverse-projection analogy, it would result in somewhat too high an estimate of the person-years lived by the population over the preceding 15 years, which in turn would give rise to too low an estimate of the birth rate. Thus, the birth rate can be adjusted upward by multiplying it by the ratio of the number of person-years lived implied by the stable growth rate to the true number of person-years actually lived by the reported population, that is, by $N_o \exp[-7.5r_s]/N_o \exp[-7.5r_o]$, where N_o is the total reported population; r_s is the stable growth rate; and r_o is the reported growth rate. If the difference between the true average growth rate over the preceding 15 years and the growth rate of the fitted stable population is 0.002 (say, a true value of 0.022 against a stable value of 0.020), for instance, the adjustment for the denominator can be made directly to the birth rate of the fitted stable population, multiplying it by the factor $\exp[7.5(0.022-0.020)]$. The adjustment would thus be about 1.5 per cent upward. A similar procedure should

also prove effective in cases of destabilization resulting from mortality change. It should be pointed out, however, that the adjustment does not always improve the initial estimate of the birth rate, particularly in the case of very recent fertility change. Further, the adjustment may not be suitable in practice if the reported growth rate is distorted by changes in enumeration completeness.

Table 141 shows the results obtained when this estimation procedure was applied to several populations with accurate birth registration. The average birth rates estimated from $I(5)$ and $C(15)$ are compared with the average birth rates registered during the 15-year period preceding each census. In the first part of table 141, $I(5)$ has been estimated from data on children ever born and surviving. In the second part, covering countries for which such data are not available, the value of $I(5)$ has been taken from official life tables referring approximately to the period 7.5 years before each census. In these cases at least, the procedure yields very good estimates of the birth rate for populations that are clearly not stable.

2. *Data required*

The following data are required for this method:

(*a*) The enumerated population classified by five-year age group and by sex. Strictly speaking, only the population of both sexes under age 15 and the total population enumerated are required. However, in order to assess the quality of the data (for example, by using the procedure described in section B), further classification by sex is necessary;

(*b*) An estimate of $I(5)$ for both sexes combined referring approximately to the period between six and eight years before the time of enumeration. (Refer to chapter III for methods yielding this estimate from data on children ever born and surviving.);

(*c*) An estimate of net migration during the period preceding the time at which the population was enumerated. If net migration is substantial, stable population analysis should be carried out only after the reported age distribution has been adjusted for its effects;

(*d*) An estimate of the sex ratio at birth;

(*e*) An estimate of the growth rate during the 15 years or so preceding enumeration (this estimate is necessary only if there is evidence suggesting that the reported population is not approximately stable and if, as a result, an adjustment of the stable birth rate for the difference between the reported and stable growth rates is desirable).

3. *Computational procedure*

Step 1: calculation of proportion under age 15 for both sexes. The calculation of this proportion, $C(15)$, is carried out according to the general principles described in step 1 in subsection B.3. Essentially,

$$C(15) = ({}_5N_0 + {}_5N_5 + {}_5N_{10})/N \qquad \text{(C.1)}$$

where ${}_5N_x$ is the population of both sexes aged from x to $x+4$; and N is the total enumerated population (excluding those of unknown age).

Step 2: identification of a mortality level consistent with the probability of surviving to age 5. The desired mortality level is identified by interpolating linearly (see annex IV) between the probabilities of surviving to age 5, $l(5)$, listed in annex VIII for the mortality pattern selected as most suited for the country concerned. Note that the mortality level should be found for males or females, but not for both sexes combined, since although $I(5)$ refers to both sexes combined, identification of a mortality level is carried out using the tables referring to a given sex (male or female). The reason for this procedure is that model stable populations referring to both sexes combined are not published in the Coale-Demeny volume.[7]

Step 3: identification of the stable population with the same probability of surviving to age 5 and proportion under age 15 as the reported population. Using the mortality level identified in step 2 and the selected family of models for a given sex (male or female), two populations at the estimated mortality level are found whose proportions under age 15 enclose the reported $C(15)$ for both sexes, denoted by $C_*(15)$, in the sense that $C_1(15) \leqslant C_*(15) \leqslant C_2(15)$. Once identified, interpolation between the parameter values of the two stable populations yields the required stable estimates (the birth rate, the growth rate and the death rate, for example). Note again that, although the observed $C_*(15)$ refers to both sexes combined, interpolation is carried out only within the stable populations referring to one selected sex (male or female). This procedure implies that one is assuming the underlying pattern of mortality of both sexes of the population under study to be similar to either the male or female model chosen as reference.

Step 4: calculation of gross reproduction rate and of total fertility for the selected stable population. The population identified in step 3 refers to both sexes combined, so that the calculation of parameters that refer only to the female population is not straightforward. To obtain their values, it is necessary to identify the female stable population associated with that already selected for both sexes combined. A serviceable choice of a female stable population is made by estimating the female birth rate as

$$b_f = b_t \frac{B_f / B_t}{N_f / N_t} \qquad \text{(C.2)}$$

where b_t is the estimated birth rate for both sexes combined; B_f / B_t is the proportion of births that are female (B stands for the number of births); and N_f / N_t is the proportion of the population that is female (N stands for number of persons). The proportion of female births can be taken as about 48.8 per cent of the total, except in tropical Africa, or in populations of primarily African origin, where a proportion of about 49.3 per cent should

[7] *Ibid*

be used. The proportion female in the population can be taken from the census, unless sex selective omission is evident.

Note that B_f/B_i can also be written as

$$B_f/B_i = 1.0/(1.0+SRB) \qquad (C.3)$$

where SRB is the sex ratio at birth defined as the number of male births per female birth. Therefore, knowledge of SRB and of the total male and female population counts is sufficient to calculate b_f from b_i.

Once b_f is available, using the mortality level identified in step 2, the value of the gross reproduction rate, GRR, can be estimated by interpolation within the female set of stable populations of the family being used. However, the printed model stable populations give only estimates of the gross reproduction rate for certain mean ages of the fertility schedule. It is recommended that a first estimate of this rate be calculated by setting this mean age, μ, to 29. Then if μ can be more accurately estimated for the population in question (see annex III), the preliminary $GRR(29)$ estimate can be adjusted by using the following relation:

$$GRR(\mu) = GRR(29)\exp[(\mu-29)(r+\delta(\mu))] \qquad (C.4)$$

where $\delta(\mu)$ is the death rate at μ and is estimated as

$$\delta(\mu) = 0.5({}_5M_{25}+{}_5M_{30}) \qquad (C.5)$$

where ${}_5M_x$ is the central death rate for females aged from x to $x+4$ in the life table at the level identified in step 2.

Lastly, an estimate of total fertility is obtained as

$$TF = GRR(1.0+B_m/B_f) = GRR(1.0+SRB). \qquad (C.6)$$

It should be mentioned that step 4 should not be carried out if there is evidence suggesting that the population being studied is not approximately stable, since the estimates obtained according to the procedures described in this step refer only to a stable population. Therefore, especially when fertility has been declining, these estimates may be misleading.

Step 5: adjustment of stable birth-rate estimate when the population is not truly stable. If there is evidence suggesting that the population has experienced important fertility and mortality changes, the stable birth rate obtained in step 3 cannot be considered an adequate estimate of the corresponding parameter of the true population. Fortunately, if the average growth rate of the population during the 15 years or so preceding the time of enumeration is known, the stable birth rate estimate may be adjusted for the effects of declining fertility. As in other instances, the required growth rate is estimated on the basis of population counts at different points in time. The equation used to calculate it is

$$r_a = [\ln(N_2)-\ln(N_1)]/(t_2-t_1) \qquad (C.7)$$

where N_i is the total population enumerated at time t_i (including those of unknown age).

When r_a is different from the stable growth rate, r_s, estimated in step 3, the stable birth-rate estimate also obtained in that step can be adjusted by

$$b^* = b_i \exp[7.5(r_a-r_s)]. \qquad (C.8)$$

4. *A detailed example*

The case of Brazil in 1960 is used as an example. A census with 1 September as reference date was carried out in that year. Its results have already been used in chapter III, subsection E.4 (b) to obtain estimates of child mortality. Furthermore, the reliability of the reported age distribution of the male population was assessed above in subsections B.4 and B.5. As is often the case, the proportion under age 15 for males appears to be overstated, while there is no unambiguous bias in the female $C(15)$ (see panels F and G of figure 19). Hence, there seems to be no reason for fitting a stable population to any $C(x)$ in preference to $C(15)$; but, bearing in mind that errors in the male and female $C(15)$ values may not cancel each other out in this case, for illustrative purposes the method described in this section is applied.

The steps of the computational procedure are given below.

Step 1: calculation of proportion under age 15 for both sexes. Table 144 presents the basic population counts in the form of cumulated numbers of persons under selected ages. Classification by sex is available. According to this table,

$${}_{15}N_0 = 15{,}157+14{,}757 = 29{,}914$$

while

$$N = 35{,}012+35{,}083 = 70{,}095$$

so that

$$C(15) = 29{,}914/70{,}095 = 0.4268.$$

TABLE 144. POPULATION UNDER EXACT AGE x, $N(x-)$ BY SEX, BRAZIL, 1960
(*Thousands*)

Age x (*a*)	Population under exact age x	
	Males $M(x-)$ (*b*)	Females $F(x-)$ (*b*)
5	5 688	5 506
10	10 829	10 494
15	15 157	14 757
20	18 609	18 480
25	21 602	21 724
30	24 147	24 425
35	26 401	26 691
40	28 375	28 684
45	30 033	30 263
50	31 426	31 584
TOTAL	35 012	35 083

Step 2: identification of a mortality level consistent with the probability of surviving to age 5. The estimation of child mortality from data on children ever born and surviving collected during the 1960 census of Brazil, was carried out in chapter III, subsection E.4 (b). Table 79 in that chapter contains the raw data, while table 80 shows the resulting child mortality estimates. The estimate of $1(5)$ for both sexes combined is 0.8222. In subsection E.4 (b) of chapter III, it was stated that if the sex ratio at birth of the Brazilian population is assumed to be 1.05 male births per female birth, this value of $1(5)$ is consistent with level 13.6 of the West model life tables for both sexes combined. However, if in the application of the methods described in this chapter one were to use consistently life tables for both sexes combined, it would be necessary to generate new stable populations using those life tables, since the Coale and Demeny tables refer only to males or females separately. Given the robustness of the methods described here to variations in the model patterns of mortality (see section C.1), it does not seem necessary to insist on absolute consistency by using stable populations referring to both sexes combined. It is more expedient and scarcely less satisfactory from the point of view of the accuracy of the estimates obtained to use the sex-specific model stable populations published in the Coale-Demeny set as if they were adequate representations of the experience of both sexes combined. This approach is equivalent to assuming that if, for example, only the female tables are used, the mortality pattern embodied in the female life tables is an adequate representation of the mortality pattern prevalent in the whole population (both sexes combined). In the case of the method at hand, this assumption is more satisfactory than it would appear at first sight because only the mortality up to age 15 is of major importance; and for the age range 0-14, differences between the mortality pattern for both sexes combined and those for each one separately are relatively small.

In the case of Brazil in 1960, the female life tables for model West are assumed to be adequate representations of true mortality patterns. Accordingly, the model $1(5)$ values listed in table 236 (see annex VIII) are used to identify the mortality level corresponding to the observed value of 0.8222. As usual, linear interpolation is used (see annex IV) to find this level. The process involves searching the values listed under label "$1(5)$" in table 236 for two values that enclose the observed $1(5)$. In this case, they are $l_{13}(15) = 0.81848$ and $l_{14}(15) = 0.84106$, where the subindices indicate the level with which each value is associated. Then, the interpolation factor Θ is obtained as

$$\Theta = (0.82220 - 0.81848)/(0.84106 - 0.81848) = 0.16.$$

Hence, the female West level desired is 13.16.

Step 3: identification of the stable population with the same probability of surviving to age 5 and proportion under age 15 as the reported population. The first task in this step is to calculate the values of the proportion under age 15, $C(15)$, at level 13.16 for West female stable populations with different birth rates (or growth rates). Referring to the printed stable age distributions, one finds that at level 13, $C_1(15) = 0.4195$ for a population with an annual growth rate of 0.025, and $C_2(15) = 0.4539$ for a population growing annually at a rate of 0.030. The corresponding values at level 14 are $\hat{C}_1(15) = 0.4141$ and $\hat{C}_2(15) = 0.4487$. Hence, in order to obtain the corresponding estimates at level 13.16, one uses the interpolation factor Θ of 0.16 as follows:

$$C_1^*(15) = 0.84(0.4195) + 0.16(0.4141) = 0.4186$$

$$C_2^*(15) = 0.84(0.4539) + 0.16(0.4487) = 0.4531$$

where the superindex * indicates that the $C(15)$ values correspond to level 13.16. It remains to find the growth rate and birth rate with which the reported $C(15) = 0.4268$ is associated. Note that the $C^*(15)$ estimates presented above enclose the reported value as desired. The reported $C(15)$ is used to interpolate between the $C^*(15)$ values, the necessary interpolation factor being

$$\Theta = (0.4268 - 0.4186)/(0.4531 - 0.4186) = 0.24.$$

The growth rate associated with the reported $C(15)$ is found using Θ:

$$r_s = 0.76(0.025) + 0.24(0.030) = 0.0262.$$

Table 145 shows some of the intermediate results of these calculations. It also gives the estimates of the birth rate. Those at level 13.16 for growth rates 0.025 and 0.030 are obtained exactly in the same way as the $C^*(15)$ estimates given above. The birth rate for $r_s = 0.0262$ is then calculated as

$$b_s = 0.76(0.04051) + 0.24(0.04538) = 0.04168.$$

TABLE 145. IDENTIFICATION OF THE STABLE POPULATION DETERMINED BY THE REPORTED PROPORTION UNDER AGE 15 AT LEVEL 13.16, BRAZIL, 1960

	Female mortality level, West model					
	13		*14*		*13.16*	
Growth rate (*r*)	Proportion under age 15, $C_1(15)$ (1)	Birth rate (*b*)	Proportion under age 15, $C_2(15)$ (2)	Birth rate (*b*)	Proportion under age 15, $C(15)$ (3)	Birth rate (*b*)
0.0250	0.4195	0.04076	0.4141	0.03921	0.4186	0.04051
0.0300	0.4539	0.04565	0.4487	0.04398	0.4531	0.04538
0.0262	–	–		–	0.4268	0.04168

This value is therefore the stable estimate of the birth rate.

Step 4: calculation of gross reproduction rate and of total fertility for the selected stable population. As is shown in the next section, the stable population identified in step 3 is not an adequate representation of the Brazilian population in 1960. Hence, it makes no sense to calculate at this point other parameters of this stable population, because they are not likely to be acceptable as estimates for the actual population. For this reason, the application of step 4 is deferred (see subsection C.5).

Step 5: adjustment of stable birth-rate estimate when the population is not truly stable. Brazil carried out censuses on 1 July 1950 as well as on 1 September of 1960, and they yielded total population counts (including persons of unknown age) amounting to 51,976,000 and 70,119,000 persons, respectively. Hence, the 1950-1960 intercensal growth rate is calculated as follows:

$$r_e = \ln[70{,}119/51{,}976]/10.167 = 0.0295.$$

This growth rate is evidently much higher than that estimated in step 3 for the fitted stable population. Such a discrepancy suggests either that the Brazilian population was not stable, as a result of mortality or fertility changes the influence of which on the estimated birth rate must be taken into account, or that some other methodological problems may exist. Ignoring for the moment the possibility of the latter situation, for the sake of illustration, an adjusted birth rate is calculated by using equation (C.8) as follows:

$$b = b_s \exp[7.5(r_e - r_s)]$$

$$= 0.04168 \exp[7.5(0.0295 - 0.0262)]$$

$$= 0.04272.$$

5. *Comments on the detailed example*

Several remarks must be made about the example just presented and, as will be seen, some will lead to the complete modification of the estimates found above. The first comment, however, is mostly of a methodological nature and refers to the question whether male or female model stable populations should be used in applying this method. In the case just analysed, if the male West model stable populations had been used, the observed $I(5)$ for both sexes combined would have implied a mortality level of 13.97, and the preliminary estimates of the stable birth rate and growth rates would have been 0.04155 and 0.0260, respectively. Adjustment of the former by $\exp[7.5(0.0035)] = 1.0265$ would have yielded a final birth rate estimate of 0.04265, in which the first three significant figures agree with those of the estimate obtained in step 5 by using the female models. This is yet another example of the robustness of the estimation procedure to changes in the model mortality pattern used.

Next to be considered is an evaluation of the estimates obtained. The difference between the estimated stable growth rate of 0.0262 and the intercensal growth rate of 0.0295 could arise either because the 1960 census was more complete than the 1950 census, or because the West mortality pattern is an inadequate representation of the mortality of the Brazilian population (higher growth rates would be expected in populations with a given $C(15)$ subject to heavier mortality below age 5 and lower mortality above age 5 than those embodied by the West model).

To assess the influence that the selection of a certain mortality model has on the final estimates and to explore the fits of other models, the estimation process is repeated using the South model. The first step is to obtain adequate child mortality estimates also referring to the South model. The appropriate coefficients given in table 47 (see chapter III) and the observed $P(1)/P(2)$ and $P(2)/P(3)$ ratios given in table 80 (see chapter III), together with $D(4)$, are used to obtain the required estimate of $l(5)$. Its value for both sexes is 0.8201, implying a mortality level of 14.85 in the female tables. Repeating step 3 with the South family, the preliminary estimate of the birth rate obtained from $C(15)$ at level 14.85 is 0.04231, and the corresponding estimate of the growth rate is 0.02859. Had male stable populations been used instead of female, the corresponding results would have been 0.04221 and 0.02805, showing that the choice between the sex-specific stable models is not important. This growth rate is comfortingly close to the intercensal growth rate, implying an adjustment factor of only 1.0068 for the stable birth rate, the adjusted birth rate estimate being therefore 0.04260. The fact that this adjusted estimate virtually coincides with that obtained earlier using a different mortality model illustrates again the robustness to choice of mortality pattern of the method that uses $l(5)$, $C(15)$ and r to estimate the birth rate. A very accurate estimate of the birth rate is possible (assuming, of course, that the underlying data are correct), regardless of whether the population is stable and without regard to the family of model stable populations employed in the calculation. In addition, the high level of agreement between the intercensal rate of increase and the estimated stable growth rate when the South family of stable populations is used supports the suitability of this model as a good representation of mortality patterns in Brazil during the 15-year period preceding 1960, provided, of course, that the intercensal estimate of the growth rate may be considered reliable.

Having established a best estimate of the birth rate for the Brazilian population during the 15 years or so preceding 1960, the rest of this section is devoted to the estimation of other parameters of the population being studied from those of the fitted stable population. For this purpose, step 4 of the detailed example is again presented below.

Step 4: calculation of gross reproduction rate and of total fertility for the selected stable population. To estimate total fertility and the gross reproduction rate, it is necessary to identify a female stable population consistent with the observed population. This identification is based on the estimated $l(5)$ for both sexes combined, the female birth rate implied by the final birth rate estimate

($b = 0.0426$), the sex ratio at birth and the sex ratio of the reported population. If one assumes that the sex ratio at birth is 1.05 male births per female birth, 48.8 per cent of all births are female, and since the fraction of the reported population that is female is

$$N_f/N = 35{,}083/(35{,}012+35{,}083) = 0.5005,$$

the birth rate for females is

$$b_f = (0.0426)(0.488)/0.5005 = 0.0415.$$

Similarly, that for males is

$$b_m = (0.0426)(1.0-0.488)/(1.0-0.5005)$$

$$= (0.0426)(0.512)/0.4995 = 0.0437.$$

The estimated $I(5)$ for both sexes is 0.8201 (based on the South model) and it implies mortality level 15.20 in the South model life tables for both sexes combined given the assumed sex ratio at birth (see table 232 in annex VIII). The female stable population of the South family with a mortality level of 15.20 and a birth rate of 0.0415 is found to have a $GRR(29)$ of 2.89 female births per woman (for details on the interpolation process leading to this estimate, see table 146). The value of 29 indicates that this estimate of the gross reproduction rate is dependent upon the assumption that the mean age of childbearing, denoted by μ, is 29 years in Brazil. Although the data necessary to obtain a direct estimate of μ for Brazil are not available, estimation method 1 described in annex III was used to obtain an estimated value of 29.1 years on the basis of the observed $P(3)/P(2)$ ratio (see table 80 in chapter III). Because of the approximate nature of this estimate of μ, the rounded value, 29, was judged to be an adequate approximation in the estimation of the gross reproduction rate. Total fertility, TF, is estimated by dividing GRR, the average number of female births per woman, by the proportion of female births:

$$TF = GRR/0.488 = 5.92.$$

That is, a Brazilian woman living to the end of her reproductive life would have had, on average, about six children, given the fertility rates in operation during the 15 years prior to 1960. It must be stressed, however, that while the final estimate of the birth rate for both sexes (0.0426) is fairly reliable, those of the gross reproduction rate and total fertility are less so, in part because of uncertainty about the sex ratio of the population and in part because of doubts about the assumed age pattern of fertility. Their values should therefore be interpreted with caution.

TABLE 146. ESTIMATION OF THE GROSS REPRODUCTION RATE BY INTERPOLATION BETWEEN THE FEMALE, SOUTH MODEL STABLE POPULATIONS, BRAZIL, 1960

Mortality level					
15		16		15.2	
Birth rate (1)	Gross reproduction rate GRR(29) (2)	Birth rate (3)	Gross reproduction rate GRR(29) (4)	Birth rate (5)	Gross reproduction rate GRR(29) (6)
0.03859	2.653	0.03730	2.564	-	-
0.04348	3.045	0.04208	2.944	-	-
0.04150	2.888	0.04150	2.898	0.04150	2.890

D. ESTIMATION OF THE EXPECTATION OF LIFE AT AGE 5 AND OF THE DEATH RATE OVER AGE 5 FROM THE PROPORTION UNDER AGE 15 AND THE RATE OF INCREASE

1. *Basis of method and its rationale*

Earlier in this chapter, it was noted that the birth rate in stable populations from different sets of model tables is much the same for given values of $C(15)$ and $I(5)$ but not in model stable populations with given values of $C(15)$ and rate of increase. The reason for the superiority of $I(5)$ in identifying stable populations with the same birth rate is that child mortality is the basic determinant of the relation between $C(15)$ and the birth rate. On the other hand, mortality over all ages helps determine the overall death rate, and hence the rate of increase when the birth rate is given. Sets of model life tables (and actual mortality schedules as well) differ most markedly in the relation they embody between mortality rates in childhood and mortality rates at older ages.

Two parameters whose values are very similar in stable populations having the same $C(15)$ and rate of increase are the death rate over age 5 and the expectation of life at age 5. Stable populations having the same proportion under age 15 have nearly the same proportion reaching age 5 annually, or nearly the same "birth rate" into the population aged 5 and over. Since the death rate in any closed population is the birth rate minus the growth rate, stable populations with the same $C(15)$ and the same rate of increase have very nearly the same death rate over age 5 and the same expectation of life at age 5, though they may have rather different age patterns of mortality. Table 147 shows the death rate over age 5 and the expectation of life at age 5 in model stable populations from the North, South, East and West sets, when the proportion under age 15 is 0.4195 and the growth rate is 0.025. Whatever the set of model life

TABLE 147. DEATH RATE OVER AGE 5 AND EXPECTATION OF LIFE AT AGE 5 IN SELECTED MODEL STABLE POPULATIONS WITH A PROPORTION UNDER AGE 15, $C(15)$ OF 0.4195 AND A RATE OF INCREASE, r, OF 0.025

Mortality	Expectation of life at age 5			Death rate over age 5		
Model (1)	Male (2)	Female (3)	Both sexes (4)	Male (5)	Female (6)	Both sexes (7)
North	55.7	56.1	55.9	0.0104	0.0104	0.0104
South	55.9	56.5	56.3	0.0101	0.0100	0.0101
East	55.3	55.7	55.4	0.0102	0.0102	0.0102
West	55.2	55.9	55.6	0.0102	0.0102	0.0102

tables, and whether the stable population is male or female, the two summary measures of mortality over age 5 are fairly similar if $C(15)$ and r are specified. In other words, for populations that are approximately stable, if the proportion under 15 and the intercensal rate of natural increase are accurately recorded, the choice of a model stable population on the basis of $C(15)$ and r provides a robust estimate of overall mortality over age 5, at least for the range of mortality patterns covered by the four Coale-Demeny families.

It is important to emphasize, however, that the requirements outlined above, namely, that the population be approximately stable and that $C(15)$ and r be accurately recorded, are necessary to assure the validity of the mortality estimates obtained. In other words, this method is fairly sensitive to deviations from stability and to errors in the basic data. In particular, the mortality estimates obtained by fitting a stable population to the $C(15)$ and r of a population whose fertility has been declining will tend to underestimate the expectation of life at age 5, and serious biases are also likely when either $C(15)$ or r are subject to error. Hence, in most cases, the results yielded by this method should be evaluated carefully in light of other evidence before being accepted as definite estimates of adult mortality.

2. *Data required*

The data required for this method are listed below:

(*a*) The population classified by five-year age group, and by sex. Strictly speaking, only the population under age 15 and the total population are required, but the availability of an age distribution further classified by sex makes possible the assessment of data quality (see section B);

(*b*) The total population at the time of the census and from at least one preceding census. Availability of population totals and age distributions at a series of dates provides evidence about the approximate stability of the population;

(*c*) An estimate of net migration during, at least, the most recent intercensal period. If net migration is substantial, the reported population will have to be adjusted before the method is applied.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of reported proportion under age 15. This proportion, $C(15)$, is calculated as the population aged 0-14 divided by the total population with recorded age. Those of unknown age should not be included in the denominator.

Step 2: calculation of intercensal growth rate. The annual rate of increase of the population during the intercensal period is calculated as

$$r_s = [\ln(N_2) - \ln(N_1)] / (t_2 - t_1) \qquad (D.1)$$

where N_i is the total population enumerated at time t_i. N_i should include all persons, even those of unknown age, and the date t_i should be expressed in terms of years, including a decimal portion.

Step 3: identification of the stable population with a proportion under age 15 and a growth rate equal to those of the reported population. The stable population that exactly matches the reported proportion under age 15, $C_s(15)$, and whose rate of growth, r_s, is equal to that estimated in step 2 must be identified. Since for a particular mortality pattern a model stable population is completely identified by a mortality level and a growth rate, the process of identification in this case consists only of estimating the mortality level associated with each pair of $C_s(x)$ and r_s values. If model stable populations for all possible mortality levels and all possible growth rates were printed, identification would be carried out by simple comparison of observed and tabulated proportions. However, because the printed stable populations correspond only to selected mortality levels and growth rates, identification usually requires two linear interpolations (see annex IV) between the printed figures. These interpolations can be carried out in any order, but probably the simplest approach consists of interpolating first with respect to r_s and then with respect to $C_s(15)$, as is shown in the detailed example.

Step 4: estimation of other demographic parameters. Once a level of mortality has been selected, its value in conjunction with the stable growth rate defines a stable population that approximates the reported population. Since most demographic measures associated with stable populations are easily obtained, these measures can be calculated for the fitted stable population and may be considered as estimates of true values. The actual computation of these parameters usually involves linear interpolation (see annex IV) with respect to two variables (first, with respect to the growth rate and then with respect to the level of mortality). Furthermore, when stable-population analysis is carried out using data for only one sex, estimates for the other sex may be obtained either by using the sex differentials embodied by the model stable populations themselves or from

knowledge of the sex ratio at birth and the actual sex ratio of the total population. The detailed example illustrates how these procedures are carried out in practice.

It must be recalled, however, that although a complete set of demographic parameters may be calculated for the fitted stable population, they will not all be equally reliable as estimates of the parameters underlying the reported population. According to the discussion in subsection D.1, at least e_5, the expectation of life over age 5, and $d(5+)$, the death rate over age 5, are likely to be trustworthy. The calculation of other parameters, although possible, should not be construed as leading necessarily to satisfactory estimates.

4. *A detailed example*

The population of Colombia in 1951 is analysed as an example of the estimation of demographic parameters from $C(15)$ and r. A series of censuses has been held in Colombia during the twentieth century. The dates and the total populations enumerated by them are shown in table 148.

TABLE 148. TOTAL POPULATION OF COLOMBIA, ACCORDING TO ITS CENSUSES

Census date (t)	Census date (years) (T)	Total population (N)	Intercensal period (n)	Growth rate (r)
15 June 1905	1905.4548	4 143 632	-	-
5 Mar. 1912	1912.1753	5 072 604	6.7205	0.0301
14 Oct. 1918	1918.7863	5 855 077	6.6110	0.0217
17 Nov. 1928	1928.8795	7 851 000	10.0932	0.0791
5 June 1938	1938.4274	8 701 816	9.5479	0.0106
9 May 1951	1951.3534	11 548 172	12.9260	0.0219
15 July 1964	1964.5370	17 484 508	13.1836	0.0315
24 Oct. 1973	1973.7890	21 070 113	9.2520	0.0202

TABLE 149. POPULATION BY AGE GROUP AND SEX, COLOMBIA, 1951
(*Hundreds*)

Age group (i)	Males (M)	Females (F)
0-1	2 097	2 009
1-4	7 416	7 209
5-9	7 944	7 690
10-14	6 841	6 578
15-19	5 451	6 054
20-24	5 324	5 506
25-29	4 093	4 472
30-34	3 423	3 373
35-39	3 176	3 342
40-44	2 483	2 398
45-49	1 936	1 967
50-54	1 831	1 756
55-59	1 104	1 057
60-64	1 073	1 169
65-69	615	655
70-74	460	572
75-79	228	274
80-84	166	248
85 +	120	187
TOTAL	55 793	56 494

Table 149 shows the population by five-year age group and sex as enumerated by the 1951 census of Colombia, and table 150 shows the cumulated proportions, $C(x)$, at five-year intervals and the values of $b(x)$ estimated as described in subsection B.3 using the female, West model stable populations of mortality level 13 as reference set. The values of $b(x)$ for each sex are plotted in figure 20. Both sequences are remarkably level, supporting the belief that the age distribution closely approximates that of a stable population. The female $b(x)$ sequence is less perfectly horizontal, rising slightly from relatively low values at 5, 10 and 15 to a slightly higher plateau from 25 to 50. The sequences of $b(x)$ values for the two sexes imply that the male age distribution may be more reliably recorded. In particular, $b(15)$ for males is representative of the sequence of the male $b(x)$ values, while $b(15)$ for females is a relatively low point in the female $b(x)$ sequence. In these circumstances, two sets of estimates should be constructed in an actual application, the conventionally recommended combination of $C(15)$ for both sexes combined and r, and an alternative that may, in this instance, be more valid: using $C(15)$ for males and r for both sexes. However, since the purpose of presenting here the detailed calculations is illustrative, only the former procedure is applied.

Step 1: calculation of reported proportion under age 15. Using the data given in table 149, one finds that

$$_{15}N_0 = 2{,}097 + 7{,}416 + 7{,}944 + 6{,}841 + 2{,}009 + 7{,}209 + 7{,}690 + 6{,}578 = 47{,}784$$

and

$$N = 55{,}793 + 56{,}494 = 112{,}287$$

Hence

$$C(15) = 47{,}784 / 112{,}287 = 0.4256.$$

The equivalent value for males only is $C_m(15) = 0.4355$.

Step 2: calculation of intercensal growth rate. The growth rate prior to 1951 is taken as equal to the intercensal rate of increase observed between 1938 and 1951. The growth rates implied by the change in population

Figure 28. Sequences of birth-rate estimates, $b(x)$, derived from female, West model stable populations of level 13, Colombia, 1951

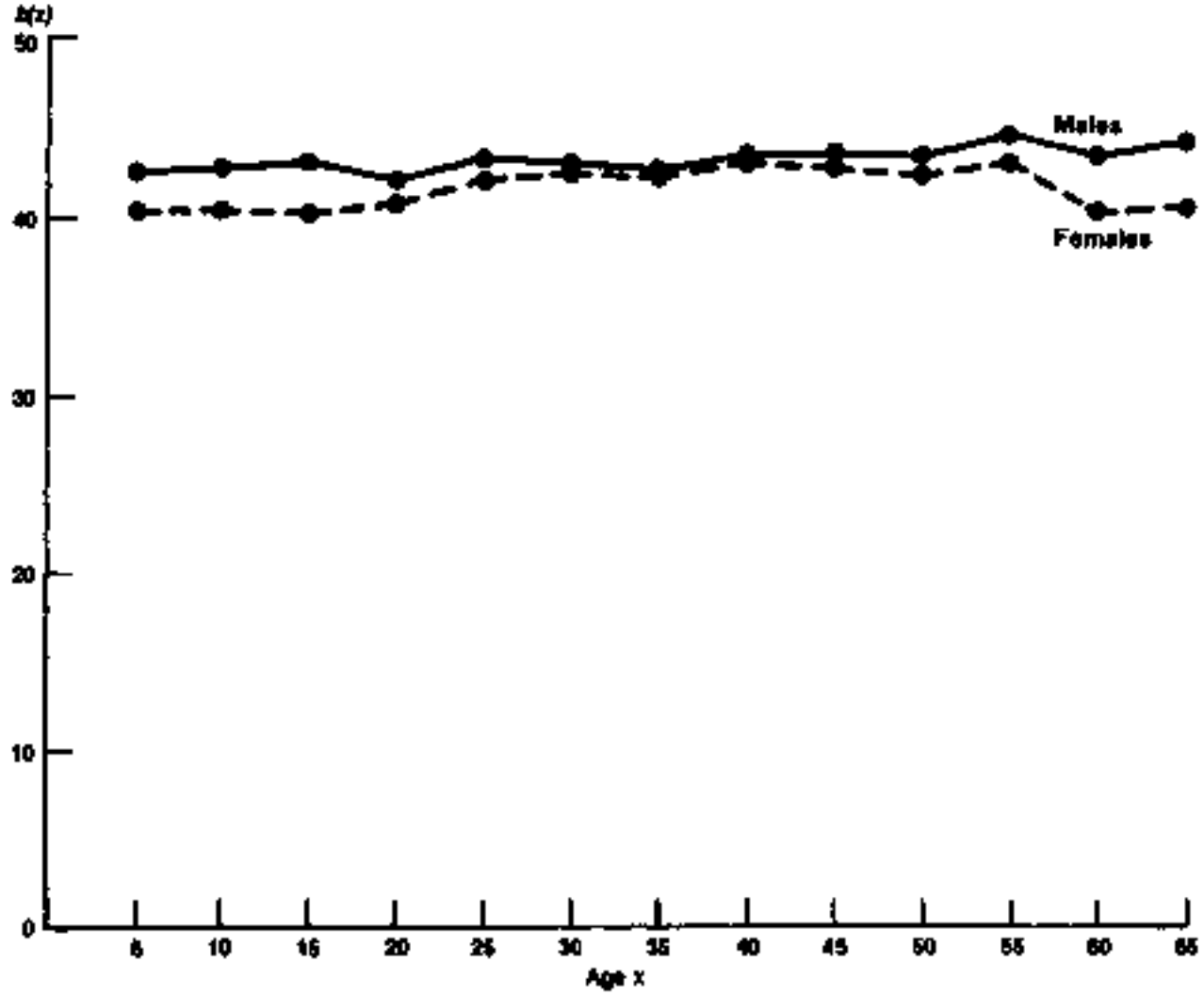


TABLE 150. CUMULATED PROPORTION UNDER AGE x FOR MALES AND FEMALES AND CORRESPONDING BIRTH RATES IN FITTED STABLE POPULATIONS, WEST MODEL, FEMALES, LEVEL 13; COLOMBIA, 1951

Age x (1)	Proportion under age x $(_xa_0)$		Stable birth rate $b(x)$	
	Males (3)	Females (4)	Males (5)	Females (6)
5	0.1705	0.1632	0.04252	0.04048
10	0.3129	0.2993	0.04281	0.04048
15	0.4355	0.4157	0.04309	0.04028
20	0.5332	0.5229	0.04225	0.04089
25	0.6286	0.6203	0.04320	0.04210
30	0.7020	0.6995	0.04305	0.04271
35	0.7633	0.7592	0.04283	0.04221
40	0.8209	0.8184	0.04355	0.04322
45	0.8648	0.8608	0.04376	0.04296
50	0.8995	0.8956	0.04350	0.04253
55	0.9323	0.9267	0.04462	0.04311
60	0.9521	0.9454	0.04340	0.04052
65	0.9713	0.9661	0.04413	0.04074

size between each pair of consecutive censuses are shown in column (5) of table 148. Equation (D.1) was used in every instance to calculate these rates of growth (see step 2 in subsection D.3); the average annual growth rate during the 1938-51 period, for example, is

$$r_a = \ln(11,548,172/8,701,816)/12.9260 = 0.0219.$$

Note that the length of the intercensal interval is calculated in years, so that months and days appear as decimal portions of a year. It is important to express time-points with these decimal portions, since rounding the length of intercensal periods to an integer number of years may have quite a large impact on the estimated growth rate.

The growth rates displayed in table 148 are far from being constant or coherent, the rate for the period 1928-1938 being especially out of line with the others. Since the results of the 1928 census were rejected by the Colombian Congress due to their poor quality, it is not surprising to find that the growth rates calculated using its results are suspect. In fact, the 1928 census should be ignored and a growth rate for the period 1918-1938

should be obtained on the basis of the information yielded by the 1918 and 1938 censuses. Thus,

$$r = \ln(8{,}701{,}816/5{,}855{,}077)/(10.0932+9.5479) = 0.0202.$$

If this value were placed in column (5) instead of those calculated on the basis of the faulty 1928 information, the growth rates observed from 1912 to 1951 would be fairly similar. In these circumstances, it seems reasonable to accept the 1951 Colombian population as approximately stable, with an annual rate of increase of 0.0219.

Step 3: identification of the stable population with proportion under age 15 and growth rate equal to those of the reported population. The process of identification is fairly simple and is carried out only once in detail. Even though, strictly speaking, model stable populations derived from life tables referring to both sexes combined should be used as the basis for identification, the use of those referring to only one sex is expedient and fairly satisfactory. The proportion under age 15, $C(15)$, for both sexes combined was calculated in step 1 as 0.4256. Since the estimated growth rate is 0.0219, the first task is to identify the levels n and $n+1$ such that, for this growth rate, $C_n(15)$ is smaller than or equal to the observed $C(15)$ and $C_{n+1}(15)$ is greater than or equal to that observed. Table 151 shows how this task is accomplished. As explained in the computational procedure, interpolation is carried out first at a given level (say, 8) using the estimated rate of growth (0.0219 in this case) in order to obtain the value of $C(15)$. This process corresponds to interpolating between the rows of table 151. Then, once the values of $C_8(15)$ and $C_9(15)$ for $r = 0.0219$ are available (0.4321 and 0.4244, respectively), the level corresponding to the reported

TABLE 151. IDENTIFICATION OF THE FEMALE STABLE POPULATION, WEST MODEL, CONSISTENT WITH A PROPORTION UNDER AGE 15, $C(15)$, OF 0.4256 AND A GROWTH RATE, r, OF 0.0219, COLOMBIA, 1951

Growth rate (1)	West mortality level: 8 $C(15)$ (2)	9 $C(15)$ (3)	8.84 $C(15)$ (4)
0.0200	0.4192	0.4114	.
0.0250	0.4532	0.4456	.
0.0219	0.4321	0.4244	0.4256

$C(15)$ is identified by interpolating along the last row of $C(15)$ values shown in table 151. The resulting level is 8.84.

Step 4: estimation of other demographic parameters. The different parameters of interest for a stable population growing at an annual rate of 0.0219 and subject to a mortality schedule equal to that of level 8.84 of the female West model life tables are calculated below:

(a) *Expectation of life at age 5, both sexes.* Using 8.84 as the representative level, e_5 may be estimated by interpolating between the e_5 values of the female, West model life tables at levels 8 and 9. Thus

$$e_5 = 0.16(48.18)+0.84(49.75) = 49.50$$

where 48.18 and 49.75 are the life expectancy values at age 5 for levels 8 and 9, respectively, of the female, West model life tables. Note that although the e_5 value presented above is obtained from "female" model tables, because it is derived from $C(15)$ for both sexes it refers to both sexes. However, its accuracy will depend upon how well the West mortality pattern for females approximates that of both sexes in the observed population.

(b) *Estimation of life expectancy at birth for both sexes.* The process followed is identical to that presented above, but using e_0 instead of e_5. In this case,

$$e_0 = 0.16(37.5)+0.84(40.0) = 39.6.$$

Again, this parameter estimate refers to both sexes because $C(15)$ corresponds to both sexes. Note that this estimate is sensitive to the relationship between child and adult mortality.

(c) *Birth rate for both sexes.* In the female stable population at levels 8 and 9, respectively, the birth rates consistent with a growth rate of 0.0219 are calculated by interpolation. For example, at level 8,

$$b_8 = 0.62(0.04572)+0.38(0.05115) = 0.04778.$$

At level 9, b_9 is 0.04532. Hence, at level 8.84,

$$b_r = 0.16(0.04778)+0.84(0.04532) = 0.04571;$$

(d) *Death rate for both sexes.* The death rate is now obtained by subtracting the growth rate from the birth rate:

$$d_r = b_r - r_r = 0.04571 - 0.02190 = 0.02381;$$

(e) *Death rate over age 5.* This parameter is obtained by following an interpolation process very similar to that used in calculating the birth rate. Thus, at level 8,

$$d_8(5+) = 0.62(0.01467)+0.38(0.01362) = 0.01427.$$

At level 9, $d_9(5+) = 0.01351$. Hence, at level 8.84,

$$d_r(5+) = 0.16(0.01427)+0.84(0.01351) = 0.01363;$$

(f) *Birth rate by sex.* The birth rate for each sex separately may be obtained from that for both sexes if the sex ratio at birth, (SRB), and the sex ratio of the population, (SRP), are known. In the case of Colombia,

TABLE 152. NUMBER OF BAPTISMS BY SEX AND YEAR OF OCCURRENCE, COLOMBIA, 1950-1952

Year (1)	Number of baptisms: Male (2)	Female (3)	Sex ratio (4)
1950	212 133	201 588	1.052
1951	215 699	203 685	1.059
1952	223 474	213 932	1.050
TOTAL	651 306	618 205	1.054

the data on baptisms by sex shown in table 152 imply that the sex ratio at birth is 1.054, similar to that observed in populations with reliable data. According to table 149, the sex ratio of the population is 0.9876. Thus, the male birth rate, b_m, is

$$b_m = b_t \frac{(1.0+1.0/SRP)}{(1.0+1.0/SRB)} = (0.04571)(1.0327) = 0.04721$$

and the female birth rate is

$$b_f = b_t \frac{(1.0+SRP)}{(1.0+SRB)} = (0.04571)(0.9677) = 0.04423;$$

(g) *Death rate by sex.* Death rates by sex are obtained by subtraction, assuming that each sex has the same growth rate, $r = 0.02190$. Hence,

$$d_m = 0.04721 - 0.02190 = 0.02531$$

and

$$d_f = 0.04423 - 0.02190 = 0.02233.$$

The sex differential implied by these death rates should, however, be interpreted with considerable caution.

5. *Comments on the detailed example*

The estimates of demographic parameters derived in this section for Colombia are probably about as good as can be obtained given the data available. However, as pointed out above, some of these estimates are likely to be more reliable than others. Using $C(15)$ and r, the summary indicators of mortality over age 5 are likely to be estimated most reliably. The indicators of overall mortality or of fertility should be treated with more caution, those estimates whose calculation depends upon the sex ratio of the population being regarded as least reliable.

Chapter VIII

ESTIMATION OF FERTILITY BY REVERSE-SURVIVAL METHODS

A. BACKGROUND OF METHODS

1. *Meaning of reverse survival*

In a closed population, children currently aged x are just the survivors of the births that occurred x years ago. From this fact it is easily inferred that the number of births occurring x years ago can be estimated by using life-table survivorship probabilities to "resurrect" numerically those no longer present among the population aged x. This method of estimation is known as "reverse survival" or "reverse projection" because the population now aged x is "survived" or "reverse-projected" to age $x - t$ by moving it, with a suitable life table, t years into the past.

It is immediately evident that if the single-year age distribution of a population enumerated at t_0 is available, it is potentially possible to estimate the number of births occurring during each of the 15 or 20 years preceding t_0. Methods that exploit this possibility are described in this chapter. However, before proceeding with their description, it is worth noting that they are all heavily dependent upon the accuracy of the reported age distribution of the population being studied. Errors in age-reporting or differential completeness of enumeration affecting certain age groups, especially the younger ones, are certain to bias the estimates obtained. Because these types of deficiencies are all too frequently characteristic of the data sets available, reverse-survival methods are often ineffective in producing reliable fertility estimates. Their usefulness depends mainly upon the fact that they often provide independent fertility estimates which can be used to assess the plausibility of those obtained by other means.

2. *Organization of this chapter*

Only two estimation methods based on reverse survival are presented in this chapter. The first method illustrates the basic principle underlying other, more sophisticated procedures. It allows the estimation of the average birth rate during the five or 10 years preceding enumeration from the population classified by five-year age group. Variations of this basic method arise mainly because of the increasing amount of data available or because of the increasing detail with which it is classified. The second method presented here depends upon the latter aspect, because it requires detailed tabulations of enumerated children classified both according to their own age and to that of their mother. To aid the user in selecting the method appropriate for a particular case, table 153 shows their data requirements and the

TABLE 153. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER VIII

Section	Type of input data	Estimated parameters
B. Estimation of birth rate by reverse survival of the population under age 10	Enumerated population under age 10 classified by five-year age group Total population enumerated at two points in time, or that enumerated at one point and its growth rate Probabilities of child survivorship ($_5l_2$, $_5l_3$ or $_5l_5$, for example)	The average birth rate during the two five-year periods immediately preceding the time of enumeration
C. The own-children method of fertility estimation	Enumerated population under age 10 or 15, classified by single year of age and by single year of age of mother Children under age 10 or 15 whose mother's age is unknown, classified by single year of own age Women aged 15-59 or 15-64, classified by single year of age Probabilities of child survivorship Probabilities of adult female survivorship (a life table for females)	Age-specific fertility rates for each of the 10 or 15 years preceding enumeration Total fertility for each of the 10 or 15 years preceding enumeration

types of estimates they yield; and the sections describing these methods are listed below:

Section B. Estimation of birth rates by reverse survival of the population under age 10. This section presents the basic reverse-survival method. It requires as input the population classified by five-year age group and survivorship probabilities referring to childhood.

Section C. The own-children method of fertility estimation. This method allows the estimation of annual age-specific fertility rates for the 10 or 15 years preceding the time of enumeration. It requires that the enumerated children be classified by age of mother at the time of enumeration.

B. Estimation of birth rates by reverse survival of the population under age 10

1. *Basis of method and its rationale*

The estimation of the number of births occurring x years before enumeration from the enumerated population aged x is a well-known possibility that has been widely exploited. However, practice has shown that the estimates obtained in this manner are often not very useful due to the severe age-reporting errors generally present in the basic data. Problems are especially acute when the reverse-projected data are classified by single year of age, since age-heaping is likely to produce spurious peaks and troughs in the estimates obtained. Furthermore, because censuses often fail to enumerate children completely, especially those aged 0 or 1, it is frequent for the estimated number of births referring to the year or two immediately preceding the time of enumeration to be too low.

In order to avoid some of these problems or to minimize their effect on the final estimates, grouped data are often used. Five-year age groups are most commonly selected; they yield estimates of the average annual number of births occurring during the two five-year periods immediately preceding the time of enumeration. According to the observations made above, the average annual number of births estimated by reverse-projecting the population aged from 0 to 4 is likely to underestimate the true number of births occurring during the five-year period immediately preceding enumeration. A better estimate may be expected from the reverse projection of the population in age group 5-9. However, estimates of numbers of births obtained from the population aged 5-9 years are likely to be more affected by errors in the estimation of mortality and may also be exaggerated by age-reporting errors (heaping at age 5, for example).

Besides being very sensitive to the presence of age-misreporting, the estimation of fertility by reverse survival is also dependent upon the type of mortality estimates used. By its very nature, reverse survival cannot be performed without a life table, covering at least the ages of childhood; and adequate information allowing the direct construction of resonable life tables is lacking in most countries where reverse-survival estimates of fertility are needed. Therefore, reverse survival is often applied in conjunction with other methods, particularly those which permit the indirect estimation of child mortality (see chapter III). All of these methods assume that the pattern of mortality in the population studied conforms to a certain model. As one would expect, the choice of model affects the fertility estimates yielded by reverse survival, the differential effects being greater the further one reverse-projects the observed population into the past.

In spite of all these caveats, the estimation of the birth rate by reverse survival is described here because of its traditional importance and because it provides a valuable tool for the detection of inconsistencies. For example, suppose that b_1, an independently obtained estimate of the birth rate for the five-year period preceding a census, is smaller than b_2, the estimate obtained by reverse-survival of the population aged 0-4. Since b_2 is usually a lower bound for the birth rate, any estimate for the same period lower than b_2 would be suspect. Of course, one should also explore the possibility of having overestimated the mortality level used in reverse surviving the population aged 0-4 years.

The value of reverse-survival procedures may also be increased if they can be applied to a series of censuses to obtain estimates for overlapping periods. In such circumstances, it may be worth while to reverse-project the population in age groups older than the traditional age groups 0-4 and 5-9 in order to obtain a range of birth-rate estimates for particular periods. Thus, if three census enumerations spaced by 10-year intervals are available, three estimates can be made of the birth rates during the periods from zero to four years and from five to nine years before the first census. For the five years before the first census, estimates can be based on the population aged 0-4 at the first census, 10-14 at the second census and 20-24 at the third census, whereas for the period from five to nine years before the first census, birth-rate estimates can be based on the population aged 5-9 at the first census, 15-19 at the second census and 25-29 at the third census. Such a procedure has been applied by Shorter and Macura[1] to the quinquennial censuses of Turkey taken between 1935 and 1975.

This extension of the basic procedure requires substantially more information than does the simpler procedure. The older the population being considered, the greater the influence of the survivorship estimates used to calculate the number of births the survivors represent, and of the growth rate estimates used to calculate the population denominators. For most developing countries, the level of child mortality in the 1940s or 1950s is known only to a very rough approximation; hence, the reverse projection of a recent census for some 30 years, or the reverse projection of an earlier census for a shorter period, is fraught with uncertainty. Consistency between estimates for earlier periods does not provide

[1] Frederic C. Shorter and Miroslav Macura, *Trends in Fertility and Mortality in Turkey, 1935-1975*, Committee on Population and Demography Report No. 8 (Washington, D.C.: National Academy Press, 1982).

any evidence in their support, because any error in specifying past mortality risks will affect all the estimates in the same direction. Migration also represents a problem; and any evidence suggesting a significant level of migration during the period being considered will make it necessary to attempt some correction, not only at the aggregate level but at the level of each age group. On the positive side, the effects of age-reporting errors should be reduced by considering data from several censuses, since different age groups determine the different estimates for the same period; however, it is possible that a sequence of 10-year intercensal intervals, combined with heavy heaping on ages ending in zero, may give rise to consistently high estimates of the birth rate for periods determined by age groups that include zero as an age ending (except for the estimate based on the population aged 0-4), while producing consistently low birth-rate estimates for the intervening periods, associated with age groups including five as an age ending (except possibly for age group 5-9). Given a range of birth-rate estimates for each period, the problem arises how to arrive at a "best" estimate; the most obvious possibility is to take the mean of the available estimates, though given the likely nature of the errors involved, the median may be a better indicator. It may be mentioned that changes in enumeration completeness from one census to the next have relatively little impact on the procedure, so long as the age distributions do not change markedly from one census to the next and intercensal survival is not used as a basis for the estimation of survivorship probabilities.

2. *Data required*

The following data are required for this method:

(*a*) The population under age 10, classified by age (five-year age groups are sufficient, though single-year data are preferable) and by sex;

(*b*) The total population at the time of enumeration, t_0;

(*c*) An estimate of the growth rate;

(*d*) Estimates of mortality parameters that would permit the construction of a life table up to age 10. A value of $l(2)$ obtained from information on children ever born and surviving (see chapter III) is adequate.

3. *Computational procedure*

Step 1: calculation of life-table estimates of person-years lived. In order to reverse-project to birth the population in age groups 0-4 and 5-9, one only needs values of ${}_5L_0$ and ${}_5L_5$, the person-years lived by the stationary population constituting the life table between birth and exact age 5, and between age 5 and exact age 10, respectively. Usually, but not necessarily, these values are obtained by assuming that the mortality level associated with $l(2)$ remained constant during the 10 years preceding enumeration. Under this assumption, the actual calculation of ${}_5L_0$ and ${}_5L_5$ is carried out by interpolating between the printed values of the Coale-Demeny model life tables.[2] Naturally, the family of models selected should be that used in estimating $l(2)$. In some cases, the mortality level selected may be the mean of those associated with the $l(2)$, $l(3)$ and $l(5)$ values yielded by the indirect estimation of child mortality (see chapter III). This mean level may represent more closely the true mortality level prevalent during the decade preceding enumeration, especially when there is evidence suggesting a recent decline in child mortality.

Step 2: estimation of mid-period populations. Since this method is directed towards the estimation of an average annual birth rate for the periods from t_0-5 to t_0 and from t_0-10 to t_0-5, t_0 being the date of enumeration, an estimate of the total population at the mid-points of these periods is required. Perhaps the simplest way of estimating these mid-period populations is by using the equation

$$N_M = N_0 \exp[r\,(t_M - t_0)] \qquad \text{(B.1)}$$

where t_M is the mid-point of the period being considered; N_0 is the total count yielded by enumeration; and r is an estimate of the growth rate. As usual, the growth rate is estimated from knowledge of the total population at two points in time, t_0 and t_1. In such a case,

$$r = \ln [N_1/N_0]/(t_1 - t_0) \qquad \text{(B.2)}$$

where N_1 is the total count at time t_1 and N_0 is that at time t_0.

Of course, there are several other ways of estimating a mid-period population; but, in general, they require far more effort than the intrinsic roughness of the method at hand would warrant. For this reason, they are not described here.

Step 3: estimation of average annual birth rates for the two five-year periods preceding the census. The average annual number of births for the first period, from t_0-5 to t_0, is

$$B_1 = {}_5N_0/{}_5L_0 \qquad \text{(B.3)}$$

where ${}_5N_0$ is the population in age group 0-4; and ${}_5L_0$ is the life-table estimate obtained in step 1.

For the period from t_0-10 to t_0-5, the equivalent average annual number of births is

$$B_2 = {}_5N_5/{}_5L_5 \qquad \text{(B.4)}$$

where ${}_5N_5$ is the population in age group 5-9. If the radix, $l(0)$, of the life table being used were not one, equations (B.3) and (B.4) would have to be modified as follows:

$$B_1 = {}_5N_0\, l(0)/{}_5L_0$$

and

$$B_2 = {}_5N_5\, l(0)/{}_5L_5. \qquad \text{(B.5)}$$

[2] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University, 1966).

Once B_1 and B_2 are calculated, the birth rate for each period is obtained by dividing these values by the corresponding mid-period populations calculated in step 2.

4. *A detailed example*

Data gathered by the 1960 census of Brazil are used to illustrate the application of this method. The steps of the procedure are described below.

Step 1: calculation of life-table estimates of person-years lived. The data on children ever born and surviving collected by the 1960 census of Brazil were used in chapter III, subsection E.4(b) to estimate probabilities of survivorship in childhood with respect to the West model. In chapter VII, subsection C.5, however, it was shown that the South model provided a better representation of child mortality in this country. Therefore, to obtain estimates of ${}_5L_0$ and ${}_5L_5$, $l(2)$ was re-estimated using the South model. Its value, 0.8491, is consistent with mortality level 15.21 in the South family of model life tables. This level is used now to estimate ${}_5L_0$ and ${}_5L_5$ by interpolation. Table 154 shows the ${}_nL_x$ values appearing in the Coale and Demeny model life tables at levels 15 and 16.

Since the Coale-Demeny model life tables only contain ${}_1L_0$ and ${}_4L_1$, one needs to calculate ${}_5L_0$ by adding these values. Therefore, for males,

$${}_5L_0^{15} = {}_1L_0^{15} + {}_4L_1^{15} = 0.9164 + 3.3204 = 4.2368$$

and

$${}_5L_0^{16} = 0.9229 + 3.3851 = 4.3080.$$

Using these values, interpolation may now be carried out as shown below:

$${}_5L_0^{M} = 0.79(4.2368) + 0.21(4.3080) = 4.2518.$$

Other values of ${}_5L_x$ are obtained in a similar fashion. They are shown in columns (2) and (5) of table 155.

TABLE 154. VALUES OF PERSON-YEARS LIVED FROM EXACT AGE x TO $x+5$ BY A STATIONARY POPULATION, SOUTH MODAL LIFE TABLES

Person-years lived (1)	Males		Females	
	Level 15 (2)	Level 16 (3)	Level 15 (4)	Level 16 (5)
${}_1L_0$	0.9164	0.9229	0.9271	0.9329
${}_4L_1$	3.3204	3.3851	3.3726	3.4360
${}_5L_5$	4.0161	4.1163	4.0832	4.1835

Step 2: estimation of mid-period populations. Table 156 shows the population counts produced by the censuses of Brazil since 1950. Intercensal growth rates for each period and sex are also given. Each growth rate has been calculated according to the equation:

$$r = \ln[N_1/N_0]/(t_1 - t_0).$$

Thus, for example, for males during the period 1950-1970, the growth rate was

$$r = \ln[45{,}754{,}659/25{,}885{,}001]/20.167 = 0.0282.$$

Examination of the set of growth rates given in table 156 shows that population growth in Brazil slowed somewhat during the period 1960-1970. The lower growth rate for males observed during that period, however, seems suspect. Sex differences in growth rates diminish when the 20-year period is considered. Because there are reasons to believe that the 1960 census might not be of comparable quality with the others, the growth rate selected in this case is based on the estimates for 1950-1970. The average of the male and female growth rates, amounting to 0.0285, is considered representative of the growth rate experienced by the Brazilian population around 1960. Therefore, the mid-year populations desired are estimated by

$$N_1 = N_0 \exp[-0.0285(2.5)]$$

and

$$N_2 = N_0 \exp[-0.0285(7.5)]$$

TABLE 155. ESTIMATION OF NUMBER OF BIRTHS BY REVERSE SURVIVAL, BRAZIL, 1960
(Thousands)

Age x (1)	Males			Females		
	Person-years lived from x to $x+5$ ${}_5L_x^M$ (2)	Reported population (thousands) ${}_5N_x^M$ (3)	Estimated number of births (thousands) B_1 (4)	Person-years lived from x to $x+5$ ${}_5L_x^F$ (5)	Reported population (thousands) ${}_5N_x^F$ (6)	Estimated number of births (thousands) B_2 (7)
0	4.2518	5 688	1 337.8	4.3142	5 506	1 276.5
5	4.0371	5 171	1 280.9	4.1043	4 988	1 215.3

TABLE 156. TOTAL POPULATION AT CENSUS DATES AND INTERCENSAL GROWTH RATES BY SEX, BRAZIL

Census date (1)	Total population		Intercensal period (4)	Growth rate	
	Males (2)	Females (3)		Males (5)	Females (6)
1 July 1950	25 885 001	26 039 396	1950-1960	0.0298	0.0294
1 Sept. 1960	35 059 546	35 131 824	1960-1970	0.0266	0.0282
1 Sept. 1970	45 754 659	46 586 897	1950-1970	0.0282	0.0288

TABLE 157. BIRTH RATES BY SEX ESTIMATED BY REVERSE SURVIVAL, BRAZIL, 1960

Period i (1)	Males: Estimated population (thousands) N_i (2)	Males: Estimated number of births (thousands) B_i (3)	Males: Estimated birth rate b_i (4)	Females: Estimated population (thousands) N_i (5)	Females: Estimated number of births (thousands) B_i (6)	Females: Estimated birth rate b_i (7)
1	32 647.9	1 337.8	0.0410	32 714.9	1 276.3	0.0390
2	28 314.5	1 280.9	0.0452	28 372.6	1 215.3	0.0428

where N_0 is the reported population at the time of the census. Values of N_0 (the population in 1960) for each sex are 35,059,546 males and 35,131,824 females. Hence, N_1 and N_2 for males (in thousands) are

$$N_1 = (35{,}060)(0.9312) = 32{,}647.9;$$

$$N_2 = (35{,}060)(0.8076) = 28{,}314.5.$$

The corresponding values for females are shown in column (5) of table 157.

Step 3: estimation of average annual birth rates for the two five-year periods preceding the census. Table 155 shows the number of males and females enumerated in age groups 0-4 and 5-9, denoted by ${}_5N_x$. Using equations (B.3) and (B.4), average annual births for each period are obtained. The case of male births is illustrated below:

$$B_1 = 5{,}688/4.2518 = 1{,}337.8;$$

$$B_2 = 5{,}171/4.0371 = 1{,}280.9.$$

Once the average annual number of births for each period is estimated, calculation of the birth rates is straightforward. For example, in the case of males:

$$b_1 = B_1/N_1 = 1{,}337.8/32{,}647.9 = 0.0410;$$

$$b_2 = B_2/N_2 = 1{,}280.9/28{,}314.5 = 0.0452.$$

The corresponding estimates for females are shown in column (7) of table 157.

Estimates for both sexes can now be obtained by adding male and female births for each period, and the male and female mid-period populations separately. Then, as usual, the birth rates are found as the ratios of total births in a period to total mid-period population. Table 158 summarizes results for both sexes.

TABLE 158. BIRTH RATES FOR BOTH SEXES COMBINED, ESTIMATED BY REVERSE SURVIVAL, BRAZIL, 1960

Period i (1)	Estimated population (thousands) N_i (2)	Estimated number of births (thousands) B_i (3)	Estimated birth rate b_i (4)
1	65 362.8	2 614.1	0.0400
2	56 687.1	2 496.2	0.0440

When the birth rates estimated by reverse survival are compared with those obtained by stable-population analysis in chapter VII, subsection C.5, $b_m = 0.0438$, $b_f = 0.0416$ and $b_i = 0.0427$, stable estimates prove in every instance to be higher than the reverse-survival estimates for the five-year period immediately preceding the census (since the census took place on 1 September 1960, this period extends from 2 September 1955 to 1 September 1960 and is denoted by 1956-1960). However, the stable estimates are much lower than the reverse-survival estimates for the period 1951-1955. If one accepts the stable estimates as true, it would appear that both the male and female populations aged 0-4 in 1960 were underenumerated by approximately 6 per cent, while those in age group 5-9 were overenumerated by about 3 per cent. Although part of the apparent excess in age group 5-9 may be caused by upward transfers of younger children, it is clear that if the stable estimates are correct, transfers alone cannot explain the deficit observed in age group 0-4. Hence, one must accept either that fertility fell during the five-year period immediately preceding the 1960 census or that the birth-rate estimates obtained by reverse-surviving the population aged from 0 to 4 constitute a lower bound for the true birth rate of the Brazilian population during the period 1956-1960. As is often the case, the latter possibility is more likely.

C. THE OWN-CHILDREN METHOD OF FERTILITY ESTIMATION

1. *Basis of method and its rationale*

The own-children method permits the estimation of age-specific fertility rates for the 10 or 15 years preceding a census or survey from information on the enumerated number of children classified by single year of age and single year of age of mother. In order to obtain the desired age-specific fertility estimates, the own-children method calls for the reverse-projection of these children to the time of their birth. It can be viewed, therefore, as a specific application of the method presented in the previous section. The point where the two methods differ most markedly is on the type of data they require, and probably the greatest innovation introduced by the proponents of the own-children method is the exploitation of seldom-used census information for fertility estimation purposes.[3] Indeed, the "own-children

[3] Lee-Jay Cho, "The own-children approach to fertility estimation: an elaboration", *International Population Conference, Liège, 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 2, pp. 263-280.

tabulation" (children by single year of age and single year of age of mother) which constitutes the basis of this method can only be made if enumerated children are linked in some way to their mothers. When the method was first proposed, censuses did not usually include a direct question making this linkage. It was therefore necessary to infer the mother-child link from information on relationship to the head of the household and from the compatibility between the age of the presumed mother and those of her children. Hence, the traditional own-children method involves more than an estimation procedure; it also includes a set of criteria to perform the mother-child linkage in any given case. Because the linkage process falls outside the scope of this *Manual* and because in recent censuses and surveys a direct question identifying the mother of each enumerated child has often been included (making the linkage unnecessary), the matching criteria are not presented here.

As mentioned above, age-specific fertility rates are estimated essentially by the reverse projection of enumerated children to the time of their birth. Therefore, these estimates are usually not as smooth as one would like because, just as in the case where reverse projection is used to estimate birth rates by single years, they are derived from enumerated children classified by single year of age, so that differential completeness of enumeration, age-misreporting and age-heaping will affect them substantially. Hence, it is not unusual to find that the fertility rates estimated for the year immediately preceding the time of enumeration are too low or that those obtained from children aged 5 or 10 are too high. Averaging the results that refer to contiguous age groups is a way of reducing the effects of age-heaping.

The own-children method is attractive because it permits the detailed estimation of fertility from data that are almost invariably collected by censuses and because it does not depend upon any assumptions about fertility trends and is not very sensitive to assumptions about recent changes in the level of mortality. However, it clearly requires detailed estimates of mortality for both children and females, referring at least to the decade preceding the time of enumeration. If only one set of mortality rates is used in reverse projection of the population, mortality is implicitly assumed to have remained constant during the period considered. Yet, the method itself does not require such an assumption and one is free to use different mortality schedules for different periods when evidence of changes in mortality exists.

The own-children method is also appealing because, in principle, it is capable of estimating fertility at different points in time and, therefore, of estimating trends as well as levels. The existence of age-heaping and differential completeness of enumeration, however, often frustrates these hopes. For instance, given the known deficiencies of census enumerations, it would be naïve to interpret the drop in fertility rates during the two or three years immediately preceding a census as an indication of the occurrence of a fertility decline. Such a spurious trend is more likely to be caused by the underenumeration of young children.

In practical applications, the own-children method provides a reasonable estimate of overall fertility level (especially when only the estimates derived from enumerated children above age 3 are considered) and a rough idea of trends. In populations where age-reporting is accurate (those of Chinese origin, for example), it performs remarkably well. In traditional applications of the own-children method (where the child-mother linkage has to be performed indirectly), the estimates it yields tend to be affected by what is known as the "grandmother effect" caused by the erroneous allocation of children to their grandmothers. This type of error is reduced when a direct question linking children to their mothers is asked.

The own-children method was first developed to study fertility differentials between different subgroups of a population;[4] and it remains an important tool for studies of this kind. However, in using it for this purpose one must bear in mind two points: first, if there is reason to believe that the subgroups considered are subject to different mortality levels or patterns, it is important to incorporate different mortality estimates for each subgroup in the application of the method; secondly, if the subpopulations under study are not closed, careful interpretation of the results obtained is required. For example, consider the estimation of fertility by the own-children method for the rural and urban populations of a country at a given point in time, t. When one reverse-projects the urban population of time t to time $t-a$, say, the result does not necessarily represent the urban population at time $t-a$. Both would be the same only if the urban population had been closed between time $t-a$ and time t, or if there were no fertility or mortality differentials between the urban and rural populations of the country in question.

Lastly, it must be pointed out that the own-children method of fertility estimation provides valuable independent estimates of total fertility that may be used to assess the quality or plausibility of those obtained by other means.

2 *Data required*

The data required for this method are described below:

(*a*) The enumerated children (persons under 15) whose mother was identified, classified according to single year of own age and single year of age of mother;

(*b*) Children (persons under 15) whose mother could not be identified, classified by single year of age;

(*c*) All women (irrespective of whether they are mothers), classified by single year of age. In general, these data are needed only for the age range from η to $\nu+\gamma$, where η and ν are the lower and upper limits of the reproductive age span, respectively, and γ is the number of age groups used in classifying the children. If one assumes that η and γ equal 15, and ν equals 49, the

[4] Lee-Jay Cho, Wilson H. Grabill and Donald J. Bogue, *Differential Current Fertility in the United States* (Chicago, University of Chicago, Community and Family Study Center, 1971).

number of women aged from 15 to 64, classified by single year of age, is sufficient;

(*d*) Estimates of child survivorship. In fact, the probabilities of surviving from birth to the age group from x to $x+1$, ${}_1L_x$, for $x = 1, 2, ..., 15$, and for the 15 or so years preceding the point of enumeration are required. In practice, however, a complete set of ${}_1L_x$ is rarely available. Therefore, the computational procedure described below includes some steps for calculating these probabilities from other indirect information;

(*e*) Estimates of female adult mortality. Again, the probabilities of surviving from age η to exact age x, for $x = \eta + 1, \eta + 2, ..., \rho + \gamma$, and for the 15 years or so preceding the time of enumeration are required. Methods for calculating them from other indirect information are described below.

3. *Computational procedure*

The steps of the computational procedure are given below:

Step 1: Redistribution of children with unidentified mother. This step can only be performed if the children whose mother could not be identified at the matching stage (probably because the mother had died or because she did not live in the same household as her child or children) are tabulated by single year of age. In general, it is important to incorporate these children in the application of the own-children method, since their elimination will certainly lead to underestimates of fertility levels. The purpose of this step is to estimate the probable distribution of the unmatched children according to age of mother on the basis of information on children whose mother could be identified.

If one denotes by U_x the number of unmatched children aged x, and by C_x^a the children aged x whose mother's age at the time of enumeration was a, then an estimate of U_x^a, the number of unmatched children whose mother's age was a can be obtained by

$$U_x^a = C_x^a U_x / \sum_{a=\eta}^{\rho+\gamma} C_x^a \qquad \text{(C.1)}$$

where η and ρ are the lower and upper limits of the childbearing ages, respectively, and γ is the number of age groups used in classifying the children.

If one extends the notation used so that

$$C_x = \sum_{a=\eta}^{\rho+\gamma} C_x^a, \qquad \text{(C.2)}$$

equation (C.1) can be rewritten as

$$U_x^a / U_x = C_x^a / C_x, \qquad \text{(C.3)}$$

that is, it states algebraically that the distribution of unmatched children of age x according to age of mother is identical to that observed among children of age x whose mothers were identified. Although this identity may not hold exactly in reality, it seems the most reasonable approximation given the data available.

Step 2: estimation of survivorship probabilities for children. The probabilities of surviving from birth to the age group from x to $x+1$ for $x = 0, 1, 2, ..., (\gamma-1)$, denoted by ${}_1L_x$, are needed to reverse-project the enumerated children to the time of their birth. According to the data available, estimates of these probabilities may be obtained by a variety of procedures, only one of which is described here. It assumes that a good estimate of the mean level of child mortality prevalent in the population under study during the 15 years or so preceding enumeration is available (see chapter III). This estimate may be expressed either as a level within any of the Coale-Demeny families of model life tables, or as a pair of parameters α and β defining a life table in one of the four logit systems generated from level 16 female life tables of the Coale-Demeny models (see chapter I, subsection B.4).

When a Coale-Demeny level is provided as an estimate of child mortality, it is possible to transform it into equivalent ${}_1L_x$ values by identifying the α and β parameters that define that level within the logit system generated by female level 16 of the life-table family selected. Table 159 presents the α and β values that identify the printed male and female levels of the life tables for the 0-15 age range. Linear interpolation between these values makes the identification of α and β straightforward.

Using these α and β values in conjunction with the suitably interpolated standard $(l(x)$ values by single year of age for female level 16 of each of the Coale-Demeny families of model life tables are listed in annex XI) and the inverse logit transformation, an estimated life table $l^*(x)$ by single year can be calculated and the desired ${}_1L_x$ values obtained.

When estimated child mortality is already expressed as a pair of α and β values, only the inversion of the logit transformation is necessary to obtain $l^*(x)$.

Although this procedure yields ${}_1L_x$ values that are slightly different from those which would be obtained by direct interpolation between the different levels of the model life tables themselves, it does not require an unmanageably large set of constants that would make its computer implementation awkward. Furthermore, since the standard used has already been interpolated by single year of age, derivation of $l^*(x)$ estimates by single year is relatively simple. These reasons recommend it for inclusion here.

Step 3: estimation of survivorship probabilities for adult females. In order to reverse-project the female population to each of the γ years preceding the time of enumeration, survivorship probabilities of the type ${}_1L_a / {}_1L_{a-\gamma}$ for $0 \leqslant x \leqslant \gamma$ and $\eta \leqslant a \leqslant \rho+\gamma$ are required. Just as in the case of children, several procedures may be used to estimate the set of female ${}_1L_a$ values by single year of age. The one presented here is analogous to that described above for children, and it also assumes that the mean level of adult mortality prevalent in the female population under consideration during the 10 or 15 years preceding enumeration is known (the estimation methods described in chapters IV and V may be used to

TABLE 159. VALUES OF PARAMETERS α AND β DETERMINING THE COALE-DEMENY MODEL LIFE TABLES FOR CHILDHOOD IN THE LOGIT SYSTEM GENERATED BY FEMALE LEVEL 16 OF THE CORRESPONDING FAMILY

Level	North model				South model			
	Females		Males		Females		Males	
	Parameter α (2)	Parameter β (3)	Parameter α (4)	Parameter β (5)	Parameter α (6)	Parameter β (7)	Parameter α (8)	Parameter β (9)
1	1.5228	1.5134	1.5159	1.4877	1.9152	2.1520	1.7322	1.9218
2	1.4007	1.4670	1.3896	1.3715	1.7623	2.0593	1.5933	1.8436
3	1.2892	1.4264	1.2749	1.3312	1.6225	1.9762	1.4660	1.7734
4	1.1857	1.3904	1.1699	1.2957	1.4924	1.9002	1.3475	1.7091
5	1.0882	1.3578	1.0698	1.2636	1.3693	1.8292	1.2354	1.6491
6	0.9952	1.3277	0.9755	1.2343	1.2512	1.7611	1.1281	1.5921
7	0.9058	1.2998	0.8849	1.2071	1.1368	1.6962	1.0240	1.5371
8	0.8177	1.2734	0.7969	1.1815	1.0235	1.6319	0.9221	1.4832
9	0.7312	1.2479	0.7104	1.1571	0.9111	1.5677	0.8210	1.4296
10	0.6450	1.2232	0.6246	1.1333	0.7981	1.4963	0.7257	1.3797
11	0.5580	1.1985	0.5385	1.1100	0.6845	1.4145	0.6284	1.3076
12	0.4695	1.1737	0.4513	1.0865	0.5676	1.3345	0.4921	1.2374
13	0.3796	1.1530	0.3461	1.0503	0.4098	1.2548	0.3757	1.1684
14	0.2811	1.0862	0.2390	1.0112	0.2790	1.1738	0.2576	1.0990
15	0.1221	1.0425	0.1296	0.9735	0.1433	1.0896	0.1361	1.0279
16	0.0000	0.9999	0.0194	0.9399	0.0000	1.0000	0.0096	0.9534
17	−0.1287	0.9560	−0.0974	0.9053	−0.1542	0.9023	−0.1246	0.8736
18	−0.2683	0.9093	−0.2242	0.8676	−0.3245	0.7920	−0.2708	0.7850
19	−0.4251	0.8552	−0.3666	0.8235	−0.4805	0.7055	−0.4555	0.6749
20	−0.6115	0.7862	−0.5332	0.7679	−0.6413	0.6200	−0.6073	0.5980
21	−0.8490	0.6882	−0.7402	0.6906	−0.8307	0.5341	−0.7683	0.5201
22	−1.1956	0.5211	−1.0234	0.5669	−0.9917	0.4484	−0.9406	0.4419
23	−1.4662	0.4225	−1.2580	0.4977	−1.1863	0.3676	−1.1277	0.3632
24	−1.7829	0.3389	−1.5493	0.4207	−1.3997	0.2805	−1.3339	0.2856

Level	East model				West model			
	Females		Males		Females		Males	
	Parameter α (10)	Parameter β (11)	Parameter α (12)	Parameter β (13)	Parameter α (14)	Parameter β (15)	Parameter α (16)	Parameter β (17)
1	1.6288	1.6374	1.5657	1.4345	1.5841	1.5351	1.4869	1.3604
2	1.5087	1.5919	1.4908	1.3937	1.4627	1.4915	1.3656	1.3210
3	1.3973	1.5506	1.3860	1.3566	1.3510	1.4528	1.2546	1.2860
4	1.2926	1.5124	1.2792	1.3223	1.2466	1.4178	1.1512	1.2543
5	1.1927	1.4766	1.1782	1.2906	1.1477	1.3856	1.0536	1.2252
6	1.0962	1.4425	1.0814	1.2604	1.0527	1.3556	0.9604	1.1981
7	1.0022	1.4097	0.9876	1.2313	0.9604	1.3270	0.8701	1.1723
8	0.9092	1.3773	0.8956	1.2029	0.8696	1.2994	0.7817	1.1475
9	0.8164	1.3450	0.7445	1.1746	0.7794	1.2724	0.6941	1.1231
10	0.7227	1.3121	0.6530	1.1461	0.6886	1.2453	0.6064	1.0987
11	0.6111	1.2634	0.5402	1.1017	0.5962	1.2177	0.5178	1.0741
12	0.4938	1.2095	0.4378	1.0587	0.5010	1.1891	0.4270	1.0486
13	0.3762	1.1577	0.3307	1.0173	0.3950	1.1528	0.3042	1.0006
14	0.2563	1.1066	0.2213	0.9763	0.2578	1.0939	0.1898	0.9608
15	0.1320	1.0548	0.1075	0.9344	0.1315	1.0469	0.0812	0.9243
16	−0.0002	0.9998	−0.0132	0.8901	−0.0002	0.9998	−0.0935	0.8871
17	−0.1436	0.9403	−0.1443	0.8411	−0.1430	0.9487	−0.1552	0.8498
18	−0.3066	0.8692	−0.2902	0.7851	−0.3024	0.8904	−0.2913	0.8064
19	−0.5002	0.7782	−0.4591	0.7164	−0.4904	0.8174	−0.4495	0.7522
20	−0.7449	0.6516	−0.6673	0.6239	−0.7269	0.7161	−0.6420	0.6794
21	−0.9619	0.5743	−0.8465	0.5734	−0.9943	0.6105	−0.8531	0.6106
22	−1.1990	0.4951	−1.0589	0.5131	−1.2530	0.5302	−1.0701	0.5550
23	−1.4724	0.4107	−1.3061	0.4497	−1.5838	0.4421	−1.3340	0.4910
24	−1.7961	0.3254	−1.6061	0.3816	−1.9451	0.3475	−1.6616	0.4211

ascertain this level). Estimates of female adult mortality can be expressed as a level within a given family of the Coale-Demeny life tables, or as a pair of α and β parameters identifying a life table within one of the four logit systems generated by the four female level 16 life tables of the Coale-Demeny families. The transformation of such mortality estimates into single-year ${}_1L_x$ values is carried out as described in step 2, but the identification of the α and β parameters on the basis of a given Coale-Demeny level must be performed by using table 160, specifically designed to cover age range 10-64 for females. Further details about this calculation procedure are presented in the example given below.

Step 4: reverse survival of children. Just as in the case of the simple reverse-survival method, the number of children born in year $t-x$ (t being the time of enumeration) to women aged $a-x$ is estimated by

$$M_{t-x}^{a-x} = (C_t^a + U_t^a)/{}_1L_x. \qquad \text{(C.4)}$$

TABLE I60. VALUES OF PARAMETERS α AND β DETERMINING THE COALE-DEMENY MODEL LIFE TABLES FOR ADULT FEMALES IN THE LOGIT SYSTEM GENERATED BY FEMALE LEVEL 16 IN THE CORRESPONDING FAMILY

Level (l)	North model		South model		East model		West model	
	Parameter α (3)	Parameter β (3)	Parameter α (4)	Parameter β (5)	Parameter α (6)	Parameter β (7)	Parameter α (8)	Parameter β (9)
1	1.3129	1.2924	1.3403	1.3948	1.2698	1.2146	1.3431	1.2829
2	1.1827	1.2331	1.2148	1.3328	1.1495	1.1675	1.2112	1.2248
3	1.0654	1.1829	1.0984	1.2796	1.0400	1.1268	1.0922	1.1737
4	0.9582	1.1401	0.9916	1.2335	0.9389	1.0919	0.9835	1.1337
5	0.8590	1.1033	0.8924	1.1952	0.8445	1.0618	0.8825	1.0978
6	0.7662	1.0718	0.7991	1.1578	0.7556	1.0359	0.7881	1.0671
7	0.6785	1.0447	0.7107	1.1268	0.6711	1.0139	0.6988	1.0408
8	0.5949	1.0216	0.6260	1.0995	0.5900	0.9952	0.6137	1.0186
9	0.5146	1.0022	0.5443	1.0755	0.5117	0.9799	0.5318	1.0001
10	0.4367	0.9862	0.4663	1.0572	0.4355	0.9677	0.4523	0.9852
11	0.3608	0.9735	0.3896	1.0434	0.3637	0.9644	0.3747	0.9727
12	0.2862	0.9641	0.3133	1.0312	0.2928	0.9664	0.2983	0.9658
13	0.2165	0.9580	0.2369	1.0208	0.2215	0.9703	0.2232	0.9631
14	0.1463	0.9767	0.1599	1.0122	0.1493	0.9768	0.1532	0.9759
15	0.0745	0.9873	0.0816	1.0059	0.0757	0.9864	0.0783	0.9867
16	0.0006	1.0007	0.0015	1.0021	0.0003	1.0003	0.0012	1.0015
17	−0.0757	1.0180	−0.0815	1.0015	−0.0777	1.0202	−0.0787	1.0218
18	−0.1552	1.0412	−0.1677	1.0067	−0.1586	1.0486	−0.1622	1.0502
19	−0.2383	1.0733	−0.2602	1.0107	−0.2432	1.0901	−0.2500	1.0911
20	−0.3258	1.1199	−0.3606	1.0105	−0.3320	1.1327	−0.3429	1.1331
21	−0.4180	1.1915	−0.4700	1.0107	−0.4317	1.2259	−0.4461	1.2397
22	−0.5137	1.3166	−0.5914	1.0109	−0.5479	1.2993	−0.5697	1.3245
23	−0.6440	1.4082	−0.7290	1.0105	−0.6815	1.3966	−0.7133	1.4410
24	−0.7919	1.5299	−0.8889	1.0084	−0.8394	1.5284	−0.8866	1.6056

Note that the number of unmatched children whose mother was estimated to be aged a at the time of enumeration must be added to the children whose mother was identified and aged a before the whole is reverse-survived. Note also that equation (C.4) must be applied for x ranging from 0 to $(\gamma-1)$ and for all age groups of mother $(\eta \leqslant a \leqslant \rho+\gamma)$.

The quantity M_{t-x}^{a-x} represents the number of births occurring during a year $(t-x-1, t-x)$ to women who are between exact ages a and $a+1$ at time t. Therefore, M_{t-x}^{a-x} are births to women whose ages ranged between $a-x-1$ and $a-x+1$ (exclusive) during the year $(t-x-1, t-x)$, so that some adjustment is necessary to obtain the births occurring during that year to women whose ages ranged only between $a-x$ and $a-x+1$. The simplest type of adjustment consists of taking the average of M_{t-x}^{a+1-x} and M_{t-x}^{a-x} to represent the desired number of births. This average is denoted by B_{t-x}^{a-x}.

Step 5: reverse survival of adult females. Women aged a at time t, W_t^a, must be reverse-survived to times $t-x$, for $0 \leqslant x \leqslant \gamma$. The equation used in this case is

$$W_{t-x}^{a-x} = W_t^a({}_1L_{a-x}/{}_1L_a) \tag{C.5}$$

or, equivalently,

$$W_{t-x}^{a-x} = {}_1L_{a-x}(W_t^a/{}_1L_a). \tag{C.6}$$

Thus, when one is performing the calculations by hand, it is easier to calculate first the set of $W_t^a/{}_1L_a$ values and then to multiply them by ${}_1L_{a-x}$ for x ranging from 0 to γ.

Note that the value of W_{t-x}^{a-x} obtained in this way represents the number of women whose ages ranged between $a-x$ and $a-x+1$ (exclusive) at exact time $t-x$. Yet, the number of women that should be used as denominator for B_{t-x}^{a-x} in calculating age-specific fertility rates is the mid-period female population during the year $(t-x-1, t-x)$. This population, denoted by M_{t-x}^{a-x}, can be estimated, as usual, by

$$M_{t-x}^{a-x} = (W_{t-x-1}^{a-x} + W_{t-x}^{a-x})/2.0. \tag{C.7}$$

Step 6: calculation of age-specific fertility rates. The calculation of fertility rates specific by single year of age and time period is now straightforward, since

$$f_{t-x}(a) = \frac{B_{t-x}^{a-x}}{M_{t-x}^{a-x}} \tag{C.8}$$

where $f_{t-x}(a)$ denotes the fertility rate corresponding to age a during the year $(t-x-1, t-x)$. However, due to age-reporting errors, rates by single year are likely to be erratic. Furthermore, since other methods of estimation usually yield fertility rates specific by five-year age group, it may be necessary, for comparison purposes, to calculate conventional five-year age group estimates. This calculation is carried out by cumulating the single-year fertility schedules just estimated to obtain $F_{t-x}(a)$, that is,

$$F_{t-x}(a) = \sum_{y=\eta}^{a-1} f_{t-x}(y) \tag{C.9}$$

and then calculating the usual five-year estimates, denoted by $f_{t-x}(i)$, by differencing

$$f_{t-x}(i) = (F(10+5(i+1)) - F(10+5i))/5.0$$
$$\text{for } i = 1, 2, \ldots, 8. \tag{C.10}$$

TABLE 161. OWN-CHILDREN DATA, WITH CHILDREN CLASSIFIED BY SINGLE YEAR OF AGE AND SINGLE YEAR AGE OF MOTHER, COLOMBIA, 1978

Age of mother	Number of children, by age of child																Number of women
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
15	13	7	0	2	2	2	1	4	3	4	2	1	3	1	3	3	755
16	12	3	0	2	0	2	1	1	1	0	0	0	2	0	0	1	696
17	23	16	6	1	0	0	0	0	0	0	1	0	0	0	0	0	686
18	56	36	17	3	0	3	1	2	0	0	0	0	0	0	0	0	706
19	66	46	24	13	11	1	3	1	0	0	0	0	1	0	0	1	538
20	77	55	45	33	19	12	2	1	0	2	2	1	1	1	0	0	602
21	78	71	56	47	48	17	7	5	3	0	1	2	1	1	0	0	488
22	84	80	76	73	46	26	18	15	3	0	0	0	0	0	1	0	534
23	84	85	80	84	61	53	29	24	7	9	1	2	0	2	1	0	488
24	93	63	78	72	56	48	45	34	17	9	8	3	0	1	1	1	451
25	91	84	87	89	69	71	55	52	31	21	5	5	2	1	0	0	464
26	73	67	65	70	66	70	61	55	41	24	17	11	1	1	2	0	393
27	58	61	70	58	63	79	64	64	47	28	27	16	11	5	2	1	339
28	83	71	77	81	94	80	87	91	80	50	42	38	16	8	3	2	442
29	48	58	52	59	68	64	77	75	61	66	48	50	23	23	6	4	530
30	46	60	70	62	82	86	86	86	82	74	69	50	45	31	20	8	403
31	42	39	42	36	44	44	55	66	63	56	57	46	43	24	12	8	243
32	45	50	67	54	66	65	73	82	79	91	78	64	63	66	38	30	343
33	51	33	37	51	46	63	53	58	65	66	65	67	55	50	58	24	272
34	34	34	42	50	46	48	50	72	62	74	68	67	63	66	57	31	257
35	33	29	33	30	38	55	60	51	80	63	73	63	77	63	66	40	317
36	31	28	40	36	39	43	40	61	71	54	70	61	83	73	69	61	272
37	28	23	37	44	43	47	49	60	61	66	74	79	80	79	74	68	291
38	25	35	28	38	49	49	56	66	65	69	69	70	83	65	87	83	345
39	17	21	25	22	38	25	38	49	48	40	52	50	62	55	68	59	216
40	31	19	33	35	44	39	49	70	65	57	66	75	79	77	80	74	347
41	8	7	22	22	22	22	27	32	30	39	38	32	46	40	46	45	173
42	5	6	11	23	18	26	26	45	38	44	50	33	62	68	47	73	242
43	13	9	15	22	21	31	37	42	46	36	60	59	50	47	66	73	221
44	2	5	1	14	17	15	28	26	24	34	45	38	43	46	42	53	182
45	6	2	9	10	16	19	21	24	42	33	52	45	77	60	55	70	227
46	4	4	6	5	11	14	15	13	31	18	29	23	31	33	36	36	158
47	2	3	1	9	13	19	24	32	38	32	28	36	52	40	40	50	215
48	3	4	4	7	7	14	22	28	44	37	40	41	58	59	47	59	283
49	0	0	2	3	2	7	8	16	15	19	21	23	40	41	39	32	156
50	3	0	2	0	4	6	7	12	11	20	29	29	43	41	47	41	302
51	1	1	1	0	2	7	2	3	10	11	13	14	25	25	21	22	127
52	0	1	1	2	2	3	7	6	8	9	12	19	22	12	29	26	169
53	0	0	0	0	0	4	1	2	2	6	5	12	12	20	19	22	150
54	0	0	0	1	2	2	4	4	3	4	9	10	14	15	19	19	146
55	1	0	2	1	0	2	1	2	1	5	4	8	10	13	17	24	183
56	0	0	0	0	0	1	1	2	1	1	3	4	3	9	8	11	133
57	0	0	0	0	1	0	0	0	2	1	2	2	3	4	9	15	138
58	0	0	0	0	0	1	1	2	1	7	4	4	2	6	10	14	126
59	0	0	0	0	0	1	0	0	1	0	1	1	2	1	2	4	82
60	0	0	0	0	0	1	1	1	1	1	2	1	3	4	6	5	207
61	0	0	0	0	0	0	0	0	0	1	0	0	1	2	1	2	86
62	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2	2	107
63	0	0	0	0	0	0	2	0	0	0	2	0	0	1	1	2	90
64	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	77
Not known[a]	33	50	84	94	301	110	145	155	145	133	171	161	244	212	226	250	-
TOTAL	1 405	1 266	1 327	1 352	1 397	1 396	1 430	1 593	1 529	1 424	1 511	1 410	1 641	1 472	1 482	1 455	-

[a] Age of mother could not be determined

In some cases, errors in the reported ages of young children may produce sequences of five-year age group fertility rates that are still fairly erratic. To obtain a smoother sequence of estimates, averages of the rates for adjacent years are often calculated. Their calculation is illustrated in the detailed example given below.

4. *A detailed example*

The data presented in table 161 serve to illustrate the application of the own-children method of fertility estimation. They were obtained from a survey conducted in Colombia in 1978. That survey included a question on identification of mother at the household level. (The data shown in table 161 have not been weighted.)

Step 1: redistribution of children with unidentified mother. The penultimate row of table 161 gives the number of children whose mother could not be found in the household in which they were enumerated. They are to be redistributed over the columns above them (representing the distribution of all other children with

respect to mother's age). The last row of table 161 gives the totals of each column. In the notation used above, these totals equal $C_x + U_x$ for $0 \leqslant x \leqslant 15$. According to equation (C.3),

$$U_x^* = C_x^*(U_x / C_x), \qquad \text{(C.11)}$$

and since, as noted in step 4 of the computational procedure, the number of children who are to be reverse-projected to birth is $C_x^* + U_x^*$, this quantity can be expressed as

$$C_x^* + U_x^* = C_x^*(1.0 + U_x / C_x), \qquad \text{(C.12)}$$

that is, for a given age of children (a column in table 161), the desired $C_x^* + U_x^*$ values can be obtained merely by multiplying every entry C_x^* by the factor $K_x = (1.0 + U_x / C_x)$.

For example, consider children aged 8 years. According to table 161, a total of 1,529 children aged 8 were enumerated. Of these, 145 had no mother present. Therefore, those with identified mother amount to 1,384 $= C_x$. Hence,

$$K_x = 1.0 + U_x / C_x = 1.0 + 145/1{,}384 = 1.1048.$$

Values of K_x for $0 \leqslant x \leqslant 15$ are given in table 162. When making the calculations by hand, it is not necessary to calculate the U_x^* values proper. It suffices to use the factors K_x to adjust at a later stage (step 4) the estimated number of births resulting from the reverse survival of children whose mothers were identified.

Step 2: estimation of survivorship probabilities for children. Assume that by other methods based on independent data sources (such as children ever born and surviving) it is known that the level of child mortality prevalent among the Colombian population during the period 1963-1978 is approximately equal to 17.2 in the North family of Coale-Demeny models for both sexes combined. According to table 159, the values of parameters α and β associated with levels 17 and 18, North model, sex female are

$$\alpha_{17}^{f} = -0.1287 \quad \text{and} \quad \beta_{17}^{f} = 0.9564;$$

$$\alpha_{18}^{f} = -0.2683 \quad \text{and} \quad \beta_{18}^{f} = 0.9093.$$

Thus, the α and β values associated with level 17.2 for females can be obtained by interpolation as follows:

$$\alpha_{17.2}^{f} = (0.2)(-0.2683) + (0.8)(-0.1287) = -0.1566$$

and

$$\beta_{17.2}^{f} = (0.2)(0.9093) + (0.8)(0.9564) = 0.9470.$$

Following the same procedure in the case of males, values of α and β for males for level 17.2 are estimated as

$$\alpha_{17.2}^{m} = -0.1228 \quad \text{and} \quad \beta_{17.2}^{m} = 0.8978.$$

Using these values of the α and β parameters and the standard selected, level 16 for females in the North family of model life tables (see annex XI), $l(x)$ values for ages from 1 to 16 can be calculated using the inverse logit transformation (see chapter I, subsection B.4). The equations needed are

$$\lambda(x) = \alpha + \beta\, \lambda_s(x) \qquad \text{(C.13)}$$

and

$$l(x) = [1.0 + \exp(2.0\, \lambda(x))]^{-1} \qquad \text{(C.14)}$$

where $\lambda(x)$ denotes the logit transformation of $l(x)$; and equation (C.14) defines the inverse of the logit transformation. The subindex s is used to denote the standard.

For example, to estimate $l(2)$ for females, one notes first that $\lambda_s(2) = -1.1332$ (copied from annex XI). Therefore,

$$\lambda_f(2) = -0.1566 + 0.9470(-1.1332) = -1.2297,$$

so that

$$l_f(2) = \frac{1.0}{1.0 + \exp(-2.4594)} = 0.9213.$$

In an analogous way, $l(2)$ for males may be calculated as follows:

$$\lambda_m(2) = -0.1228 + 0.8978(-1.1332) = -1.1402,$$

so

$$l_m(2) = \frac{1.0}{1.0 + \exp(-2.2804)} = 0.9072.$$

TABLE 162. VALUES OF THE EXPANSION FACTORS, K_x, USED IN ADJUSTMENT OF OWN-CHILDREN DATA FOR THE EXISTENCE OF CHILDREN WITHOUT MOTHER, COLOMBIA, 1978

Age of child (x)	Expansion factor (K_x)
0	1.0241
1	1.0411
2	1.0507
3	1.0747
4	1.0779
5	1.0855
6	1.1111
7	1.1078
8	1.1048
9	1.1030
10	1.1276
11	1.1289
12	1.1747
13	1.1683
14	1.1799
15	1.2075

Hence $l_b(2)$, the probability of surviving from birth to exact age 2 for both sexes combined may be calculated as

$$l_b(2) = \frac{(1.05)(0.9072)+(0.9213)}{2.05} = 0.9141,$$

assuming that the sex ratio at birth is 1.05 males per female.

Table 163 gives other $l(x)$ values for males, females and both sexes combined. Once these values have been calculated, the desired survivorship probabilities, ${}_1L_x$, are obtained by averaging contiguous $l(x)$ values. For example,

$${}_1L_7 = \frac{l(7)+l(8)}{2.0} = \frac{0.8819+0.8786}{2.0} = 0.8803.$$

TABLE 163. ESTIMATES OF PROBABILITIES OF SURVIVING, $l(x)$ AND ${}_1L_x$, FOR CHILDREN: NORTH MODEL, COLOMBIA, 1978

Age x	Probability of surviving to exact age x: Female $l(x)$ (2)	Male $l(x)$ (3)	Both sexes $l(x)$ (4)	Probability of surviving in age group from x to $x+1$: Both sexes ${}_1L_x$ (5)
0	1.0000	1.0000	1.0000	0.9478
1	0.9365	0.9242	0.9302	0.9222
2	0.9213	0.9072	0.9141	0.9090
3	0.9114	0.8965	0.9038	0.8999
4	0.9038	0.8882	0.8959	0.8928
5	0.8979	0.8818	0.8897	0.8877
6	0.8939	0.8776	0.8856	0.8838
7	0.8904	0.8738	0.8819	0.8803
8	0.8872	0.8704	0.8786	0.8772
9	0.8845	0.8674	0.8757	0.8745
10	0.8820	0.8648	0.8732	0.8721
11	0.8799	0.8625	0.8710	0.8700
12	0.8779	0.8605	0.8690	0.8681
13	0.8762	0.8586	0.8672	0.8663
14	0.8745	0.8568	0.8654	0.8646
15	0.8728	0.8551	0.8637	0.8627
16	0.8709	0.8530	0.8617	.

The only exception is ${}_1L_0$, the calculation of which requires an estimate of the separation factor for deaths under age one. This factor can be estimated using the equations proposed by Coale and Demeny[5]; the form and coefficients for these equations are shown in table 164. In the case at hand, $q_f(1) = 1.0 - l_f(1) = 0.0635$ and $q_m(1) = 0.0758$, so both are less than 0.100. Hence, the coefficients for case A are used to estimate S for each sex separately as follows:

$$S_f = 0.05 + 3.0(0.0635) = 0.2405$$

$$S_m = 0.0425 + 2.875(0.0758) = 0.2604,$$

so that

$${}_1L_0^f = S_f + (1.0 - S_f)(l_f(1))$$
$$= 0.2405 + (1.0 - 0.2405)(0.9365)$$
$$= 0.9518$$

$${}_1L_0^m = S_m + (1.0 - S_m)(l_m(1))$$
$$= 0.2604 + (1.0 - 0.2604)(0.9242)$$
$$= 0.9439,$$

and

$${}_1L_0^b = [(1.05)(0.9439) + 0.9518]/2.05$$
$$= 0.9478.$$

This and other values of ${}_1L_x$ for both sexes combined are given in column (5) of table 163.

TABLE 164. COEFFICIENTS FOR ESTIMATION OF SEPARATION FACTORS FOR AGE GROUP 0-1, COALE-DEMENY MODEL LIFE TABLES

Model (1)	Female: Coefficients a_1 (2)	Female: Coefficients b_1 (3)	Male: Coefficients a_1 (4)	Male: Coefficients b_1 (5)
Case A: $q(1) < 0.100$				
North, South and West	0.0500	3.000	0.0425	2.875
East	0.0100	3.000	0.0025	2.875
Case B: $q(1) \geq 0.100$				
North, South and West	0.3500	0.000	0.3300	0.000
East	0.3500	0.000	0.2900	0.000
Equation: $S = a_1 + b_1 q(1)$				

Step 3: estimation of survivorship probabilities for adult females. The procedure followed in estimating these probabilities is very similar to that illustrated in the previous step. The main difference is that only females need to be considered and the translation of a given mortality level into values of α and β is made by using table 160. In the case at hand, assume that independent information (orphanhood data, for example) has shown that female adult mortality in Colombia during the period of interest (1963-1978) is well represented by level 16.6 of the West model life tables. Then, according to table 160, the α and β values associated with levels 16 and 17 are, respectively,

$$\alpha_{16} = 0.0012 \quad \text{and} \quad \beta_{16} = 1.0015;$$

$$\alpha_{17} = -0.0787 \quad \text{and} \quad \beta_{17} = 1.0218.$$

The values corresponding to level 16.6 are obtained by interpolation as follows:

$$\alpha_{16.6} = 0.4(0.0012) + 0.6(-0.0787) = -0.0467,$$

$$\beta_{16.6} = 0.4(1.0015) + 0.6(1.0218) = 1.0137.$$

[5] *Op. cit.*

Once these α and β values have been calculated, equations (C.13) and (C.14) can be used to estimate $l(x)$ by single year of age. For example, for $x = 35$, annex XI shows that $\lambda_s(35) = -0.6661$ for the West model. Hence,

$$\lambda(35) = -0.0467 + 1.0137(-0.6661) = -0.7219$$

and

$$l(35) = \frac{1.0}{1.0 + \exp(2.0(-0.7219))} = 0.8091.$$

Other values of $l(x)$ are shown in columns (2) and (5) of table 165. From these values, ${}_1L_x$ values are calculated by taking successive averages. For example,

$${}_1L_{35} = (0.8091 + 0.8048)/2.0 = 0.8070.$$

The complete sequence of ${}_1L_x$ values is given in columns (3) and (6) of table 165.

Step 4: reverse survival of children. Using the C_x^* values shown in table 161 and the ${}_1L_x$ estimates presented in table 163, the reverse-projection of enumerated children is straightforward. For example, to estimate the births occurring in 1970 to women aged 25, one proceeds as follows:

$$M_{1970}^{25} = K_2(C_1^{25}/{}_1L_1) = 1.1048(65/0.8772) = 81.87,$$

$$M_{1970}^{26} = K_2(C_1^{26}/{}_1L_1) = 1.1048(62/0.8772) = 78.09,$$

so that

$$B_{1970}^{25} = (M_{1970}^{25} + M_{1970}^{26})/2.0$$

$$= (81.87 + 78.09)/2.0 = 79.98.$$

The full set of B_{1-x}^* values is given in table 166. Note, however, that the headings used in table 166 do not correspond exactly to the notation used so far. The B_{1970}^* estimates, for example, appear in the column labeled "1969/70". This heading is used to suggest explicitly that the births being estimated occurred during parts of both the 1969 and the 1970 calendar years. The exact portions involved depend upon the date on which enumeration (the survey in this case) took place in 1978. Only if enumeration had taken place 31 December 1978 would the B_{1970}^* estimates refer exclusively to 1970. Although the table headings used may be slightly confusing for the beginner, they have been adopted because they match those appearing in the output of computer programs that implement the own-children method and because, in fact, they are more accurate than the algebraic notation being used. The latter does not refer explicitly to two contiguous calendar years in each case only because such practice would render it even more awkward and imposing.

Lastly, note should be taken that in table 166 the entries for women aged 14 are not the result of averaging births belonging to two different age groups. They are just the values of M_{1-x}^{a-x} when $a - x = 15$. Since they represent boundary values, there is always some uncer-

TABLE 165. ESTIMATES OF PROBABILITIES OF SURVIVING, $l(x)$ AND ${}_1L_x$, FOR ADULT FEMALES, WEST MODEL, COLOMBIA, 1978

Age x (1)	Probability of surviving to exact age x, $l(x)$ (2)	Probability of surviving to age group from x to $x+1$, ${}_1L_x$ (3)	Age x (4)	Probability of surviving to exact age x, $l(x)$ (5)	Probability of surviving to age group from x to $x+1$, ${}_1L_x$ (6)
15	0.8735	0.8725	41	0.7824	0.7800
16	0.8714	0.8703	42	0.7776	0.7751
17	0.8691	0.8679	43	0.7726	0.7701
18	0.8667	0.8655	44	0.7675	0.7649
19	0.8642	0.8629	45	0.7622	0.7595
20	0.8616	0.8602	46	0.7567	0.7539
21	0.8587	0.8573	47	0.7510	0.7480
22	0.8558	0.8542	48	0.7449	0.7418
23	0.8526	0.8510	49	0.7386	0.7353
24	0.8494	0.8478	50	0.7320	0.7285
25	0.8461	0.8445	51	0.7250	0.7214
26	0.8428	0.8411	52	0.7177	0.7138
27	0.8394	0.8380	53	0.7099	0.7059
28	0.8366	0.8345	54	0.7018	0.6975
29	0.8323	0.8305	55	0.6931	0.6885
30	0.8286	0.8268	56	0.6839	0.6791
31	0.8249	0.8230	57	0.6742	0.6690
32	0.8211	0.8192	58	0.6638	0.6583
33	0.8172	0.8152	59	0.6527	0.6469
34	0.8132	0.8112	60	0.6410	0.6348
35	0.8091	0.8070	61	0.6285	0.6218
36	0.8048	0.8027	62	0.6151	0.6081
37	0.8005	0.7983	63	0.6010	0.5935
38	0.7961	0.7939	64	0.5859	0.5779
39	0.7916	0.7893	65	0.5698	-
40	0.7870	0.7847			

TABLE 166. ESTIMATED NUMBER OF BIRTHS IN SINGLE YEAR PERIODS BY SINGLE YEAR OF AGE OF MOTHER, COLOMBIA, 1963/64-1977/78

Age of mother at birth of child	Year of birth of child and estimated number of births, B_i^a														
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1976/1977	1977/1978
14	8.19	10.79	14.89	14.28	6.47	11.35	8.82	18.88	8.80	14.68	13.29	3.58	6.94	3.39	14.05
15	17.75	20.91	18.27	17.52	14.23	18.92	15.12	24.55	15.22	17.74	18.12	9.56	13.29	10.73	13.51
16	21.84	26.42	26.40	32.45	28.45	26.39	30.23	36.51	29.56	26.30	40.46	27.48	23.70	29.36	18.91
17	34.13	37.10	46.02	54.52	64.62	32.80	45.35	54.13	46.53	48.32	56.76	47.79	39.88	46.30	43.76
18	65.53	60.70	59.56	64.90	58.20	55.31	55.43	67.35	62.88	61.77	64.61	71.68	58.34	57.03	66.99
19	78.49	78.24	71.74	62.30	75.66	79.48	79.99	74.90	72.95	72.78	70.65	93.78	76.30	71.14	77.26
20	83.96	64.75	79.86	71.39	81.48	88.31	88.81	97.56	78.61	86.24	75.48	93.19	90.17	85.26	83.74
21	92.15	73.52	81.22	85.02	87.30	82.01	90.03	104.49	94.95	91.13	81.52	92.59	91.33	93.16	87.52
22	97.61	91.75	96.11	86.96	92.45	92.73	91.31	101.34	103.13	97.35	77.90	91.39	95.37	83.56	90.77
23	109.39	102.52	108.29	83.03	86.01	99.04	89.44	95.67	102.50	88.07	94.81	76.46	87.86	83.00	95.63
24	105.80	97.12	110.32	79.11	91.18	88.31	90.70	93.16	88.67	91.24	97.83	83.03	78.03	85.26	99.41
25	101.02	80.94	111.67	90.86	92.47	86.42	79.99	88.12	80.49	79.51	90.58	83.63	84.97	72.27	88.60
26	86.00	89.03	99.49	96.70	93.17	73.81	89.44	81.83	79.23	66.67	76.09	72.28	74.57	74.53	70.78
27	63.48	78.91	95.43	77.84	92.47	75.70	95.11	77.42	64.77	78.29	66.43	58.54	70.52	72.83	76.18
28	77.13	72.84	84.60	81.12	78.25	85.16	83.14	70.50	69.17	67.89	67.63	53.76	64.74	66.62	70.78
29	73.72	77.56	73.09	69.44	76.31	68.76	79.36	76.16	62.88	63.00	55.56	62.72	63.01	55.90	50.79
30	66.21	62.73	75.80	62.18	67.25	61.19	71.18	79.31	55.97	59.94	62.80	60.33	60.12	50.25	47.54
31	62.11	71.49	62.94	59.71	56.91	60.56	71.18	72.39	66.03	55.05	58.58	47.79	45.66	46.86	47.00
32	51.87	62.73	81.22	62.95	71.13	52.36	59.84	74.90	59.11	58.72	49.52	39.42	43.35	37.83	51.87
33	59.38	49.24	73.09	53.87	66.61	50.47	42.83	64.20	54.71	45.26	55.56	47.79	42.20	35.57	45.92
34	58.70	66.77	36.17	44.13	61.43	44.16	52.91	47.21	47.79	39.14	32.54	48.98	44.51	32.18	36.20
35	58.70	67.45	74.45	38.29	52.38	42.26	44.09	53.50	53.33	37.31	49.52	35.84	37.57	28.80	34.58
36	46.41	55.31	66.33	49.97	36.86	32.17	41.57	42.80	39.62	29.36	39.86	34.00	30.64	32.75	31.88
37	34.13	44.51	56.17	41.54	43.97	31.54	45.98	31.47	40.27	34.86	24.15	34.05	33.53	31.62	28.63
38	32.76	24.96	46.02	33.75	39.45	43.53	43.46	23.29	37.10	28.13	23.59	26.88	31.79	22.58	22.69
39	25.94	21.38	31.81	27.91	32.33	35.33	51.65	28.32	28.93	20.80	22.95	26.88	19.07	14.68	25.93
40	24.57	23.61	23.01	21.42	25.87	24.60	37.16	37.77	24.52	20.18	19.93	21.50	15.03	7.34	21.07
41	17.06	18.88	17.60	20.12	14.87	19.56	16.38	27.70	28.93	20.18	16.30	14.34	9.25	8.47	7.02
42	11.60	14.84	16.24	14.28	10.99	12.62	13.23	17.62	18.87	22.63	14.49	8.96	5.78	7.90	9.72
43	12.97	8.77	8.80	11.68	9.05	9.46	11.34	9.44	9.43	15.29	12.08	8.36	8.67	3.95	8.10
44	8.19	6.74	4.06	7.79	8.41	6.31	6.30	5.66	5.66	7.95	5.43	9.56	4.05	3.39	4.32
45	5.46	4.72	5.38	3.89	4.53	5.68	3.15	5.04	5.66	4.89	3.62	5.97	2.89	3.95	5.40
46	4.78	3.37	2.71	3.89	3.23	3.78	2.52	3.78	5.03	3.06	3.62	1.79	3.47	3.95	3.24
47	1.37	4.05	3.38	3.24	3.88	1.26	1.26	3.78	3.14	4.28	2.42	0.00	2.31	2.26	2.70
48	1.37	1.35	2.71	1.30	3.23	5.05	1.89	2.52	3.14	3.67	1.21	1.19	1.73	0.00	1.62
49	0.68	0.67	0.68	0.65	1.94	4.42	1.89	1.26	1.26	2.45	1.21	1.19	1.16	0.56	1.62

Note: The figures in this table were generated by computer. They may not coincide with those given in the text due to rounding and truncation in intermediate calculations.

TABLE 167. ESTIMATED NUMBER OF WOMEN IN SINGLE-YEAR PERIODS, BY SINGLE-YEAR AGE OF WOMAN, COLOMBIA, 1963/64–1977/78

Age of women	Estimated number of women, $_{1}P^{a}_{x-x}$														
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1976/1977	1977/1978
14	405.24	408.38	381.10	444.44	452.11	467.60	523.96	522.15	554.84	528.52	629.14	702.12	695.17	727.96	756.64
15	385.94	404.36	407.49	380.27	443.48	451.13	461.60	522.82	521.02	553.63	577.27	627.78	700.59	695.66	726.36
16	340.54	384.96	403.34	406.47	379.31	442.36	448.99	460.43	526.51	519.70	552.23	575.81	626.19	698.83	691.91
17	309.85	339.65	383.95	402.28	405.39	378.31	441.19	448.80	459.22	520.13	518.33	550.78	574.29	624.54	696.96
18	325.61	308.99	338.71	382.89	401.16	404.27	377.26	439.97	447.56	457.95	518.69	516.89	549.25	572.70	622.81
19	280.86	324.63	308.06	337.69	381.73	399.96	403.06	376.13	438.65	446.21	456.57	517.13	515.34	547.60	570.96
20	393.21	279.75	323.57	307.06	336.59	380.50	398.66	401.75	374.91	437.22	444.77	455.09	515.45	513.67	545.83
21	313.61	304.16	278.81	322.48	306.03	335.46	379.21	397.32	400.39	373.64	435.75	443.27	453.56	513.72	511.94
22	300.41	312.52	303.12	277.83	321.36	304.95	334.28	377.89	395.93	398.99	372.34	434.22	441.72	451.97	511.92
23	340.01	299.29	311.35	301.98	276.79	320.15	303.81	333.03	376.47	394.44	397.50	370.94	432.60	440.06	450.28
24	300.22	338.73	298.16	310.17	300.85	275.75	318.95	302.67	331.78	375.05	392.96	396.00	369.55	430.97	438.41
25	302.27	299.05	337.41	297.00	308.97	299.68	274.68	317.71	301.49	330.49	373.59	391.43	394.46	368.11	429.29
26	279.23	301.06	297.84	336.06	295.81	307.72	298.47	273.57	316.43	300.28	329.16	372.09	389.85	392.87	366.63
27	223.75	278.22	299.95	296.75	334.82	294.72	306.59	297.37	272.57	315.26	299.17	327.94	370.72	388.42	391.43
28	250.01	222.82	277.05	298.70	295.51	333.42	293.49	305.31	296.13	271.43	313.94	297.92	326.57	369.17	386.79
29	217.97	248.81	221.75	275.73	297.26	294.09	331.82	292.08	303.85	294.71	270.13	312.44	296.49	325.01	367.40
30	254.56	217.00	247.71	220.76	274.50	295.94	292.78	330.34	290.78	302.49	293.40	268.92	311.05	295.17	323.56
31	241.75	253.41	216.00	246.57	219.75	273.24	294.58	291.43	328.82	289.44	301.10	292.05	267.69	309.62	293.82
32	203.58	240.60	252.21	214.98	245.40	218.70	271.94	293.18	290.05	327.27	288.07	299.67	290.66	266.42	308.15
33	261.69	202.61	239.46	251.01	213.96	244.23	217.66	270.65	291.79	288.67	325.71	286.70	298.25	289.28	265.15
34	229.83	260.37	201.59	238.25	249.75	212.88	243.00	216.57	269.29	290.32	287.22	324.07	285.26	296.75	287.82
35	252.85	228.64	259.02	200.55	237.02	248.45	211.78	241.74	215.45	267.89	288.82	285.73	322.39	283.78	295.21
36	237.05	251.53	227.43	257.67	[99.50	235.79	247.16	210.67	240.49	214.33	266.50	287.31	284.25	320.71	282.30
37	164.78	235.75	250.15	226.20	256.26	198.41	234.49	245.80	209.52	239.17	213.15	265.04	285.74	282.69	318.96
38	178.34	163.87	234.45	248.77	224.95	254.85	197.31	233.20	244.45	208.37	237.85	211.98	263.58	284.16	281.13
39	166.49	177.31	162.92	233.09	247.33	223.65	253.37	196.17	231.85	243.03	207.16	236.47	210.75	262.05	282.52
40	186.42	165.52	176.28	161.97	231.73	245.89	222.35	251.89	195.03	230.50	241.62	205.95	235.09	209.52	260.52
41	180.05	183.31	164.53	175.22	161.00	230.34	244.42	221.01	250.39	193.86	229.12	240.17	204.72	233.69	208.26
42	155.85	178.92	184.14	163.50	174.12	159.99	228.90	242.88	219.62	248.81	192.64	227.68	238.66	203.43	232.22
43	153.14	154.85	177.77	182.95	162.44	173.00	158.96	227.42	241.31	218.21	247.21	191.40	226.21	237.12	202.12
44	121.70	152.10	153.80	176.57	181.72	161.35	171.83	157.89	225.84	239.68	216.73	245.54	190.11	224.68	235.52
45	171.99	120.84	151.03	152.72	175.32	180.44	160.21	170.62	156.77	224.29	237.99	215.20	243.80	188.76	223.10
46	175.05	170.70	119.93	149.90	151.57	174.00	179.08	159.00	169.34	155.60	222.61	236.21	213.59	241.98	187.35
47	117.53	173.68	169.37	118.99	148.72	150.39	172.64	177.68	157.76	168.01	154.38	220.86	234.36	211.92	240.08
48	121.53	116.57	172.26	167.99	118.02	147.51	149.16	171.23	176.23	156.47	166.84	153.12	219.06	232.45	210.19
49	104.76	120.46	115.53	170.75	166.51	116.99	146.22	147.85	169.73	174.69	155.10	165.18	151.78	217.14	230.41

Note: The figures in this table were generated by computer. They may not coincide with those given in the text due to rounding and truncation in intermediate calculations.

tainty about their accuracy, and it is perhaps better to ignore them when computing total fertility.

Step 5: reverse survival of adult females. Using the ${}_1L_x$ values shown in table 165, the reverse projection of females given in table 161 is carried out by using equation (C.5), or (C.6) when the calculations are being made systematically in two steps by hand. As an example, the female population aged 25 at the middle of a year (1969/70) is calculated. As intermediate steps one needs to compute W^{25}_{1969} and W^{25}_{1970} by

$$W^{25}_{1970} = W^{25}_{1975}({}_1L_{25}/{}_1L_{30}) = 272(0.8445/0.8152) = 281.78$$

$$W^{25}_{1969} = W^{25}_{1974}({}_1L_{25}/{}_1L_{34}) = 257(0.8445/0.8112) = 267.55$$

and then to compute their average as

$$N^{25}_{1970} = (281.78 + 267.55)/2.0 = 274.66.$$

All values of N^x_{t-x} are given in table 167.

Step 6: calculation of age-specific fertility rates. The calculation of age-specific fertility rates by single years, denoted by $f_{t-x}(a)$, using equation (C.8), is simple. For example,

$$f_{t-x}(25) = B^{25}_{1970}/N^{25}_{1970} = 79.98/274.66 = 0.2912.$$

Other values of $f_{t-x}(a)$ are shown in table 168. From these values, the calculation of five-year age-specific fertility rates, denoted by $f_{t-x}(i)$, is carried out using a modification of equation (C.10) as illustrated below:

$$f_{1970}(1) = (f_{1970}(15) + f_{1970}(16) + f_{1970}(17) +$$

$$+ f_{1970}(18) + f_{1970}(19))/5.0$$

$$= (0.0327 + 0.0672 + 0.1028 + 0.1469 +$$

$$+ 0.1985)/5.0 = 0.1096.$$

Other estimates of $f_{t-x}(i)$ are given in table 169. They are obtained by averaging the relevant $f_{t-x}(a)$ values, as specified by the next general equation:

$$f_{t-x}(i) = \sum_{a=10i+5}^{14+5i} f_{t-x}(a) \qquad \text{(C.15)}$$

Tables 168 and 169 also show values of total fertility. In both cases, they are obtained by adding the $f_{t-x}(a)$ values or the $f_{t-x}(i)$ values multiplied by five to take into account the fact that the latter values represent five-year averages. The difference between the total fertility values appearing in table 168 and those shown in table 169 is due to the fact that the values given in table 168 include the fertility rates for age 14, whereas those shown in table 169 do not.

Note that every set of fertility rates and total fertility estimates is labelled with a pair of years, indicating the period to which the rates refer. According to the notation used in this section, the second or last year of each period is that denoted by $(t-x)$.

Consider the sequence of total fertility estimates obtained by the application of the own-children method (given in table 169). In general, these estimates tend to increase as one moves into the past. There are, however, some exceptions to a monotonical increase. The estimates for the single-year periods 1965/66 and 1967/68 are higher than those of the neighbouring years. They are estimates derived from children whose reported ages in 1978 were 12 and 10 years, respectively. Obviously, age-heaping at these preferred ages is the cause of the relatively high total fertility estimates associated with 1965/66 and 1967/68. The relatively low total fertility estimated for 1966/67 is probably also due to age-reporting errors: avoidance of age 11 in 1978. To smooth out some of the peaks and troughs observed in the raw total fertility estimates, averages of the estimates for contiguous years may be calculated. The estimates shown in table 170 are obtained in this way; for example,

$$f_{1967-1969}(3) = (f_{1966/67}(3) + f_{1967/68}(3) + f_{1968/69}(3))/3.0$$

$$= (0.2759 + 0.2823 + 0.2549)/3.0 = 0.2710$$

and

$$TF_{1967-1969} = (TF_{1966/67} + TF_{1967/68} + TF_{1968/69})/3.0$$

$$= (5.8168 + 6.1170 + 5.5807)/3.0 = 5.8382.$$

The smoothed total fertility estimates given in table 170 increase steadily as one moves further into the past; that is, they are consistent with the existence of a long-term decline in fertility. The levels they imply are, on the whole, satisfactory but should not be interpreted too strictly. In particular, it appears likely that the strong attraction of age 12 may bias the 1964-1966 estimate upward, and that selective omission of young children may bias downward the estimate for 1976-1978. Comparison of these estimates with those derived from other sources and by other methods is necessary in order to validate them.

To conclude, it must be pointed out that the own-children method, as described here, uses unchanging mortality schedules for the entire period under consideration. The modification of the method to the case in which mortality changes is straightforward, but it is not common to have the information required to estimate with some confidence a time-series of mortality schedules. Therefore, the version of the method described here is suited for many of the cases encountered in practice.

TABLE 168. ESTIMATED SINGLE-YEAR FERTILITY RATES BY SINGLE-YEAR OF AGE OF WOMAN, COLOMBIA, 1963/64-1977/78

Age of woman	Annual fertility rates, $f_{x,x+1}$													
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1977/1978
14	0.0202	0.0264	0.0391	0.0321	0.0143	0.0245	0.0168	0.0362	0.0159	0.0254	0.0211	0.0051	0.0100	0.0047	0.0186
15	0.0460	0.0517	0.0448	0.0461	0.0321	0.0419	0.0327	0.0470	0.0302	0.0330	0.0314	0.0152	0.0190	0.0155	0.0186
16	0.0691	0.0946	0.0634	0.0798	0.0750	0.0642	0.0672	0.0795	0.0567	0.0506	0.0733	0.0417	0.0378	0.0420	0.0273
17	0.1101	0.1092	0.1199	0.1355	0.1101	0.0867	0.1028	0.1206	0.1013	0.0929	0.1095	0.0868	0.0694	0.0741	0.0628
18	0.2012	0.1965	0.1758	0.1695	0.1451	0.1373	0.1469	0.1531	0.1405	0.1349	0.1246	0.1387	0.1063	0.0996	0.1076
19	0.2797	0.2410	0.2329	0.1845	0.1982	0.1987	0.1985	0.1991	0.1663	0.1631	0.1547	0.1814	0.1481	0.1299	0.1353
20	0.2751	0.2314	0.2468	0.2325	0.2421	0.2321	0.2228	0.2428	0.2097	0.1972	0.1697	0.2048	0.1749	0.1660	0.1534
21	0.2938	0.2417	0.2915	0.2636	0.2853	0.2445	0.2375	0.2630	0.2372	0.2439	0.1871	0.2089	0.2014	0.1815	0.1710
22	0.3249	0.2935	0.3171	0.3130	0.2878	0.3041	0.2752	0.2682	0.2605	0.2437	0.2092	0.2105	0.2159	0.1849	0.1773
23	0.3232	0.3425	0.3478	0.2751	0.3107	0.3093	0.2944	0.2873	0.2723	0.2235	0.2585	0.2066	0.2031	0.1886	0.2124
24	0.3524	0.2867	0.3700	0.2553	0.3031	0.3203	0.2844	0.3078	0.2672	0.2446	0.2489	0.2097	0.2112	0.1978	0.2268
25	0.3342	0.2706	0.3310	0.3059	0.2993	0.2884	0.2912	0.2774	0.2670	0.2406	0.2425	0.2136	0.2154	0.1963	0.2064
26	0.3080	0.2957	0.3340	0.2877	0.3148	0.2998	0.2997	0.2991	0.2504	0.2220	0.2312	0.1943	0.1913	0.1897	0.1930
27	0.2837	0.2836	0.3182	0.2624	0.2762	0.2569	0.3102	0.2604	0.2376	0.2453	0.2220	0.1785	0.1902	0.1875	0.1946
28	0.3085	0.3269	0.3054	0.2716	0.2648	0.2554	0.2833	0.2309	0.2336	0.2503	0.2154	0.1803	0.1982	0.1805	0.1830
29	0.3362	0.3117	0.3296	0.2519	0.2567	0.2338	0.2392	0.2608	0.2070	0.2138	0.2057	0.2007	0.2125	0.1720	0.1782
30	0.2601	0.2891	0.3060	0.1911	0.2450	0.2068	0.2431	0.2401	0.1925	0.1982	0.2141	0.2243	0.1933	0.1702	0.1469
31	0.2569	0.2821	0.2914	0.2422	0.2590	0.2216	0.2416	0.2484	0.2008	0.1902	0.1945	0.1636	0.1706	0.1514	0.1600
32	0.2548	0.2607	0.3220	0.2928	0.2899	0.2394	0.2200	0.2555	0.2038	0.1794	0.1819	0.1316	0.1491	0.1420	0.1683
33	0.2269	0.2430	0.3052	0.2146	0.3113	0.2066	0.1968	0.2372	0.1875	0.1568	0.1706	0.1667	0.1415	0.1230	0.1732
34	0.2534	0.2565	0.2787	0.1852	0.2460	0.2074	0.2177	0.2180	0.1775	0.1348	0.1829	0.1511	0.1560	0.1085	0.1258
35	0.2522	0.2950	0.2874	0.1909	0.2210	0.1701	0.2082	0.2213	0.1547	0.1393	0.1714	0.1254	0.1165	0.1015	0.1171
36	0.1958	0.2199	0.2916	0.1939	0.1845	0.1384	0.1682	0.2032	0.1647	0.1370	0.1496	0.1185	0.1078	0.1021	0.1129
37	0.2071	0.1888	0.2246	0.1836	0.1716	0.1590	0.1961	0.1280	0.1951	0.1458	0.1133	0.1285	0.1173	0.1118	0.0898
38	0.1837	0.1523	0.1963	0.1357	0.1254	0.1708	0.2203	0.0999	0.1518	0.1350	0.0990	0.1268	0.1206	0.0795	0.0807
39	0.1558	0.1217	0.1952	0.1197	0.1307	0.1580	0.2038	0.1444	0.1248	0.0856	0.1108	0.1137	0.0905	0.0560	0.0918
40	0.1318	0.1426	0.1305	0.1322	0.1116	0.1001	0.1671	0.1499	0.1258	0.0876	0.0825	0.1044	0.0639	0.0350	0.0809
41	0.0948	0.1019	0.1070	0.1148	0.0924	0.0849	0.0650	0.1253	0.1155	0.1041	0.0712	0.0597	0.0452	0.0362	0.0337
42	0.0745	0.0829	0.0882	0.0873	0.0631	0.0789	0.0578	0.0726	0.0959	0.0910	0.0752	0.0394	0.0242	0.0389	0.0419
43	0.0847	0.0566	0.0495	0.0639	0.0557	0.0547	0.0713	0.0415	0.0319	0.0701	0.0489	0.0457	0.0383	0.0167	0.0401
44	0.0673	0.0443	0.0264	0.0441	0.0463	0.0391	0.0367	0.0359	0.0251	0.0332	0.0251	0.0389	0.0213	0.0151	0.0184
45	0.0317	0.0391	0.0224	0.0255	0.0258	0.0315	0.0197	0.0295	0.0361	0.0218	0.0152	0.0278	0.0119	0.0209	0.0242
46	0.0273	0.0198	0.0226	0.0260	0.0213	0.0218	0.0141	0.0238	0.0297	0.0197	0.0163	0.0076	0.0162	0.0163	0.0173
47	0.0116	0.0233	0.0200	0.0273	0.0261	0.0084	0.0073	0.0213	0.0199	0.0255	0.0156	0.0000	0.0099	0.0107	0.0113
48	0.0112	0.0116	0.0157	0.0077	0.0274	0.0042	0.0127	0.0147	0.0178	0.0235	0.0072	0.0078	0.0079	0.0000	0.0077
49	0.0065	0.0056	0.0059	0.0058	0.0117	0.0077	0.0129	0.0085	0.0074	0.0140	0.0078	0.0072	0.0076	0.0026	0.0070
TOTAL FERTILITY	6.8333	6.4407	7.0557	5.8489	6.1313	5.6052	5.8832	5.8516	5.3086	4.8168	4.6279	4.2691	4.0144	3.5487	3.7752

Note: The figures in this table were generated by computer. They may not coincide with those given in the text due to rounding and truncation in intermediate calculations.

TABLE 169. ESTIMATED SINGLE-YEAR FERTILITY RATES BY FIVE-YEAR AGE GROUP, COLOMBIA, 1963/64-1977/78

Age group	Estimated fertility rate, f_{t-x} [1]													
	1963/1965	1964/1966	1965/1967	1966/1968	1967/1969	1968/1970	1969/1971	1970/1972	1971/1973	1972/1974	1973/1975	1974/1976	1975/1977	1976/1978
15-19	0.1402	0.1386	0.1278	0.1231	0.1121	0.1058	0.1096	0.1198	0.0990	0.0947	0.0987	0.0959	0.0761	0.0722	0.0703
20-24	0.3139	0.2792	0.3146	0.2679	0.2858	0.2821	0.2625	0.2738	0.2494	0.2306	0.2107	0.2080	0.2013	0.1837	0.1882
25-29	0.3145	0.2977	0.3236	0.2759	0.2823	0.2549	0.2847	0.2657	0.2391	0.2350	0.2233	0.1935	0.2015	0.1852	0.1831
30-34	0.2508	0.2663	0.3007	0.2252	0.2702	0.2164	0.2239	0.2398	0.1924	0.1719	0.1868	0.1675	0.1621	0.1390	0.1548
35-39	0.1949	0.1955	0.2390	0.1648	0.1767	0.1589	0.1993	0.1594	0.1582	0.1285	0.1288	0.1226	0.1106	0.0902	0.0985
40-44	0.0906	0.0857	0.0803	0.0885	0.0738	0.0715	0.0800	0.0850	0.0783	0.0772	0.0606	0.0572	0.0386	0.0284	0.0430
45-49	0.0177	0.0199	0.0173	0.0181	0.0225	0.0267	0.0133	0.0195	0.0222	0.0209	0.0124	0.0101	0.0107	0.0101	0.0135
TOTAL FERTILITY	6.6123	6.4143	7.0166	5.8164	6.1170	5.5807	5.8664	5.8155	5.1928	4.7934	4.6068	4.2640	4.0045	3.5441	3.7566

TABLE 170. ESTIMATED THREE-YEAR FERTILITY RATES BY FIVE-YEAR AGE GROUP, COLOMBIA, 1964-1978

Age group	1964-1966	1967-1969	1970-1972	1973-1975	1976-1978
15-19	0.1355	0.1136	0.1095	0.0958	0.0729
20-24	0.3026	0.2786	0.2619	0.2164	0.1911
25-29	0.3120	0.2710	0.2632	0.2173	0.1899
30-34	0.2726	0.2373	0.2187	0.1754	0.1520
35-39	0.2098	0.1668	0.1723	0.1266	0.0997
40-44	0.0855	0.0779	0.0811	0.0630	0.0366
45-49	0.0183	0.0224	0.0184	0.0145	0.0114
TOTAL FERTILITY	6.6814	5.8382	5.6249	4.5547	3.7684

Chapter IX

ESTIMATION OF ADULT MORTALITY USING SUCCESSIVE CENSUS AGE DISTRIBUTIONS

A. BACKGROUND OF METHODS

1. *Use of a sequence of population age distributions*

The value for demographic estimation purposes of having basically similar information about a population for two points in time has already been stressed, for example, in the discussion of the use of overlapping fertility estimates based on reverse-survival techniques (chapter VIII) and in the sections dealing with hypothetical-cohort methods (see chapters II, III and IV). The age and sex distributions from successive enumerations of a population also provide a basis for estimating intercensal mortality. In a closed population with two accurate censuses t years apart, the population aged $x+t$ at the second census represents the survivors of the population aged x at the first census, so that the intercensal survivorship probability from age x to age $x+t$ can be calculated. Traditional mortality measures can then be obtained from the sequence of survivorship probabilities for successive initial ages x.

This method of mortality estimation from intercensal survival is appealingly simple and straightforward, requiring only the most basic of census information, making no assumptions about the age pattern of mortality, and providing estimates of mortality for a clearly defined time period. The trouble is that these advantages are nullified by the requirements that the censuses be accurate and that the population be closed. In practice, the application of this method very often gives disappointing results. Migration can affect a population as much as mortality; and, in particular, at young adult ages, its influence on population size may be more important than that of mortality. Age-misreporting can also distort the results severely: a marked preference for certain digit endings when declaring age, which will introduce considerable variability into the estimated survivorship ratios, can be reduced to some extent by the use of grouped data, but systematic overreporting or underreporting of age can cause insuperable problems. Changes in enumeration completeness from one census to the other can, if no adjustment is feasible, completely swamp the effects of mortality, giving rise to very misleading results; indeed, it may be stated without exaggeration that, in many cases, intercensal survival estimates are better indicators of the comparability of two census enumerations than of the level of intercensal mortality.

Despite these problems, however, it is worth applying the method where possible, because if the errors in the basic data are not overwhelming, one can obtain useful estimates of mortality by using suitable age groups and certain smoothing techniques; and even if the data errors are severe, the calculated survivorship probabilities may be useful indicators of the nature of the errors involved.

2. *Organization of this chapter*

The methods described in this chapter are all based essentially on the use of two successive age (and preferably sex) distributions of a population. The age distributions should be obtained from complete enumerations, because sampling errors would greatly distort the results were data from sample surveys used; and the population should ideally be closed to migration, failing which, one population or the other should be adjusted for the effects of net migration. The calculations are greatly simplified if the length of the intercensal period is an exact multiple of five years, though other periods can be accommodated; the period should not, in general, exceed 15 years. The main features of the methods available are described below (for data requirements and parameters estimated, see table 171):

Section B. Estimation of mortality from intercensal survivorship probabilities. The traditional procedure for estimating adult mortality from two successive census age distributions by calculating cohort survivorship probabilities for the intercensal period is described. Different procedures for smoothing[1] the calculated probabilities, using the Coale-Demeny model life tables or the logit life-table system in order to estimate a single mortality parameter, are also presented. Variants based on cumulated and uncumulated age distributions are included;

Section C. Intercensal survival with additional information on the age pattern of mortality. In a closed population, the proportionate reduction in cohort size from one census to another can be compared with the proportionate reduction expected on the basis of cohort deaths as recorded by a vital registration system or retrospective survey question. Since the age patterns of the popula-

[1] The term "to smooth" is used in this *Manual* in its most general sense to mean the elimination or minimization of irregularities often present in reported data or in preliminary estimates obtained from them. In this sense, the set of possible "smoothing techniques" encompasses a wide variety of procedures, ranging from the fitting of models to simple averaging. The traditional smoothing techniques applied to age distributions and to observed age-specific mortality rates are part of this set, but they do not exhaust it. The somewhat rougher procedures described in this *Manual* are necessary because the basic data available are both deficient and incomplete.

TABLE 17E. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER IX

Section	Subsection and method	Type of input data	Parameters estimated
B. Estimation of mortality from intercensal survivorship probabilities	B.2 Survival ratios for five-year age cohorts smoothed by using the Coale-Demeny life tables	Population classified by age and sex from two censuses 15 years apart or less. If the intercensal period is not an exact multiple of five, at least one age distribution must be by single years An estimate of net intercensal migration	A life table for the intercensal period from age 10 or so onward
	B.3 Survival ratios for five-year age cohorts smoothed by using the logit system	Population classified by age and sex from two censuses 15 years apart or less. If the intercensal period is not an exact multiple of five, at least one age distribution must be by single years. An estimate of child survivorship An estimate of net intercensal migration	A life table for the intercensal period from age 10 or so onward
	B.4 Survival ratios smoothed by cumulation and by using the Coale-Demeny life tables	Population classified by age and sex from two censuses 15 years apart or less. If the intercensal period is not a multiple of five, at least one age distribution must be by single years. An estimate of net intercensal migration	A life table for the intercensal period from age 10 or so onward
C. Intercensal survival with additional information on the age pattern of mortality		Population classified by age and sex from two censuses 15 years apart or less. If the intercensal period is not an exact multiple of five, at least one age distribution must be by single years. Registered deaths during the intercensal period classified by age and sex Estimates of net intercensal migration (to adjust raw data, if necessary)	Completeness of coverage of recorded deaths with respect to the completeness of the first census Completeness of coverage of the first census in relation to that of the second
D. Estimation of a post-childhood life table from an age distribution and intercensal growth rates		Population classified by age and sex from two censuses 15 years apart or less. The same age classification must be used for both populations Estimates of net intercensal migration (to adjust raw data, if necessary)	Estimates of ${}_5L_x$ from age 10 or 15 onward

tion and of deaths are very different, it is possible to disentangle the effects of changes in enumeration completeness from undercoverage of deaths. Specifically, if enumeration completeness and coverage of deaths do not vary with age, at least after childhood, the enumeration completeness of one census in relation to the other, and the completeness of death recording in relation to either, can be estimated. The procedure is severely affected, however, by systematic age errors;

Section D. Estimation of a post-childhood life table from an age distribution and intercensal growth rates. Two census enumerations provide the data necessary to calculate intercensal growth rates for five-year age groups. These growth rates can then be used to convert the average age distribution from the two censuses into stationary population form, that is, a life-table ${}_5L_x$ function. This method is computationally attractive if the length of the intercensal period is not a multiple of five years.

B. ESTIMATION OF MORTALITY FROM INTERCENSAL SURVIVORSHIP PROBABILITIES

1. *Basis of methods and their rationale*

The methods described in this section are all based on the same, very simple information, namely, the change in size of successive age cohorts of a population from one census to the next. The methods differ only in the ways in which this basic information is smoothed to reduce the effects of errors and converted into a mortality parameter. Therefore subsections B.3 and B.4 only cover those steps which are different from the steps described in subsection B.2.

2. *Intercensal survivorship ratios for five-year age cohorts smoothed using the Coale-Demeny life tables*

(a) *Data required*

The data required for this method are two census

enumerations with populations classified by age and sex. (Classification by sex is not necessary, but since it is generally available, it is useful to consider it whenever possible.) Although ω, the highest age reached, is the theoretical limit for the value of the intercensal interval t (measured in years), in practice, with intervals longer than 15 years the information available is scanty and the calculations are more likely to be affected by changes other than those caused by mortality. If t is divisible by five, both age distributions can be classified by five-year age groups and cohorts from the first census can be identified at the second census; if t is not a multiple of five, it is convenient to have one of the age distributions by single year of age, so that comparable cohorts may be constructed.

(b) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: adjustment for net intercensal migration and territorial coverage. Substantial net migration during the intercensal period will generally render the method of intercensal survival unusable. Nevertheless, if it is possible to adjust one age distribution or the other on an age-specific basis for the effects of migration, the method may be applied after such adjustment has been made. However, it is most unusual for adequate information about migration to be available, and no general procedures for carrying out an adjustment can be expounded here. Problems introduced by changes in territorial coverage may not be quite so serious. By the judicious aggregation of subnational information from one census or the other, it is usually possible to arrive at age distributions for comparable populations. No general procedures for so doing need to be stated, however, beyond pointing out the necessity of making suitable adjustments if changes in territorial coverage have occurred.

Step 2: grouping of data from the two censuses by cohort. Simple intercensal survival techniques generally disregard the effects of age distribution within cohort groupings, assuming in effect that the population is distributed within each age group in the same way as if it were a stationary or life-table population. As a result of this simplifying assumption, the width of the cohorts should not be too large (probably not more than five years). Groupings that are five years in size are also convenient because most model life-table systems are tabulated for five-year age groups, though other intervals can be used if necessary. If the intercensal interval t is divisible by five, conventional five-year age groups from x to $x+4$ at the first census will survive to become conventional five-year age groups from $x+t$ to $x+t+4$ at the second census; and no regrouping is required. If t is not divisible by five, a single-year age distribution from either the first or the second census can be used in order to create groups corresponding to conventional five-year cohorts at the other census. In cases where there is substantial age-heaping and a danger that it may introduce systematic age exaggeration, unconventional five-year age groups (such as $3-7$, $8-12$ and $13-17$) centred on preferred-digit endings may be used.

Step 3: adjustment for intercensal interval that is not an exact number of years. When the intercensal interval is not an exact number of years, a small adjustment should be made to one population or the other, by moving it forward or backward in order to approximate the population corresponding to the nearest date defining an interval with an exact number of years and thus to remove the slight effect that normal population growth would have on the intercensal survivorship estimates. The intercensal growth rate r can be calculated as

$$r = [\ln(N_2) - \ln(N_1)]/t$$

where N_2 is the total population recorded by the second census; N_1 is the total population recorded by the first census; and t is the intercensal period measured in years. This growth rate can then be used to move either the first or second age distribution over the required length of time. If the decimal portion of t is less than 0.5, the interval should be shortened to t exact years, whereas if t is greater than 0.5, the interval should be lengthened to $t+1$ exact years. If the decimal portion of t is denoted by z, the interval can be shortened to t years either by multiplying each age group at the first census by a factor $\exp[rz]$ or by multiplying each age group at the second census by a factor $\exp[-rz]$. The interval can be lengthened either by multiplying each age group at the first census by a factor $\exp[r(z-1.0)]$ or by multiplying each age group at the second census by a factor $\exp[r(1.0-z)]$.

Step 4: calculation of cohort survivorship ratios. Cohort survivorship probabilities or ratios during the intercensal period, denoted by ${}_tS_{x,x+t}$, can now be calculated by dividing the cohort size at the second census by its size at the first census. These survivorship ratios approximate life-table (or stationary-population) survivorship probabilities, provided the effects of deviations of the actual age distribution within cohorts from that corresponding to the stationary population are small (as is usually the case). Thus,

$${}_tS_{x,x+t} = {}_5N^2_{x+t}/{}_5N^1_x = {}_5L_{x+t}/{}_5L_x$$

where t is the adjusted length of the intercensal interval after applying step 3; ${}_5N^1_x$ is the population aged from x to $x+4$ enumerated by the first census; and ${}_5N^2_{x+t}$ is the population aged from $x+t$ to $x+t+4$ enumerated by the second census.

Step 5: fitting of a Coale-Demeny model life table. The consistency of the cohort survivorship ratios calculated in step 4 may be conveniently examined by finding the mortality level, in the Coale-Demeny model life tables, to which each ratio corresponds. A best estimate of mortality level can then be obtained by discarding any detectable outliers and basing the estimate on the remaining levels (by taking their average, for example). If the adjusted intercensal period t is divisible by five, stationary-population ratios of the type ${}_5L_{x+t}/{}_5L_x$ can

be calculated directly for relevant levels of the selected regional family of model life tables. If x is not divisible by five, however, additional steps become necessary, since the Coale and Demeny life tables do not provide stationary-population age distributions for non-standard age groups. The simplest procedure is to calculate stationary-population distributions for non-standard age groups simply by weighting adjacent standard five-year values by the proportions of the age groups covered. Thus, ${}_5L_{19}$, the stationary population aged from 19 to 23, covers one fifth of age group 15-19 and four fifths of age group 20-24; it can be approximated as

$${}_5L_{19} = (\frac{1}{5})\,{}_5L_{15} + (\frac{4}{5})\,{}_5L_{20}$$

In fact, if the $l(x)$ function is linear with age, the approximation is exact.

If somewhat more precision is required, ${}_5L_{x+n}$ values can be estimated from tabulated values of $l(x)$, $l(x+5)$ and $l(x+10)$ using equation (B.1):

$${}_5L_{x+n} = a(n)\,l(x) + b(n)\,l(x+5) + c(n)\,l(x+10) \qquad \text{(B.1)}$$

where the coefficients $a(n)$, $b(n)$ and $c(n)$, for n ranging from 0 to 4, are calculated by fitting a second-order polynomial to the $l(x)$ values. Values of these coefficients are shown in table 172.

Step 6: completion of the life table. Intercensal survival provides no information about the mortality experience

TABLE 172. COEFFICIENTS FOR ESTIMATION OF STATIONARY-POPULATION AGE DISTRIBUTION ${}_5L_x$ FOR UNCONVENTIONAL AGE GROUPS

Index n	Coefficients		
	$a(n)$ (I)	$b(n)$ (II)	$c(n)$ (III)
0	2.083	3.333	−0.417
1	1.583	4.133	−0.717
2	0.483	4.533	−0.017
3	−0.017	4.533	0.483
4	−0.317	4.133	1.183
5	−0.417	3.333	2.083

Estimation equation:
$${}_5L_{x+n} = a(n)\,l(x) + b(n)\,l(x+5) + c(n)\,l(x+10)$$
where x is exactly divisible by five.

of those born between the censuses, since the first census does not provide their initial number (accurate birth registration could supply this want, but where births are completely registered, better estimates of mortality would probably be available from other sources). In order to obtain a complete life table, therefore, it is necessary to supply further information about child mortality. The most satisfactory source of such estimates is information about children ever born and surviving (see chapter III). If estimates of this type are available, the methods described in chapter VI for linking estimates of child and adult mortality can be used to obtain a complete life table.

A problem remains, however, if no independent estimate of child mortality is available. Since the Coale-Demeny life-table system has been used in selecting a model life table (see step 5), the mortality pattern of the model used can be adopted by taking the life table associated with the average mortality level of the intercensal survival probabilities as representative.

(c) *First detailed example: Panama, 1960-1970*

The first detailed example illustrates a fairly simple case of a 10-year interval and reasonably good age-reporting. Population censuses were held in Panama on 11 December 1960 and 10 May 1970. This example examines only the intercensal survival of the female population, though in a complete study, this analysis should be carried out for both males and females. Table 173 shows the female population enumerated by the two censuses classified by five-year age group.

The computational procedure for this example is given below.

Step 1: adjustment for net intercensal migration and territorial coverage. As no information on intercensal migration by age is available, no adjustment can be made. No change in territorial coverage occurred between 1960 and 1970; therefore, no adjustment for coverage is needed.

Step 2: grouping of data from the two censuses by cohort. Because the interval between the two censuses is about nine and one-half years, one of the populations has to be moved slightly to bring cohorts into alignment. The exact interval, 9.41 years, is somewhat closer to nine years than 10, so the adjustment for dates would be minimized by moving the first population forward slightly, or by moving the second one back, to create an intercensal period of nine years. However, there is also an advantage to working with intervals divisible by five, and since the actual interval was only slightly less than nine and one-half years, the convenience factor outweighs that of a marginal gain in accuracy. Thus, the first census will be moved back to approximate the female population on 10 May 1960 (the results would be precisely the same if the second census were moved forward to 11 December 1970). Standard five-year age groups will therefore define cohorts, and no regrouping is required.

Step 3: adjustment for length of the intercensal interval. The total female population in 1960 was 529,767, and in 1970 it was 704,333; thus, the exponential rate of population growth during the intercensal period 1960-1970 is

$$r = \frac{\ln[N_2] - \ln[N_1]}{t_2 - t_1} = \frac{13.4650 - 13.1802}{9.41} = 0.0303.$$

The growth factor k needed to adjust the 1960 population for 0.59 of a year's growth is then obtained as

$$k = \exp[(0.0303)(9.41 - 10.00)] =$$

$$\exp(-0.0179) = 0.9823.$$

Column (3) of table 173 shows the adjusted population.

TABLE 173. ENUMERATED AND ADJUSTED FEMALE POPULATION IN 1960, ENUMERATED POPULATION IN 1970 AND COHORT SURVIVORSHIP RATIOS, PANAMA

Age group (1)	1960 population: Enumerated (2)	1960 population: Adjusted[a] (3)	1970 population enumerated (4)	Cohort survivorship ratio (cohort age in 1970) (5)
0-4	90 071	88 477	114 017	-
5-9	76 598	75 242	106 944	-
10-14	63 635	62 509	85 253	0.9636
15-19	54 431	53 468	73 381	0.9753
20-24	45 634	44 826	63 010	1.0080
25-29	37 818	37 149	50 924	0.9524
30-34	32 179	31 609	40 885	0.9121
35-39	28 724	28 216	36 115	0.9122
40-44	23 974	23 550	29 409	0.9304
45-49	20 618	20 253	25 360	0.8988
50-54	15 068	14 801	21 775	0.9246
55-59	11 999	11 787	17 632	0.8706
60-64	10 283	10 101	13 004	0.8786
65-69	6 737	-	10 061	0.8536
70-74	5 342	18 403	6 690	0.6623
75 +	6 756	-	9 873	0.5365
TOTAL	529 767		704 333	

[a] Enumerated population adjusted by a factor of 0.9823 to move it from 11 December 1960 to 10 May 1960.

Step 4: calculation of cohort survivorship ratios. Cohort survivorship ratios or probabilities are calculated by dividing the number in each cohort at the second census by the corresponding number in the same cohort at the first census, using, of course, the date-adjusted numbers in columns (3) and (4) of table 173. Thus, for example, the survivors of the cohort aged 20-24 at the first census are aged 30-34 at the second census, and the 10-year survivorship probability for the cohort, ${}_{10}S_{20,\,24}$, is calculated as

$$ {}_{10}S_{20,\,24} = \frac{{}_5N_{30}^2}{{}_5N_{20}^1} = \frac{40{,}885}{44{,}826} = 0.9121 $$

where ${}_5N_x^1$ and ${}_5N_{x+10}^2$ are the populations aged from x to $x+4$ at the first census and from $x+10$ to $x+14$ at the time of the second census, respectively. Results for all age groups are shown in column (5) of table 173. Note that the female population was classified by five-year age group only up to age 74, with an open-ended age group 75 and over. Since those over 75 in 1970 are the survivors of those over 65 in 1960, the last survivorship ratio is a 10-year survivorship probability for those 65 and over in 1960.

Step 5: fitting of a Coale-Demeny model life table. The cohort survivorship ratios given in column (5) of table 173 show a certain amount of variability; and one ratio even has a value that, in the absence of migration, would be impossible (greater than 1.0). The fitting of a model life table is therefore desirable. The mortality level associated with each estimate (excluding those which are impossible or out of range) can be found in a family of Coale-Demeny model life tables. In the case of Panama, the West family is selected as the most suitable.

It is assumed that the cohort survivorship probabilities, ${}_{10}S_{x,\,x+10}$, are equivalent to stationary-population survivorship ratios, ${}_5L_{x+10}/{}_5L_x$. These stationary survivorship probabilities or ratios are not tabulated in the Coale-Demeny tables, but their values are shown in the second half of annex X (tables 271-278). Table 174 shows the cohort survivorship ratios (taken from column (5) of table 173), the stationary-population survivorship ratios (hereafter also called "model ratios") for a range of mortality levels of the West family of model life tables, and the levels implied by the cohort ratios, obtained by interpolating between the model values. The interpolation is straightforward: if the cohort ratio falls between levels r and $r+1$, the interpolated level z is found as

$$ z(x) = r + \frac{{}_{10}S_{x,\,x+10} - [{}_5L_x^r{}_{+10}/{}_5L_x^r]}{[{}_5L_x^{r+1}{}_{+10}/{}_5L_x^{r+1}] - [{}_5L_x^r{}_{+10}/{}_5L_x^r)} \qquad (8.2) $$

where r and $r+1$ indicate the mortality levels of the ratios. If the interval between the levels to which the model ratios correspond is two levels, as shown in table 174, the term to be added to r has to be multiplied by two.

It will be seen that two of the implied levels shown in table 174 are high (above 20) and two are below 14. Discarding these outlying values, an estimate of overall level can be obtained by averaging the remaining estimates; in this case, the estimate obtained is 16.1. This level is then taken as a best estimate of the level of mortality after age 10 for females in Panama, on the basis of the two census enumerations.

Step 6: completion of the life table. So far, the level of adult mortality has been estimated, but not the level of child mortality. If no information is available about child mortality, the best that can be done is to assume that the adult level also applies in childhood and to adopt the complete life table of the estimated level; in the case of Panama, this level is 16.1.

If some information on child mortality is available, estimates of adult survivorship obtained from a model life table of the estimated level can be linked with the independent estimate of survivorship to age 5. In chapter III, the level of female child mortality in Panama in the 1960s was estimated to be 18.05 (according to the West model), so a life table at this level is adopted up to age 5. Thus, $l(1) = 0.9405$ and $l(5) = 0.9165$. Probabilities of survival from age 5 onward are then calculated for level 16.1 and are used to extend the life table from age 5, as shown in table 175.

For comparison purposes, the life-table $l(x)$ function, which will be generated in subsection B.3 using the logit life-table system, is also included. As can be seen, the two life tables are slightly different in detail but very similar in broad shape and level

(d) *Second detailed example: Colombia, 1951-1964*

The second detailed example considers a less tidy case, where age-misreporting is more extensive and the intercensal period is not a convenient multiple of five.

The computational procedure is described below.

TABLE 174. DETERMINATION OF THE MORTALITY LEVEL IMPLIED BY EACH COHORT SURVIVORSHIP RATIO, PANAMA, 1960-1970

Age x (1)	Cohort survivorship ratio $10^{5}L_{x+5}/10^{5}L_{x}$ (2)	Survivorship ratio according to model life tables (3)	(4)	(5)	(6)	(7)	Implied mortality level (8)
0	0.9636	0.9562	0.9568	0.9723	0.9854	0.9941	16.9
5	0.9753	0.9668	0.9758	0.9836	0.9905	0.9956	14.9
10	1.0080	0.9602	0.9705	0.9796	0.9875	0.9941	[a]
15	0.9524	0.9497	0.9623	0.9734	0.9831	0.9919	14.4
20	0.9121	0.9415	0.9556	0.9683	0.9794	0.9897	<14.0
25	0.9722	0.9341	0.9493	0.9631	0.9754	0.9868	18.5
30	0.9304	0.9260	0.9417	0.9563	0.9697	0.9822	14.6
35	0.8988	0.9160	0.9316	0.9465	0.9604	0.9743	<14.0
40	0.9246	0.8987	0.9147	0.9303	0.9454	0.9613	17.3
45	0.8706	0.8685	0.8866	0.9045	0.9220	0.9414	14.2
50	0.8786	0.8210	0.8422	0.8639	0.8855	0.9101	19.4
55	0.8536	0.7489	0.7741	0.8002	0.8266	0.8580	21.7
60	0.6623	0.6473	0.6750	0.7042	0.7342	0.7319	15.1
65	0.5365[b]	[c]	[c]	[c]	[c]	[c]	[c]

Note: Mean level (excluding two lowest and two highest values) = 16.1.

[a] Survivorship ratio in excess of 1.0

[b] $10^{5}L_{65+}$.

[c] Not computed.

TABLE 175. COMPLETION OF AN INTERCENSAL LIFE TABLE USING THE CHIANG-DEMENY MODEL LIFE TABLES, PANAMA, 1960-1970

Age x (1)	Values of probability of surviving to age x, $l(x)$, West female model life tables: Level 14 (2)	Level 17 (3)	Level 16.1 (4)	Level 16.1 $l(x+5)/l(x)$ (5)	Final estimates of probability of surviving to age x: Chiang-Demeny (6)	Logit system (7)
0	[a]	[a]	[a]	[a]	1.0000	1.0000
1	[a]	[a]	[a]	[a]	0.9405	0.9323
5	0.8804	0.8985	0.8822	0.9870	0.9165	0.9046
10	0.8687	0.8887	0.8707	0.9899	0.9046	0.8954
15	0.8598	0.8811	0.8619	0.9854	0.8955	0.8883
20	0.8470	0.8701	0.8493	0.9808	0.8824	0.8777
25	0.8305	0.8558	0.8330	0.9781	0.8655	0.8638
30	0.8119	0.8394	0.8147	0.9748	0.8463	0.8478
35	0.7912	0.8211	0.7942	0.9710	0.8262	0.8297
40	0.7680	0.8001	0.7712	0.9663	0.8013	0.8088
45	0.7438	0.7760	0.7462	0.9585	0.7743	0.7841
50	0.7107	0.7466	0.7143	0.9434	0.7422	0.7533
55	0.6702	0.7075	0.6739	0.9222	0.7002	0.7121
60	0.6177	0.6560	0.6215	0.8838	0.6457	0.6575
65	0.5454	0.5843	0.5493	0.8287	0.5707	0.5817
70	0.4515	0.4889	0.4552	0.7399	0.4729	0.4813
75	0.3335	0.3663	0.3368	0.6173	0.3499	0.3547
80	0.2055	0.2297	0.2079	-	0.2160	0.2179

[a] Not calculated.

Step 1: adjustment for net intercensal migration and territorial coverage. Once again, no basis exists for adjusting for migration, and no adjustment for territorial coverage is necessary.

Step 2: grouping of data from the two censuses by cohort. Population censuses were held in Colombia on 9 May 1951 and on 15 July 1964, the intercensal interval thus being 13.185 years. Since the population distribution by single year of age is available from the 1964 census, standard five-year age groups from x to $x+4$ identified at the first census can be reidentified at the second census as age groups from $x+13$ to $x+17$. Thus, survivors of those aged 0-4 at the first census are aged 13-17 years at the second census. Suitably grouped data for the female population, based on a single-year age distribution for the population enumerated in 1964, are shown in columns (2) and (4) of table 176.

Step 3: adjustment for length of the intercensal interval. As the intercensal interval was 13.185 years, the population enumerated at the second census can be moved back 0.185 of a year to improve comparability. The total female population in 1964 was 8,869,856, whereas that in 1951 was 5,649,250. Thus, the overall growth rate of the female population was

$$r = \frac{\ln(8{,}869{,}856) - \ln(5{,}649{,}250)}{13.185} = 0.0342.$$

The adjustment factor k for the second census is thus

$$k = \exp[(0.0342)(-0.185)] = 0.9937.$$

Column (5) of table 176 shows the 1964 population systematically multiplied by the factor k (constant with respect to age).

Step 4: calculation of cohort survivorship ratios. Survivorship ratios for each cohort are calculated by dividing the number of survivors at the second census (after adjustment) by the corresponding number at the time of the first census. Thus, in table 176, the numbers in column (5) are divided by the numbers in column (2). The results are shown in column (6). Note that the survivorship ratios for the first three age groups in 1951 exceed 1.0, indicating the existence of problems related to coverage or to age-reporting.

Step 5: fitting of a Coale-Demeny model life table. The cohort survivorship ratios given in column (6) of table 176 are for five-year cohorts over a period of 13 years. Comparable ratios for stationary populations are not published in the Coale-Demeny life tables, nor can they be calculated directly from information which is published. It is possible to estimate them, but the calculations necessary are rather heavy, particularly if the variations in level are substantial. Time can be saved by finding the approximate mortality level for each cohort survivorship ratio and then estimating more accurately the model survivorship ratios (referring to stationary populations) for adjacent mortality levels in order to perform the final interpolation.

The first step is to find the approximate mortality level to which each cohort ratio corresponds. In the stationary populations, a 13-year survivorship ratio should lie almost half-way between the 10-year and 15-year ratios, both of which can be calculated directly from the published tables. To give an example, consider the cohort aged 40-44 in 1951 whose 13-year survivorship ratio is estimated to be 0.7885. By trial and error, one can find the approximate level to which this cohort ratio corresponds in the West family of model life tables. At level 10, the 10-year female survivorship ratio, ${}_5L_{50}/{}_5L_{40}$, has a value of 235,666/273,796 or 0.8607, whereas the 15-year ratio, ${}_5L_{55}/{}_5L_{40}$, has a value of 0.7719. The average of the two, 0.8163, is higher than the cohort survivorship ratio, so the approximate mor-

TABLE 176. FEMALE POPULATION BY AGE GROUP, 1951 AND 1964; ADJUSTED FEMALE POPULATION, 1964; AND COHORT SURVIVORSHIP RATIOS, COLOMBIA

Age group (1)	1951 census female population (2)	Age group (3)	1964 census female population (4)	Adjusted female population[a] (5)	Cohort survivorship ratio (6)
0-4	921 747	13-17	988 879	982 649	1.0661
5-9	768 958	18-22	810 294	805 189	1.0471
10-14	657 753	23-27	676 158	671 898	1.0215
15-19	605 411	28-32	566 038	562 472	0.9291
20-24	550 555	33-37	499 191	496 046	0.9010
25-29	447 242	38-42	419 842	417 197	0.9328
30-34	337 311	43-47	307 196	305 261	0.9050
35-39	334 197	48-52	284 416	282 624	0.8457
40-44	239 771	53-57	190 266	189 067	0.7885
45-49	196 659	58-62	175 311	174 207	0.8858
50-54	175 580	63-67	124 648	123 863	0.7055
55-59	105 721	68-72	91 170	90 596	0.8569
60-64	116 939	73-77	49 108	48 799	0.4173
65-69	63 339	78-82	39 882	39 631	0.6257
70-74	57 175	83-87	16 949	16 842	0.2946
75-79	27 398	88-92	10 213	10 149	0.3704
80-84	24 807	93-97	3 642	3 619	0.1459
85+	18 687	98+	3 199	3 179	0.1701

[a] Adjusted for the difference between the observed intercensal period, 13.185 years, and 13 exact years.

tality level of this ratio should be lower. At level 8, the equivalent 10-year, 15-year and average survivorship ratios are 0.8377, 0.7380 and 0.7879, the last value being very close to the cohort ratio; hence, level 8 is selected as the approximate level for the cohort ratio. Proceeding in this way, approximate levels for all the cohort ratios are estimated. They are shown in column (3) of table 177.

The next step is to make a more accurate estimate of the 13-year model survivorship ratios for the approximate levels determined above. Thus, for the cohort aged 15-19 in 1951, the approximate level is 13, so a model survivorship ratio, ${}_5L_{20}/{}_5L_{15}$, is to be calculated for level 13.

The value of ${}_5L_{15}$ is tabulated in the published tables (for West females, level 13, it has the value of 3.89403). The constants and equation given in table 172 can be used to estimate ${}_5L_{20}$. The age range from 20 to 22 is covered by the tabulated $l(x)$ values for 25, 30 and 35, and n is equal to 20 minus 25, that is, 3. In the West female model life table of level 13, $l(25) = 0.74769$, $l(30) = 0.72326$, and $l(35) = 0.69647$; thus,

$$
\begin{aligned}
{}_5L_{20} &= a(3)\,l(25) + b(3)\,l(30) + c(3)\,l(35) \\
&= (-0.017)(0.74769) + (4.533)(0.72326) + \\
&\quad (0.483)(0.69647) \\
&= 3.60222.
\end{aligned}
$$

The model survivorship ratio is then calculated as

$${}_{13}S^{m}_{15,19} = {}_5L_{20}/{}_5L_{15} = 3.60222/3.89403 = 0.9251.$$

For the next cohort, aged 20-24 in 1951, the approximate mortality level is 11; therefore, the 13-year model survivorship ratio, ${}_5L_{15}/{}_5L_{20}$, is calculated for level 11. The value of ${}_5L_{20}$ is obtained directly from the model life tables as 3.51543; that of ${}_5L_{25}$ is estimated from values of $l(30)$, $l(35)$ and $l(40)$ for level 11 as follows:

$$
\begin{aligned}
{}_5L_{20} &= (-0.017)l(30) + (4.533)l(35) + (0.483)l(40) \\
&= (-0.017)(0.66224) + (4.533)(0.63186) + \\
&\quad (0.483)(0.59963) \\
&= 3.14258.
\end{aligned}
$$

The model survivorship ratio is then calculated as

$${}_{13}S^{m}_{20,24} = 3.14258/3.51543 = 0.8939.$$

Results for each cohort are shown in column (4) of table 177. Note that for the open-ended cohort, a cruder simplification is adopted. The model survivorship ratio required can be approximated as

$${}_{13}S^{m}_{65+} = T_{75}/T_{65},$$

that is, the stationary population over 75 divided by the stationary population over 65. The value for T_{65} is tabulated in the model tables, and T_{75} can be estimated by weighting the values of T_{75} and T_{80} in the following manner:

$$T_{75} = 0.6\,T_{80} + 0.4\,T_{75}.$$

The precision of the estimate for the open-ended interval need not be high, since the survivorship ratio for the open-ended cohort is likely to be distorted anyway by age-misreporting and age-distribution effects.

Once 13-year model survivorship ratios have been estimated for each approximate level given in column (3), the model survivorship ratio for an adjacent level has to be computed in order that the level of the cohort survivorship ratio can be found by interpolation. If the cohort survivorship ratio exceeds that computed for the approximate level, a model ratio should be estimated for the next higher mortality level; whereas if the cohort

TABLE 177 STEPS IN ESTIMATION OF THE MORTALITY LEVEL TO WHICH EACH COHORT SURVIVORSHIP RATIO CORRESPONDS IN THE WEST MODEL, COLOMBIA, 1951-1964

Age group in 1951 (1)	Cohort 13-year survivorship ratio (2)	West model, 13-year survivorship ratio: Approximate level (3)	Estimated ratio (4)	Adjacent level (5)	Estimated ratio (6)	Estimated mortality level (7)
0-4	1.0061	a	a	a	a	a
5-9	1.0471	a	a	a	a	a
10-14	1.0215	a	a	a	a	a
15-19	0.9291	13	0.9251	14	0.9321	13.6
20-24	0.9010	11	0.8939	12	0.9042	11.7
25-29	0.9328	16	0.9318	17	0.9410	16.1
30-34	0.9050	14	0.9014	15	0.9115	14.4
35-39	0.8457	10	0.8406	11	0.8533	10.4
40-44	0.7885	8	0.7805	9	0.7958	8.5
45-49	0.8358	20	0.8376	19	0.8758	19.8
50-54	0.7055	11	0.6998	12	0.7175	11.3
55-59	0.8369	24	0.8473	a	a	>24.0
60-64	0.4173	8	0.3983	9	0.4209	8.8
65+	0.3860	24	0.3245	a	a	>24.0

[a] Cannot be calculated.

ratio is lower than that computed for the approximate level, a model ratio should be estimated for the next lower level. Thus, for age group 15-19 the cohort ratio, ${}_{13}S_{15,19}$, is 0.9291, whereas the model ratio, ${}_{13}S^{m}_{15,19}$, for the approximate level 13 is 0.9251. In order to bracket the cohort ratio between estimated model ratios, it is therefore necessary to compute a model survivorship ratio for the next higher level, 14. (Had the cohort ratio ${}_{13}S_{15,19}$ been lower than 0.9251, a model ratio would instead have been computed for the next lower level, 12.) The steps to follow to perform this computation are the same as those followed in calculating the model ratios corresponding to the approximate levels, except that now the adjacent level is used. Thus, in the case at hand, the life table of level 14 is used instead of that of level 13. The model ratio obtained is 0.9321, and it is listed together with all other model ratios corresponding to the selected adjacent levels in column (6) of table 177. It is worth noting that, in this instance, the cohort survivorship ratios are generally higher than the model ratios corresponding to the approximate levels because the method used to select an approximate level approximates a 12.5-year interval rather than the actual 13-year interval. Fortunately, in every case, the cohort ratio was less than one level away from the first approximation selected.

The last step consists of interpolating between the model ratios for the approximate and adjacent levels in order to find the mortality level of the cohort ratio. As always, the amount to be added to the lower mortality level is equal to the difference between the cohort survivorship ratio and the model ratio corresponding to the lower mortality level, divided by the difference between the model ratios corresponding to the higher and lower mortality levels. Thus, if ν is the lower mortality level, $z(x)$ is the estimated level and $\nu+1$ is the upper level,

$$z(x)=\nu+({}_{5}S_{x,x+t}-{}_{5}S^{\nu}_{x,x+t})/({}_{5}S^{\nu+1}_{x,x+t}-{}_{5}S^{\nu}_{x,x+t}),$$

so that, for the cohort aged 15-19 in 1951,

$$z(15)=13.0+(0.9291-0.9251)/(0.9321-0.9251)$$

$$=13.57.$$

Complete results, rounded to one decimal place, are shown in column (7) of table 177.

Step 6: completion of the life table. The life table may be completed by adding information on child mortality in exactly the same way as for the example for Panama. However, the results obtained in step 5 are so erratic that they require some comment. For the first three cohorts, more survivors were recorded in 1964 than had been enumerated in 1951. It is probable that two factors were mainly responsible for this outcome: the general tendency to underenumerate young children or to exaggerate their ages; and the tendency to shift the ages of women into the peak reproductive years from either side. The first factor would reduce the initial numbers in the cohorts, and the second would increase the apparent numbers of survivors. Two older cohorts also show implausibly high survivorship ratios, the cohort aged 55-59 in 1951, whose survivors include the number heaped on age 70, and the cohort corresponding to the open-ended interval (65 and over), whose survivors are probably inflated by age exaggeration. In order to obtain some overall average estimate of intercensal mortality, these five cases should be excluded; and to balance their exclusion, the five lowest levels should also be disregarded. Such Draconian elimination leaves as the only "acceptable" estimates the set 13.6, 16.1, 14.4 and 19.8, the average of which is 16.0. However, because of the fairly wide range of levels covered by these estimates, their average cannot be considered a reliable indicator of intercensal mortality.

To conclude, note that no independent information on child mortality is available for the period 1951-1964, so that if the average mortality level estimated above were reliable, a life table could only be completed by assuming that the mortality pattern embodied in the model used (West) adequately represents that experienced by the population being studied (in terms of both adult and child mortality).

3. *Intercensal survivorship ratios for five-year age cohorts smoothed by use of the logit system*

(a) *Data required*

The data required for this method are listed below:

(*a*) Two census enumerations separated by t years with populations classified by five-year age group (and sex);

(*b*) An independent estimate of child mortality. Such estimates are generally derived from information on children ever born and children surviving analysed according to the procedures described in chapter III.

(b) *Computational procedure*

The steps of the computational procedure are described below.

Steps 1-4. These steps, by which cohort survivorship ratios analogous to those for a stationary population, ${}_{5}L_{x+t}/{}_{5}L_{x}$, are calculated, are identical to those described in subsection B.2(b) and are not repeated here.

Step 5: smoothing cohort survivorship ratios by use of the logit life-table system. Somewhat more flexibility in the model pattern of mortality used can be introduced by smoothing through the logit life-table system (see chapter I, subsection B.4). The cohort survivorship ratios, analogous to ${}_{5}L_{x+t}/{}_{5}L_{x}$, are transformed into estimates of ${}_{5}L_{x+t}$ by multiplying each by the corresponding ${}_{5}L_{x}$; the first value or values of ${}_{5}L_{x}$ are estimated on the basis of information about child mortality, and subsequent values are obtained from previous estimates of ${}_{5}L_{x+t}$. The calculations thus form a chain: the first value of ${}_{5}L_{x+t}$ is calculated by assuming a value of ${}_{5}L_{0}$; and if t is greater than 5, the second value of ${}_{5}L_{x+t}$ can be obtained by assuming a value of ${}_{5}L_{5}$; but once x is greater than t, the denominators ${}_{5}L_{x}$ will be provided by earlier estimates of ${}_{5}L_{x+t}$.

Once a series of ${}_{5}L_{x+t}$ values has been obtained, it is

assumed that the proportion of the stationary population aged from $x+t$ to $x+t+5$ approximates the probability of surviving from birth to age $y=x+t+2.5$, $l(x+t+2.5)$. The logit transformations of each of these $l(y)$ estimates can then be calculated and compared with the logit transformations of equivalent values derived from an appropriate standard life table. Then, the α and β parameters defining the linear relationship between the logit transformations of the estimated and the standard survivorship probabilities can be estimated by using a suitable line-fitting procedure, and a complete $l(x)$ survivorship function can be generated. It must be noted, however, that for childhood ages the $l(x)$ values generated in this way will not, in general, coincide with those used as input in applying the method. The magnitude of the differences between the input and output child mortality estimates depends, among other things, upon the appropriateness of the mortality pattern used as standard and upon the quality of the intercensal survivorship estimates. If the magnitude of such differences is unacceptably large, the use of a different standard should be considered. When changes in the standard fail to reduce the differences observed, one may either have to discard entirely the intercensal life table or one may adopt a life table built by linking in a manner similar to that described in step 6 in subsection B.2(b), the child mortality estimates used as input with the estimated $l(x)$ values over age 10.

(c) *First detailed example: Panama, 1960-1970*

The computational procedure for this example is described below.

Steps 1-4. These steps have already been presented in subsection B.2(c).

Step 5: smoothing cohort survivorship ratios by use of the logit life-table system. The starting-point of this smoothing procedure is the cohort survivorship ratios shown in column (5) of table 173. It is assumed that a cohort survivorship ratio approximates a life-table survival probability from the central age of the cohort at the first census to its central age at the second census. In the case in hand, therefore, it is assumed that

$$ {}_{10}S_{x,\,x+4} = \frac{{}_5L_{x+10}}{{}_5L_x} = \frac{5.0\,|l(x+12.5)|}{5.0\,|l(x+2.5)|} \qquad (B.3) $$

The one age group for which $5.0|l(x+2.5)|$ is not an adequate approximation to ${}_5L_x$ is group 0-4. If estimates of $l(2)$, $l(3)$ and $l(5)$ are available from child survival data, the mortality levels associated with these three estimates in a selected family of Coale-Demeny model life tables can be averaged; and the value of ${}_5L_0$ for that level, sex and family can be read off from the relevant table. If an estimate of ${}_4L_1$ is required, it should be taken also from this model table.

In the case of Panama, no information relevant to the estimation of child mortality was collected by either the 1960 or the 1970 census. However, child mortality estimates are available from the Retrospective Demographic Survey conducted in 1976, and the reference period for the estimates derived from information pertaining to women aged from 30-34 to 45-49 effectively covers the intercensal period 1960-1970 (see table 55). The average female mortality level for this period is 18.05 in the West family of model life tables. For level 18, ${}_1L_0$ is 0.95377 and ${}_4L_1$ is 3.69742; for level 19, the corresponding values are 0.96004 and 3.75407. Therefore, interpolating linearly,

$$ {}_1L_0^{18.05} = (0.95)(0.95377) + (0.05)(0.96004) = 0.9541 $$

and

$$ {}_4L_1^{18.05} = (0.95)(3.69742) + (0.05)(3.74507) = 3.7003. $$

Interpolating in a similar fashion for ${}_5L_5^{18.05}$,

$$ \begin{aligned} {}_5L_5^{18.05} &= (0.95)({}_5L_5^{18}) + (0.05)({}_5L_5^{19}) \\ &= (0.95)(4.55832) + (0.05)(4.64405) \\ &= 4.5626. \end{aligned} $$

The chaining of survivorship probabilities can then begin. It is assumed that

$$ {}_{10}S_{0,\,5} = \frac{5.0\,|l(12.5)|}{{}_5L_0} $$

so

$$ l(12.5) = \frac{1}{5}({}_{10}S_{0,\,5})({}_5L_0) $$

and

$$ {}_5L_x = 5.0\,l(x+2.5) \quad \text{for } x = 10, 15, \ldots, 60. $$

Table 178 shows the full calculations.

The essence of the logit life-table system lies in the comparison of an estimated $l(x)$ survivorship function with a standard $l_s(x)$ function on the logit scale (see chapter I, subsection B.4). In this case, a Coale-Demeny West model life table of level 18 for females has been selected as standard on the basis of the child mortality estimates available. The $l(x)$ function in the Coale-Demeny model life tables is given only for ages 0, 1, 5, 10 and so on, whereas for comparison with the estimated probabilities, $l(y)$ values are required for ages $y = 12.5$, 17.5, 22.5 and so on. These values can be obtained by averaging the standard $l_s(x)$ values for adjacent ages x (multiples of five) and then calculating the logit transformations of these averages. Thus, to obtain an estimate of $l_s(12.5)$,

$$ l_s(12.5) = 0.5(l_s(10) + l_s(15)) = 0.5(0.90762 + $$

$$ 0.90136) = 0.90449. $$

Therefore, the logit transformation of the standard at 12.5, $\lambda_s(12.5)$, is

$$ \lambda_s(12.5) = 0.5 \ln\left|\frac{1.0 - 0.90449}{0.90449}\right| = -1.1241. $$

TABLE 178. APPLICATION OF SMOOTHING PROCEDURES BASED ON THE LOGIT SYSTEM TO THE COHORT SURVIVORSHIP RATIOS FOR THE PERIOD 1960-1970, PANAMA

Age x (1)	Cohort survivorship ratio $b(_5P_{x,x+4})$ (2)	Stationary population: Five-year age groups $_5L_x$ (3)	Stationary population: Exact age $l(x+2.5)$ (4)	Logit transformation: Estimated $\lambda(x+2.5)$ (5)	Logit transformation: Standard[a] $\lambda_s(x+2.5)$ (6)
0	0.9636	(4.8544)[b]	0.8970	−1.0822	−1.1241
5	0.9753	(4.3626)[b]	0.8900	−1.0454	−1.0807
10	1.0080	4.4850	0.9042	−1.1224	−1.0252
15	0.9524	4.4499	0.8476	−0.8580	−0.9633
20	0.9721	4.3209	0.8247	−0.7743	−0.8988
25	0.9722	4.2381	0.8241	−0.7722	−0.8314
30	0.9304	4.1235	0.7673	−0.5966	−0.7595
35	0.8988	4.1203	0.7407	−0.5248	−0.6795
40	0.9246	3.8365	0.7094	−0.4462	−0.5852
45	0.8706	3.7033	0.6448	−0.2981	−0.4713
50	0.8786	3.5472	0.6233	−0.2518	−0.3302
55	0.8536	3.2241	0.5504	−0.1011	−0.1529
60	0.6623	3.1166	0.4128	0.1762	0.0715
65	(0.5365)	2.7521	c	c	c
70	c	2.0643	c	c	c
75	c	1.4765	c	c	c

[a] Female, level 18, West model.
[b] Obtained as described in the text.
[c] Not computed

Note that computing $l_s(x+2.5)$ by directly averaging $l_s(x)$ and $l_s(x+5)$ has an advantage over the use of closer approximations, since it is being assumed in the case of the estimated $l(x)$ values that $_5L_x$ is equal to $5.0(l(x+2.5))$. If the $l(x)$ and $l_s(x)$ functions are similar, but not linear, between x and $x+5$, the logit transformations of the estimated and the standard $l(y)$ values calculated as shown above will still be comparable, but will not refer to exact age $x+2.5$. Yet, the form of their relationship (its linearity and the parameters that define it) will not be greatly affected.

Columns (5) and (6) of table 178 show the logit transformations of the estimated and standard $l(x)$ functions, respectively. The points defined by $[\lambda_s(x),\lambda(x)]$ are plotted in figure 21. It can be seen that these points follow a generally linear trend, though there are outliers, notably the point associated with the cohort aged 10-14 in 1960, which had an apparent survivorship ratio from 1960 to 1970 greater than 1.0. A straight line has been fitted to the points by using group means (see chapter V, subsection C.4). This line has a slope of 1.022 (an estimate of β in the logit system) and an intercept of 0.094 (an estimate of α in the logit system). The intercensal cohort survivorship probabilities thus indicate an age pattern of mortality similar to that of the standard, since β is roughly equal to one, but an overall level of mortality somewhat heavier than that of the standard, since α is slightly larger than zero.

A complete life table can now be calculated by inverting the logit transformation estimated by means of α and β. Thus,

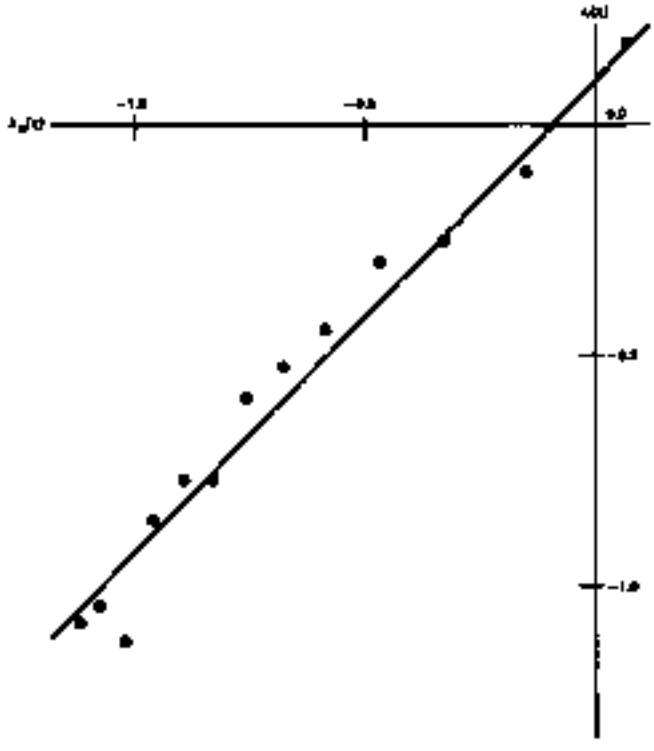
$$\lambda^*(x) = \alpha + \beta\,\lambda_s(x)$$

and

$$l(x) = [1.0 + \exp(2.0\,\lambda^*(x))]^{-1}.$$

The results obtained in this case are shown in column (7) of table 175. Note that, as mentioned earlier, the child mortality estimates obtained by this procedure differ from those used as input (the latter are shown as $l(1)$ and $l(5)$ in column (6) of table 175). In terms of infant mortality, for example, the logit estimate is $l(1) = 0.9323$, while the value used as input is 0.9405

Figure 21. Plot of the logit transformation of the estimated survivorship function, $l(x)$, against that of the standard, West model for females, level 18; Panama



(corresponding to level 18.05 in the Coale-Demeny West models for females). Hence, whereas according to the life table generated by using the logit system, some 68 out of every 1,000 births die before reaching age one, according to the estimates derived from data on children ever born and surviving, about 60 deaths per 1,000 births are expected. If the latter estimate were correct, the one obtained through the logit fit overestimates infant mortality by about 13 per cent. This outcome is due to the fact that intercensal adult mortality is substantially higher than the child mortality used as input in terms of the West mortality pattern. In the case of Panama during the period 1960-1970, the fairly low child mortality estimates derived from reports of older women are likely to be biased downward. Therefore, the estimates yielded by the logit fit are probably acceptable.

It should also be pointed out that the chaining of survivorship ratios used in this procedure introduces a substantial element of smoothing into the results even before the smoothing action of the logit system is introduced. Each link in the chain depends upon one or more of the earlier links, and each $l(x+2.5)$ estimate is determined both by an earlier estimate of $l(x-7.5)$ and by the intercensal cohort survivorship probability ${}_{10}S_{x-10,x-5}$. Thus, the final estimates yielded by this procedure are likely to be smoother than those obtained directly from each survivorship probability, as was done in subsection B.2(c).

(d) *Second detailed example: Colombia, 1951-1964*

The computational procedure for this example is described below.

Steps 1-4. These steps have already been covered in subsection B.2(d). Therefore, this example begins with the application of this procedure once the 13-year cohort survivorship ratios shown in table 176 are available.

Step 5: smoothing the cohort survivorship ratios by use of the logit life-table system. To use the smoothing procedure based on the logit system, it is necessary to have some estimate of child mortality in order to begin the chaining of survivorship probabilities. A recent study by Somoza,[3] based on the results of the Colombian National Fertility Survey (part of the World Fertility Survey), found that mortality among female children born during the period 1941-1959 could be approximated by level 14.5 of the West model life tables; for female children born during the period 1960-1967, the same procedure yielded a level of 16.2. Therefore, it is estimated that, for the intercensal period 1957-1964, female child mortality was, on average, equal to that of level 15.35 (the arithmetic average of the two levels estimated by Somoza). For this level, ${}_5L_0$ is equal to 4.4699, ${}_5L_5$ to 4.3076 and ${}_5L_{10}$ to 4.2508, these values being obtained by interpolating linearly between the tabulated values for levels 15 and 16, respectively.

The first cohort survivorship ratio, ${}_{13}S_{0,4}$, is regarded as being equivalent to the ratio ${}_5L_{13}/{}_5L_0$. Therefore, multiplying by the assumed value of ${}_5L_0$, one obtains an estimate of ${}_5L_{13}$:

$${}_5L_{13} = ({}_{13}S_{0,4})({}_5L_0) = (1.0661)(4.4699) = 4.7654.$$

Similar calculations for ${}_5L_{18}$ and ${}_5L_{23}$ give results of 4.5105 and 4.3422, using the assumed values of ${}_5L_5$ and ${}_5L_{10}$, respectively. The next survivorship ratio, ${}_{13}S_{15,19}$, is regarded as equivalent to ${}_5L_{28}/{}_5L_{15}$, but no value has been assumed for ${}_5L_{15}$, which therefore has to be estimated. Estimates of ${}_5L_{13}$ and ${}_5L_{18}$ have already been obtained, and ${}_5L_{15}$ can be estimated from them by weighting them suitably. The age interval from 13 to 17 shares three years, or 60 per cent, with age group 15-19; and the interval from 18 to 22 shares two years, or 40 per cent, with age group 15-19. An estimate of ${}_5L_{15}$ can therefore be obtained by summing 60 per cent of ${}_5L_{13}$ and 40 per cent of ${}_5L_{18}$:

$${}_5L_{15} = 0.6\,{}_5L_{13} + 0.4\,{}_5L_{18} = (0.6)(4.7654) +$$

$$(0.4)(4.5105) = 4.6634.$$

An estimate of ${}_5L_{20}$ can be obtained in a similar way from the estimated values of ${}_5L_{18}$ and ${}_5L_{23}$:

$${}_5L_{20} = 0.6\,{}_5L_{18} + 0.4\,{}_5L_{23} = (0.6)(4.5105) +$$

$$(0.4)(4.3422) = 4.4432.$$

These values of ${}_5L_{15}$ and ${}_5L_{20}$ can now be used to estimate ${}_5L_{28}$ and ${}_5L_{33}$ from the cohort survivorship ratios ${}_{13}S_{15,19}$ and ${}_{13}S_{20,24}$, whereupon the values of ${}_5L_{23}$, ${}_5L_{28}$ and ${}_5L_{33}$ can be used to estimate ${}_5L_{25}$ and ${}_5L_{30}$, which can in turn be used to estimate ${}_5L_{35}$ and ${}_5L_{40}$ from the cohort survivorship ratios ${}_{13}S_{25,29}$ and ${}_{13}S_{30,34}$, and so on until all but the last of the cohort ratios have been used (the last ratio, for the open-ended cohort aged 65 and over in 1951, cannot be equated with an ${}_nL_x$ value and therefore cannot be used). The results of the various sets of calculations are shown in table 179.

It is assumed that the probability of surviving from birth to the mid-point of each age group can be approximated by one fifth of the ${}_5L_x$ values. That is,

$$l(x+2.5) = (0.2)_5L_x.$$

Column (5) of table 179 shows estimates of ${}_5L_x$ for values of x of 13, 18, 23, 28 and so on up to 73. Each can be divided by five to estimate values of $l(x+2.5)$, or survival probabilities from birth to ages 15.5, 20.5, 25.5 and so forth up to 75.5. Thus, for example, $l(20.5)$, is estimated as

$$l(20.5) = \frac{1}{5}\,{}_5L_{18} = \frac{1}{5}(4.5105) = 0.9021.$$

Full results are shown in column (6) of table 179.

The final stage of the smoothing process is the comparison of the logit transformations of the estimated sur-

[3] Jorge L. Somoza, *Illustrative Analysis: Infant and Child Mortality in Colombia*, World Fertility Survey Scientific Report No. 10 (Voorburg, The Hague, International Statistical Institute, 1980).

TABLE 179. SMOOTHING OF FEMALE COHORT SURVIVORSHIP RATIOS BY USE OF THE LOGIT LIFE-TABLE SYSTEM: COLOMBIA, 1951-1964

Age group of cohort in 1951 (1)	Age x (2)	Cohort survivorship ratio $_{15}S_{x,x+4}$ (3)	Stationary population: Standard five-year age group $_5L_x$ (4)	Stationary population: Non-standard five-year age group $_5L_{x+15}$ (5)	Stationary population: All-ages ratio $A(x+15.5)$ (6)	Logit transformation: Estimated $\lambda(x+15.5)$ (7)	Logit transformation: Standard $\lambda_s(x+15.5)^a$ (8)
0-4	0	1.0661	(4.4699)[b]	4.7654	0.9531	−1.5059	−0.8142
5-9	5	1.0471	(4.3076)[b]	4.5105	0.9021	−1.1104	−0.7615
10-14	10	1.0215	(4.2508)[b]	4.3422	0.8684	−0.9434	−0.6997
15-19	15	0.9291	4.6634	4.3328	0.8666	−0.9356	−0.6358
20-24	20	0.9010	4.4432	4.0033	0.8007	−0.6953	−0.5702
25-29	25	0.9328	4.3384	4.0469	0.8094	−0.7231	−0.5207
30-34	30	0.9050	4.2010	3.8019	0.7604	−0.5774	−0.4323
35-39	35	0.8657	4.0207	3.4003	0.6801	−0.3771	−0.3549
40-44	40	0.7885	3.9489	3.1137	0.6227	−0.2505	−0.2806
45-49	45	0.8858	3.6413	3.2255	0.6451	−0.2988	−0.1459
50-54	50	0.7055	3.2857	2.3181	0.4636	0.0729	0.0035
55-59	55	0.8369	3.1588	2.7064	0.5413	−0.0828	0.1936
60-64	60	0.4173	2.8625	1.1945	0.2389	0.5794	0.4457
65+	65	0.3860	2.4734	.	.	.	.

[a] Females, West model of level 15

[b] Obtained independently from child mortality estimates for the period.

vivorship probabilities (equivalent to the stationary population at exact ages) with those of an adequate standard. The logit transformations of the $l(x)$ values shown in column (6) of table 179 are easily calculated; for instance, for $l(20.5)$,

$$\lambda(20.5) = 0.5 \ln[(1.0 - l(20.5))/l(20.5)] =$$

$$0.5 \ln(0.10852) = -1.1104.$$

The full set of $\lambda(x)$ values is shown in column (7) of table 179. The selection of a suitable standard and the calculation of $l_s(x)$ values that match the estimated values present a problem, however, since the life tables published in the Coale-Demeny set do not include the values of x required in this case. Estimates of $l_s(x)$ must be obtained for $x = 15.5$, 20.5 and so on, so that they may be compared with the estimated $l(x)$ function on the logit scale. In this application, a West model life table of level 15 for females has been adopted as standard. The required estimates of $l_s(x)$ are obtained by interpolating linearly between the published $l_s(y)$ values. Thus, for $x = 15.5$,

$$l_s(15.5) = 0.9\, l_s(15) + 0.1\, l_s(20)$$

$$= (0.9)(0.83740) + (0.1)(0.82284)$$

$$= 0.83394.$$

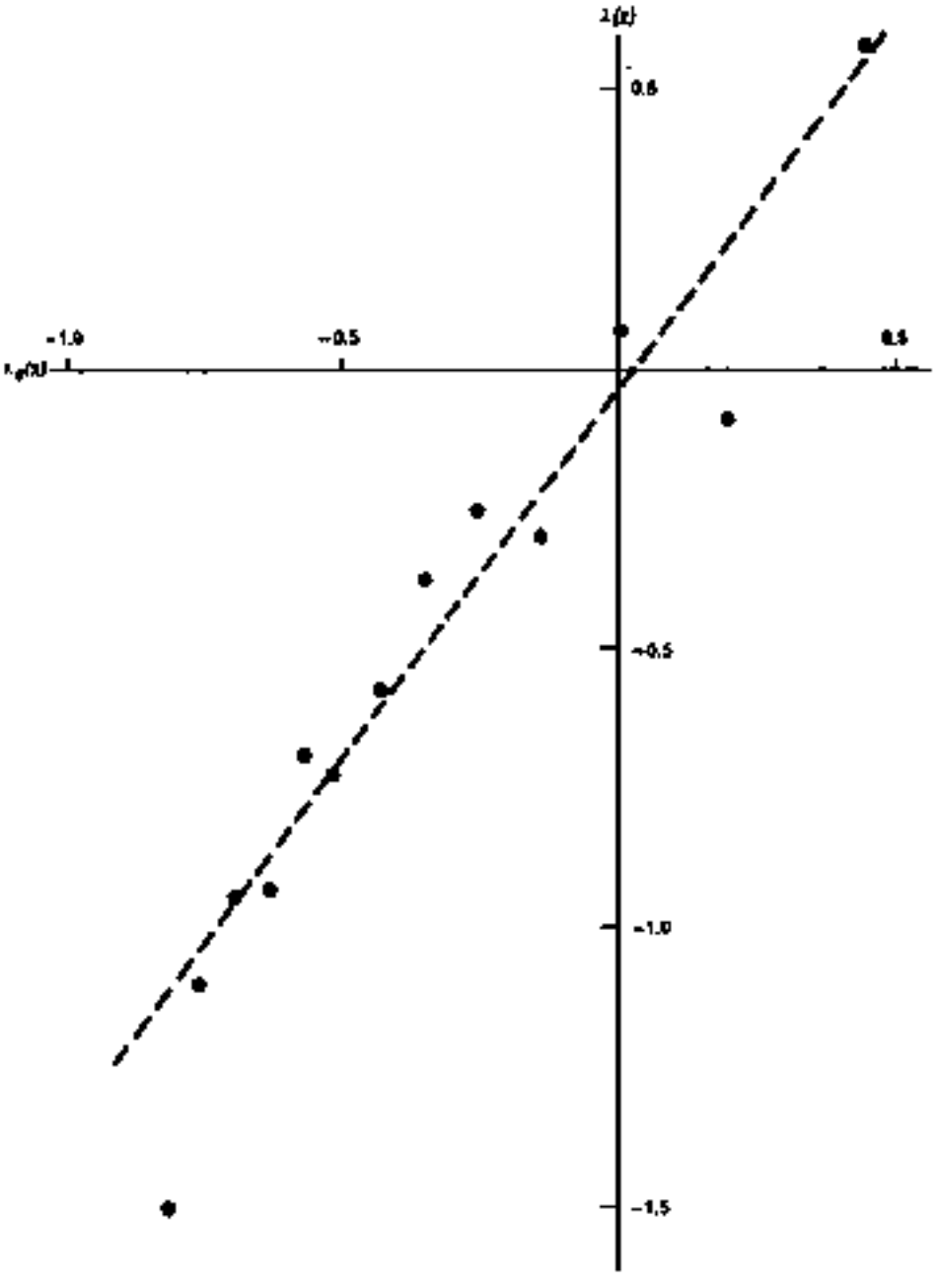
The logit transformations of these $l_s(x)$ values are calculated next, the results being shown in column (8) of table 179. The logit transformations of the estimated and standard $l(x)$ functions are plotted one against the other in figure 22. The resulting points fluctuate substantially, that for the cohort aged 0-4 in 1951 being particularly divergent and the remainder following a broadly linear trend. The straight line shown in the figure has been fitted by group means (see chapter V, subsection C.4) calculated on the basis of all the points except the first; it has an intercept (α) of -0.03 and a slope (β) of 1.32, indicating that the level of mortality in the population in question is similar to that of the standard, but that observed mortality increases much more rapidly with age than it does in the standard. However, little confidence can be placed in the final results, given that the slope of the line is heavily affected by points referring to young women for whom age-reporting and coverage errors are substantial. The chaining process used by this smoothing procedure reduces the independence of the estimates derived from different cohorts, thus increasing the possibility of serious biases in the slope of the fitted line. Because of these problems, the calculation of a final life table is omitted. If desired, it would be calculated just as explained in step 5 of subsection B.3(c).

4. *Intercensal mortality estimated by using projection and cumulation*

(a) *General characteristics of method*

Some of the effects of age-misreporting on intercensal mortality estimates can be eliminated by the use of cumulation. Instead of calculating survivorship ratios for cohorts, the initial population is projected forward to the date of the second census using a range of mortality levels. For each mortality level, the projected population over ages x from 10 or 15 to 50 or 55 is obtained by cumulation, and the observed population at the second census, $N2(x^+)$, is used to interpolate within the projected values in order to determine the mortality level consistent with it. In this way, the effects of age-misreporting at the second census are limited to the effects of transfers across each age boundary x. Unfortunately, the effects of age-misreporting in the initial age distribution are not reduced.

Figure 22. Plot of the logit transformation of the estimated survivorship function, $Y(x)$, against that of the standard, West model for females, level 15: Colombia



It should be noted in passing that there is no theoretical reason to prefer the forward projection of the first age distribution to make it comparable to the second age distribution over the reverse projection of the second age distribution to make it comparable to the first. The two procedures may, however, be expected to give somewhat different results because of different errors in the two distributions. The analyst might wish to carry out the calculations both ways, compare them and possibly obtain a final estimate by averaging the two results; but this discussion is confined to the description of the usual forward-projection procedure. Backward projection is carried out in an analogous way.

(b) *Data required*

The only data required are two census enumerations separated by t years with populations classified by age and sex.

(c) *Computational procedure*

The steps of the computational procedure are described below.

Steps 1-3. These steps, by which two comparable age distributions for time-points separated by an exact number of years are obtained, are identical to those described in subsection B.2(b) and are not repeated here.

Step 4: cumulation of the second age distribution. The age distribution from the first census is not cumulated, but the second age distribution is cumulated by summation from the uppermost age group downward. The ages above which populations should be cumulated will depend upon the intercensal interval t; normally, the oldest initial population that should be used is the population aged 45 and over at the first census, and this population will be aged $45+t$ and over at the second census; the next age group will be aged $40+t$ and over, the next $35+t$ and over, and so on.

Step 5: projection of initial population with different mortality levels. A suitable family of Coale-Demeny model life tables is selected, and life tables from different levels are used to project the initial population, five-year age group by five-year age group. If the intercensal interval is t years, and the initial population aged from x to $x+4$ is ${}_5N1_x$, the population projected by using level ν of the life tables, ${}_5NP^{\nu}_{x+t}$, is given by:

$$ {}_5NP^{\nu}_{x+t} = ({}_5N1_x)({}_5L^{\nu}_{x+t}/{}_5L^{\nu}_{x}) $$

where ${}_5L^{\nu}_{x}$ and ${}_5L^{\nu}_{x+t}$ come from a model life table of level ν. If t is not a multiple of five, values of ${}_5L^{\nu}_{x+t}$ can be estimated by using the technique described in step 5 of subsection B.2(b). Each age group has to be projected using several levels ν, and the populations over each age $x+t$ are then obtained by cumulation. The observed population aged $x+t$ and over at the time of the second census, $N2((x+t)+)$, is then used to determine the level consistent with it by interpolating linearly between the projected estimates $NP^{\nu}((x+t)+)$. Once mortality levels have been determined in this way for each initial age $x = 5, 10, 15, \ldots, 45$, the median of these levels can be used as an estimate of adult mortality during the intercensal period.

(d) *A detailed example: Panama, 1960-1970*

The steps of the computational procedure are described below.

Steps 1-3. These steps have already been performed in subsection B.3(c) and need not be repeated here. Therefore, the starting-point of this example is the 1960 and 1970 female age distributions for Panama given in columns (3) and (4), respectively, of table 173.

Step 4: cumulation of the second age distribution. The Coale-Demeny model life tables tabulate the stationary-population function, ${}_5L_x$, only up to age 80, the final category being the stationary population aged 80 and over. For a 10-year survival period, therefore, the highest age group for which a model survivorship ratio can be calculated is the initial open-ended age group 70 and over. The initial age distribution therefore needs to be tabulated by five-year age group up to age group 65-69, with the last age group being 70 and over. For the final age distribution, however, less detail is required, since the highest initial age group to be used is that aged 45 and over. Thus, the final population in 1970 is required in age groups 55+ (survivors of the initial population aged 45 and over), 50+, 45+ and so on down to 15+ (survivors of the initial population aged 5 and over). The easiest way to cumulate the age distribution is to begin with the number observed in the oldest age group and add in successively the number corresponding to the age group immediately below it. Thus, given the 1970 age distribution in column (4) of table 173, the population over age x, $N2(x+)$, is calculated as

$$ N2(x+) = N2((x+5)+) + {}_5N2_x $$

where ${}_5N2_x$ is the population aged from x to $x+4$ in 1970. For example, for $x = 75$,

$$ N2(75+) = 9,873; $$

and for $x = 70$,

$$ N2(70+) = N2(75+) + {}_5N2_{70} = 9,873 + 6,690 = 16,563. $$

Then for $x = 65$,

$$ N2(65+) = N2(70+) + {}_5N2_{65} = 16,563 + $$

$$ 10,061 = 26,624. $$

Full results are shown in column (15) of table 180.

Step 5: projection of initial population with different mortality levels. For a 10-year intercensal interval, the probability of surviving from the age group from x to $x+4$ to the age group from $x+10$ to $x+14$ is approximated by ${}_5L_{x+10}/{}_5L_x$, and such model survivorship ratios can be calculated for each level of any family of Coale-Demeny model life tables. The last survivorship ratio, that for the population aged 70 and over to 80 and over 10 years later, is approximated as T_{80}/T_{70}, that is, $T_{80}/({}_5L_{70}+{}_5L_{75}+T_{80})$. In a growing population with an age distribution 70 and over that is younger than the equivalent stationary population, this model survivorship ratio is likely to be somewhat lower than the true value, but an adjustment would not generally be worthwhile. Table 174 shows 10-year model survivorship probabilities for initial age groups up to 60-64 and for a range of mortality levels in the West family of model life tables; in order to project the entire population over any age, 10-year model survivorship probabilities for the initial age groups 65-69 and 70+ need to be added. For level 16, for example,

$${}_{10}S^{16}_{65,60} = {}_5L^{16}_{75}/{}_5L^{16}_{65} = 134,741/249,225 = 0.5406$$

and

$$ {}_{10}S^{16}_{70+} = T^{16}_{80}/T^{16}_{70} = T^{16}_{80}/({}_5L^{16}_{70} + {}_5L^{16}_{75} + T^{16}_{80}) $$

$$ = 102,911/(196,255 + 134,741 + 102,911) $$

$$ = 0.2372. $$

In the application of forward projection with cumulation, there is no alternative to carrying out the calcula-

TABLE 130. PROJECTION OF THE INITIAL FEMALE POPULATION OVER 10 YEARS USING DIFFERENT MORTALITY LEVELS, PANAMA, 1960-1970

Age x (1)	Female population aged x to $x+4$ in 1960 (2)	Projection at West level 16: Stationary-population age group ratio $5L_{x+10}/5L_x$ (3)	Projection at West level 16: Projected 1970 population aged From $x+10$ to $x+14$ (4)	Projection at West level 16: Projected 1970 population aged $x+10$ and over (5)	Projection at West level 14: Stationary-population age group ratio $5L_{x+10}/5L_x$ (6)	Projection at West level 14: Projected 1970 population aged From $x+10$ to $x+14$ (7)	Projection at West level 14: Projected 1970 population aged $x+10$ and over (8)	Projection at West level 20: Stationary-population age group ratio $5L_{x+10}/5L_x$ (9)	Projection at West level 20: Projected 1970 population aged From $x+10$ to $x+14$ (10)	Projection at West level 20: Projected 1970 population aged $x+10$ and over (11)	Projection at West level 22: Stationary-population age group ratio $5L_{x+10}/5L_x$ (12)	Projection at West level 22: Projected 1970 population aged From $x+10$ to $x+14$ (13)	Projection at West level 22: Projected 1970 population aged $x+10$ and over (14)	Actual 1970 population aged $x+10$ and over (15)	Intercensal mortality level (16)
5	75 242	0.9758	73 421	393 970	0.9836	74 008	339 759	0.9905	74 527	405 151	0.9956	74 911	410 558	398 119	17.45
10	62 509	0.9705	60 665	320 549	0.9796	61 234	323 721	0.9875	61 728	330 624	0.9941	62 140	335 647	324 738	17.62
15	53 468	0.9623	51 452	259 884	0.9734	52 046	264 518	0.9831	52 564	268 896	0.9919	53 035	273 506	261 728	16.80
20	44 826	0.9536	42 736	208 432	0.9683	43 405	212 472	0.9794	43 902	216 332	0.9897	44 364	220 471	210 804	17.81
25	37 149	0.9493	35 266	165 696	0.9631	35 778	169 067	0.9754	36 235	172 429	0.9868	36 659	176 107	169 919	18.51
30	31 609	0.9417	29 766	130 331	0.9563	30 228	133 289	0.9697	30 653	136 194	0.9822	31 046	139 449	133 804	18.35
35	28 216	0.9316	26 286	100 565	0.9465	26 706	103 061	0.9604	27 099	105 543	0.9743	27 491	108 402	104 395	19.07
40	23 550	0.9147	21 541	74 278	0.9303	21 913	76 354	0.9454	22 264	78 444	0.9613	22 639	80 911	79 035	20.48
45	20 253	0.8866	17 956	52 737	0.9045	18 319	54 441	0.9220	18 673	56 180	0.9414	19 066	58 273	57 260	21.03
50	14 801	0.8422	12 465	34 781	0.8639	12 787	36 122	0.8855	13 106	37 507	0.9101	13 470	39 207	39 628	>22
55	11 787	0.7741	9 124	22 316	0.8002	9 432	23 336	0.8266	9 743	24 400	0.8580	10 113	25 736	26 624	>22
60	10 101	0.6750	6 818	13 191	0.7042	7 113	13 904	0.7342	7 416	14 657	0.7719	7 797	15 621	16 563	>22
65	6 618	0.5406	3 578	6 373	0.5704	3 775	6 791	0.6016	3 981	7 241	0.6426	4 253	7 826	.	.
70	11 785	0.2372	2 795	2 795	0.2559	3 016	3 016	0.2766	3 260	3 260	0.3032	3 573	3 573	.	.

tions for a range of mortality levels. For each mortality level v, the projected population of each age group from x to $x+5$ in 1960, ${}_5NP^v_{x+10}$, is obtained by applying the survivorship ratio, ${}_{10}S^v_{x,x+5}$, to the initial population, ${}_5N1_x$. Thus, for age group 15-19 and mortality level 16, the initial population is 53,468, and the survivorship probability is 0.9623, so

$${}_5NP^{16}_{25} = ({}_5N1_{15})({}_{10}S^{16}_{15,19}) = (53{,}468)(0.9623) = 51{,}452,$$

whereas for the same age group but at mortality level 22,

$${}_5NP^{22}_{25} = (53{,}468)(0.9919) = 53{,}035.$$

The 1960 population of each five-year age group, from 5-9 upward, has been projected forward using survivorship probabilities from each mortality level (note that in practice it is sufficient to work with steps of two levels, that is, to use levels 16, 18, 20 and 22, and not to repeat the calculations for the intermediate levels), the results being shown in columns (4), (7), (10) and (13) of table 180. Each projected population can then be cumulated from the oldest age group towards the youngest to find the population over ages 55, 50, 45 and thus down to the population over 15. The results for the mortality levels being used are shown in columns (5), (8), (11) and (14) of table 180. The reported populations over each of these ages can then be compared with the projected populations, and linear interpolation can be used to estimate the mortality level implied by each. Thus, for instance, the reported population over age 20 in 1970 is 324,738; the projected population over age 20 for mortality level 16 (column (5) of table 180) is 320,549, whereas for mortality level 18 it is 325,721 (column (8) of table 180). Therefore, the mortality level implied by the observed population is obtained as

$$v(20+) = 16.0 + (2.0)(324{,}738 - 320{,}549)/$$

$$(325{,}721 - 320{,}549) = 17.62$$

where 16 is the level associated with the smaller projected population; and the interpolation factor has to be multiplied by two because the levels used, 16 and 18, are two units apart. Column (15) of table 180 shows the cumulated populations observed in 1970, and column (16) shows the mortality levels yielding projected populations over each age consistent with the observed.

The mortality levels shown in column (16) are certainly less variable than those obtained for individual age groups in table 174. However, after four rather consistent estimates of levels in the range from 16.8 to 17.8 associated with the populations over 15, 20, 25 and 30 in 1970, the estimates show a steady tendency to rise as the lower age boundary increases. One possible cause of this outcome is that the West mortality pattern is not a good representation of adult mortality in Panama, but it seems more likely that systematic age-reporting errors may be distorting the second age distribution (and probably the first as well, though the method provides information only about relative differences). In the circumstances, the best estimate of mortality level that one can obtain from these data is the average of the first four values, 17.4, rather than the median of all the values, which is more likely to be affected by the apparent tendency towards age exaggeration. Note that this value of 17.4 is substantially higher than the final estimate based on individual age groups, 16.1. The level based on cumulated data is probably the better of the two estimates, although it should be remembered that the cumulation procedure is really only applied to the second age distribution, not to the first, so the results are still dependent upon the age detail of the initial age distribution. The greater consistency of the results obtained from cumulated data should not be interpreted as necessarily indicating greater accuracy.

C. Intercensal survival with additional information on the age pattern of mortality

1. *Basis of method and its rationale*

The two most serious problems affecting intercensal-survival techniques are age-misreporting and different levels of census coverage. A procedure often used to reduce the effects of age-misreporting is cumulation; the population over age x is affected only by erroneous transfers of people across the boundary x and not by errors over or under x. The second problem, that of coverage changes between the first and second censuses, can play havoc with mortality estimates derived from intercensal survival, since the coverage change will appear either as excess deaths (when the second census is the less complete) or as a deficiency of deaths (when the second census is the more complete). However, a change in coverage that is more or less constant by age will inflate or deflate intercensal deaths by amounts proportional to the population at each age, rather than by amounts proportional to the number of deaths at each age. A change in coverage will therefore have much more effect on deaths at younger ages, where there are in reality few deaths but large numbers of people, than at older ages, where there are many more deaths but small numbers of people.

A technique that is simple to understand but rather laborious to apply makes it possible to use what is essentially an intercensal-survival procedure while employing cumulated data and also making allowance for differential census coverage.[3] The method is based on the simple idea that, in a closed population, the number of people in a particular age group at a first census should be equal to the number of survivors of the same cohort at the second census plus the deaths of cohort members during the intercensal period. It can be simply shown that if the coverages of the first and second censuses and of intercensal deaths are invariant with age and are denoted by C_1, C_2 and k, respectively, then

$${}_nN1_x / {}_nN2_{x+t} = C_1/C_2 + (C_1/k)\,{}_nD_x / {}_nN2_{x+t} \qquad \text{(C.1)}$$

[3] S. Preston and K. Hill, "Estimating the completeness of death registration", *Population Studies*, vol. XXXIV, No. 2 (July 1980), pp. 349-366.

where n is the cohort width in years; t is the length of the intercensal period; ${}_nN1_x$ and ${}_nN2_{x+t}$ are the enumerated cohort populations at the first and second censuses, respectively; and ${}_nD_x$ is the registered number of intercensal deaths to the cohort aged from x to $x+n$ at the first census. Equation (C.1) defines a straight line with slope C_1/k and intercept C_2/C_1; that is, its slope is the completeness of the first census in relation to the completeness of death registration and the intercept is the completeness of the first census in relation to that of the second. The fitting of a straight line to points $[{}_nN1_x/{}_nN2_{x+t}, {}_nD_x/{}_nN2_{x+t}]$ for different cohorts should therefore provide estimates of the relative completeness of the two censuses and of the completeness of death registration in relation to that of the first census.

Equation (C.1) is valid for any cohort, be it an initial five-year age group, the initial population over some age x, or even the population between two ages, x and y, at the first census. All that is required is that the range of the ratios ${}_nN1_x/{}_nN2_{x+t}$ and ${}_nD_x/{}_nN2_{x+t}$ be wide enough for the robust estimation of the parameters (mainly the slope) of the straight line to be possible. Cumulation can therefore be used to reduce the effects of some age errors, though the procedure remains sensitive to systematic age exaggeration on the part of the elderly.

2. *Data required*

The data listed below are required for this method:

(*a*) Two census enumerations with populations classified by age (and sex) for two points in time not more that 15 years apart. (It may be necessary that the age classification be by single year for at least one census if the intercensal interval is not a multiple of five years);

(*b*) Information on deaths by age (and sex) for the intercensal period; registered deaths for each intercensal year can be used but the calculations are lengthy, and deaths for every fifth year are adequate. If no information on deaths is available, a model life table can be used to supply the deficiency.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: adjustment for net intercensal migration and territorial coverage. See step 1 in subsection B.2(b). Note, however, that before applying this method, intercensal deaths should also be adjusted for migration and coverage changes, though if the age pattern of deaths is not much affected, the adjustment is not crucial to the final mortality estimates.

Step 2: grouping of data from the two censuses by cohort. See step 2 in subsection B.2(b).

Step 3: adjustment of intercensal interval that is not an exact number of years. See step 3 in subsection B.2(b).

Step 4(a): cumulation of cohort deaths from registration data. Registered deaths are normally tabulated by calendar year, five-year age group and sex. Given that the two censuses being used probably do not have reference dates at the beginning of a year and that a cohort will be continually moving across standard five-year age groups as it moves through the intercensal period, the task of cumulating intercensal cohort deaths is tedious and imprecise. Since the value of the information on intercensal deaths lies in their age pattern, not in their precise overall level, a degree of simplification is in order.

If the first census was held in year a and initial cohorts are defined by standard five-year age group, deaths to the cohort aged from x to $x+4$ in year a over the first five years of the intercensal period (that is, between a and $a+5$) can be approximated by summing the deaths in year a to persons aged from x to $x+4$ and the deaths in year $a+5$ to persons aged from $x+5$ to $x+9$, and multiplying the sum by 2.5. Thus, if $D^j_{x,x+4}$ denotes, in general, the number of deaths to the cohort aged from x to $x+4$ at the beginning of the period (that is, in year a) and recorded during year j, and ${}_5D^j_x$ denotes the number of deaths to persons aged from x to $x+4$ in year j, then

$$\sum_{j=a}^{a+5} D^j_{x,x+4} = 2.5\,[{}_5D^a_x + {}_5D^{a+5}_{x+5}]. \qquad \text{(C.2)}$$

Similar approximations can be applied for a second five-year period between $a+5$ and $a+10$:

$$\sum_{j=a+5}^{a+10} D^j_{x,x+4} = 2.5\,[{}_5D^{a+5}_{x+5} + {}_5D^{a+10}_{x+10}]. \qquad \text{(C.3)}$$

Cohort deaths for intercensal periods that are multiples of five can therefore be approximated rather simply from registered deaths for calendar years five years apart.

The case of an intercensal interval that is not a multiple of five years is slightly more complicated, but adequate approximations can be arrived at by suitable weighting of registered deaths. If the interval is between five and 10 years, cohort deaths for the first five years can be approximated as described above using the deaths registered in years a and $a+5$. Cohort deaths over the period from $a+5$ to $a+t$, where t is the length of the intercensal interval, are approximated by averaging the number of deaths recorded in years $a+5$ and $a+t$ belonging to the appropriate age groups and then weighting the averages according to the number of years between $a+5$ and $a+t$. Thus, letting

$${}_5\bar{D}_{x+5} = 0.5\,[{}_5D^a_{x+5} + {}_5D^{a+t}_{x+5}], \qquad \text{(C.4)}$$

$$\sum_{j=a+5}^{a+t} D^j_{x,x+4} = w(t-5)\,{}_5\bar{D}_{x+5} + (t-5-w(t-5))\,{}_5\bar{D}_{x+10}, \qquad \text{(C.5)}$$

Values of $w(t-5)$ are shown in table 181.

If the intercensal interval is between 10 and 15 years, cohort deaths for the first 10 years can be obtained from equations (C.2) and (C.3). Cohort deaths for the extra period can then be obtained by using equations (C.4)

and (C.5) with $a+5$ substituted by $a+10$, $t-5$ by $t-10$; and $x+5$ and $x+10$ replaced by $x+10$ and $x+15$, respectively. The necessary weights can still be calculated from table 181 using as point of entry $(t-10)$ years.

TABLE 181. WEIGHTING FACTORS FOR APPROXIMATION OF COHORT DEATHS FOR INTERVALS THAT ARE NOT MULTIPLES OF FIVE

Interval (years) $t-1$ (1)	Weighting factor $w(t-5)$ (2)	Weighting factor $w(t-5)-w(t-10)$ (3)
1	0.9	0.1
2	1.6	0.4
3	2.1	0.9
4	2.4	1.6

Step 4(b): approximation of cohort deaths from a model life table. If no information is available about the age distribution of deaths over the intercensal period, model life tables can be used to fill the gap. The exact level of mortality is not critical, since the method will estimate the "completeness" of death registration; but because the age pattern of mortality is important, a model that adequately represents the pattern of mortality experienced by the population being studied and that roughly approximates its level should be chosen.

What is required as input is an estimate of the number of deaths occurring to an initial cohort during the intercensal period. The life-table ${}_5L_x$ function can conveniently be used to synthesize the required number. If the intercensal period is five years in length, the number of deaths occurring to the cohort aged from x to $x+4$ at the time of the first census is given by ${}_5L_x - {}_5L_{x+5}$; the proportion of the initial cohort dying during the period is given by $({}_5L_x - {}_5L_{x+5})/{}_5L_x$. In general, for any intercensal interval t, the number of deaths occurring during the intercensal interval to the cohort aged from x to $x+4$ at the first census can be estimated as

$$\sum_{j=a}^{a+t} D^j_{x,x+4} = ({}_5N1_x)({}_5L_x - {}_5L_{x+t})/{}_5L_x \qquad (C.6)$$

where ${}_5N1_x$ is the population aged from x to $x+4$ at the time of the first census (year a). If t is not a multiple of five, ${}_5L_{x+t}$ values cannot be read directly from the Coale-Demeny model life tables, though they can be estimated from the tabulated values of $l(x)$ for adjacent ages that are multiples of five, as described in step 5 of subsection B.2(b), using the coefficients and equation given in table 172.

Step 5(a): calculation and plotting of population and death ratios for five-year cohorts. Working with five-year age groups, one can calculate the ratios of the cohort aged from x to $x+4$ at the first census to its survivors t years later, ${}_5N1_x/{}_5N2_{x+t}$, and of the intercensal deaths of its members to its survivors t years later,

$$\sum_{j=a}^{a+t} D^j_{x,x+4}/{}_5N2_{x+t}.$$

These points can then be plotted. Typically, they will be very erratic, often to the extent that no linear trend can be plausibly associated with them. If an examination of their plot suggests that they do represent a line, its parameters may be estimated by using group means (see chapter V, subsection C.4). The slope can then serve as an adjustment factor for the recorded number of deaths, adjustment that will make them consistent with the coverage of the first census, while the intercept is an estimate of the coverage of the first census in relation to that of the second.

Step 5(b): calculation and plotting of population and death ratios for open-ended cohorts. Cohorts can also be defined in terms of open-ended age intervals, that is, as all those of aged x and over at the time of the first census; and the ratios $N1(x+)/N2((x+t)+)$ and $\sum_{j=a}^{a+t} D^j_{x,\omega}/N2((x+t)+)$ can be calculated for values of x of 5, 10, 15 and so on. Many of the irregularities observed in the five-year ratios will be smoothed out by this cumulation, and a group mean procedure (see chapter V, subsection C.4) can be used to fit a straight line to these points.

Step 5(c): calculation and plotting of population and death ratios for truncated cohorts. It is often useful to exploit the advantages of cumulation without using information for the elderly, whose age-misreporting may be substantial. Therefore, initial cohorts aged between x and 60, or x and 65, can be used for values of x that are multiples of five. The use of truncated cumulation will have a substantial smoothing effect, but the slope of the resulting line may be highly sensitive to the upper age-limit chosen.

Step 6: interpretation of results. Often, selective migration, age exaggeration and other reporting problems may distort the slope of the fitted line. In such cases, this method may not be very useful for mortality estimation purposes. The estimate of the intercept, on the other hand, appears to be more robust, so that the main value of this method lies in the assessment it provides of the relative coverage of successive censuses.

4. *A detailed example*

The method is applied to the case of Panama, 1960-1970, since some of the necessary calculations have already been made. The steps of the procedure are given below.

Step 1: adjustment for net intercensal migration and territorial coverage. As no basis exists for making the necessary adjustments, the basic population data used are those presented in table 173. Deaths do not require any adjustment either.

Step 2: grouping of data from the two censuses by cohort. This step has been fully described in subsection B.2(c); standard five-year age groups in both 1960 and 1970 are used to define cohorts, since the intercensal interval is approximately 10 years.

Step 3: adjustment for length of the intercensal interval. As described in subsection B.2(c), the 1960 female popu-

TABLE 182. FEMALE POPULATION, 1960 AND 1970; AND REGISTERED DEATHS OF FEMALES, 1960, 1965 AND 1970; BY AGE GROUP, PANAMA

Age group (1)	Female population		Registered deaths		
	1960[a] (2)	1970 (3)	1960 (4)	1965 (5)	1970 (6)
0-4	88 477	114 017	1 670	1 497	1 608
5-9	75 242	106 944	138	122	167
10-14	62 509	85 253	63	64	71
15-19	53 468	73 381	76	84	95
20-24	44 826	63 010	106	102	99
25-29	37 149	50 924	103	79	104
30-34	31 609	40 885	87	91	94
35-39	28 216	36 115	98	112	116
40-44	23 550	29 409	99	111	110
45-49	20 253	25 360	123	119	137
50-54	14 803	21 775	111	141	161
55-59	11 787	17 632	122	115	186
60-64	10 101	13 004	147	188	233
65-69	10 403	10 061	152	174	268
70-74		6 690	184	239	270
75+		9 873	513	742	913

[a] Adjusted to approximate an intercensal interval of 10 years.

lation was moved back from the actual census date, 11 December 1960, to a date exactly 10 years before the 1970 census, 10 May 1960. Results are shown in column (2) of table 182.

Step 4(a): cumulation of cohort deaths from registration data. Registered deaths by age and sex are available for Panama for 1960, 1965 and 1970; and these data can be used to estimate intercensal deaths for each cohort. The numbers of female deaths for each year and age group are shown in table 182.

The approximate procedure for estimating cohort deaths is too crude to give reasonable results for the cohort aged 0-4 in 1960, so the cohort aged 5-9 is the starting-point. Deaths occurring over the first five years, from mid-1960 to mid-1965, to this cohort are estimated as

$$\sum_{j=1960}^{1965} D^j_{5,9} = 2.5\,[{}_5D_5^{1960} + {}_5D_{10}^{1965}] = 2.5\,(138+64) = 505.$$

Deaths for the cohort aged 5-9 in 1960 over the second five years, from mid-1965 to mid-1970, are estimated as

$$\sum_{j=1965}^{1970} D^j_{5,9} = 2.5[{}_5D_{10}^{1965} + {}_5D_{15}^{1970}] =$$

$$2.5(64+95) = 397.5.$$

Cohort deaths for the 10-year period are then obtained by summing the deaths during the two five-year periods:

$$\sum_{j=1960}^{1970} D^j_{5,9} = \sum_{j=1960}^{1965} D^j_{5,9} + \sum_{j=1965}^{1970} D^j_{5,9} =$$

$$505 + 397.5 = 902.5.$$

The results for each cohort are shown in table 183. The only age group that requires special treatment is the open-ended interval 65 and over. For the period 1960-1965, all the annual deaths of persons over 70 belong to this cohort, as do some of the deaths of persons aged 65-69. The total number of deaths of persons over 70 during the period 1960-1965 can be estimated by summing the deaths over age 70 in 1960 and 1965 and multiplying the sum by 2.5:

$$\sum_{j=1960}^{1965} {}_{\omega-70}D^j_{70} = 2.5({}_{\omega-70}D_{70}^{1960} + {}_{\omega-70}D_{70}^{1965})$$

$$= 2.5(184+513+239+742)$$

$$= 2.5(1\,678) = 4\,195$$

where, as usual, ${}_{\omega-70}D^j_{70}$ denotes the number of deaths occurring during year j to persons aged 70 and over.

For age group 65-69, the average number of deaths per annum between 1960 and 1965 is estimated as 0.5(152 + 174), or 163; and since the cohort aged 65-69 in 1960 averaged 2.5 years of exposure to the risk of dying during the period 1960-1965, the deaths of persons aged 65-69 belonging to that cohort are estimated as 2.5(163) = 407.5 during that period. Hence, the total

TABLE 183. ESTIMATED COHORT DEATHS, FEMALE POPULATION, PANAMA, 1960-1970

Cohort age group (1960) (1)	Number of cohort deaths		
	1960-1965 (2)	1965-1970 (3)	1960-1970 (4)
5-9	505.0	397.5	902.5
10-14	367.5	457.5	825.0
15-19	465.0	515.0	980.0
20-24	462.5	432.5	895.0
25-29	485.0	517.5	999.5
30-34	497.5	555.0	1 052.5
35-39	522.5	620.0	1 142.5
40-44	545.0	700.0	1 245.0
45-49	660.0	817.5	1 477.5
50-54	565.0	870.0	1 435.0
55-59	800.0	1 140.0	1 940.0
60-64	602.5	1 110.0	1 912.5
65+	4 602.5	4 773.8	9 376.3

number of deaths during the period 1960-1965 for the cohort aged 65 and over in 1960 is

$$\sum_{j=1960}^{1965} {}_5D_{65}^{j} + \sum_{j=1960}^{1965} {}_{\omega-70}D_{70}^{j} = 4,195 + 407.5 = 4,602.5.$$

Between 1965 and 1970, all the deaths at age 75 and over belong to the initial 65+ cohort, as do a proportion of the deaths of persons aged 70-74. Deaths at age 75 and over are estimated as

$$\sum_{j=1965}^{1970} {}_{\omega-75}D_{75}^{j} = 2.5({}_{\omega-75}D_{75}^{1965} + {}_{\omega-75}D_{75}^{1970})$$

$$= 2.5(742 + 913) = 4,137.5.$$

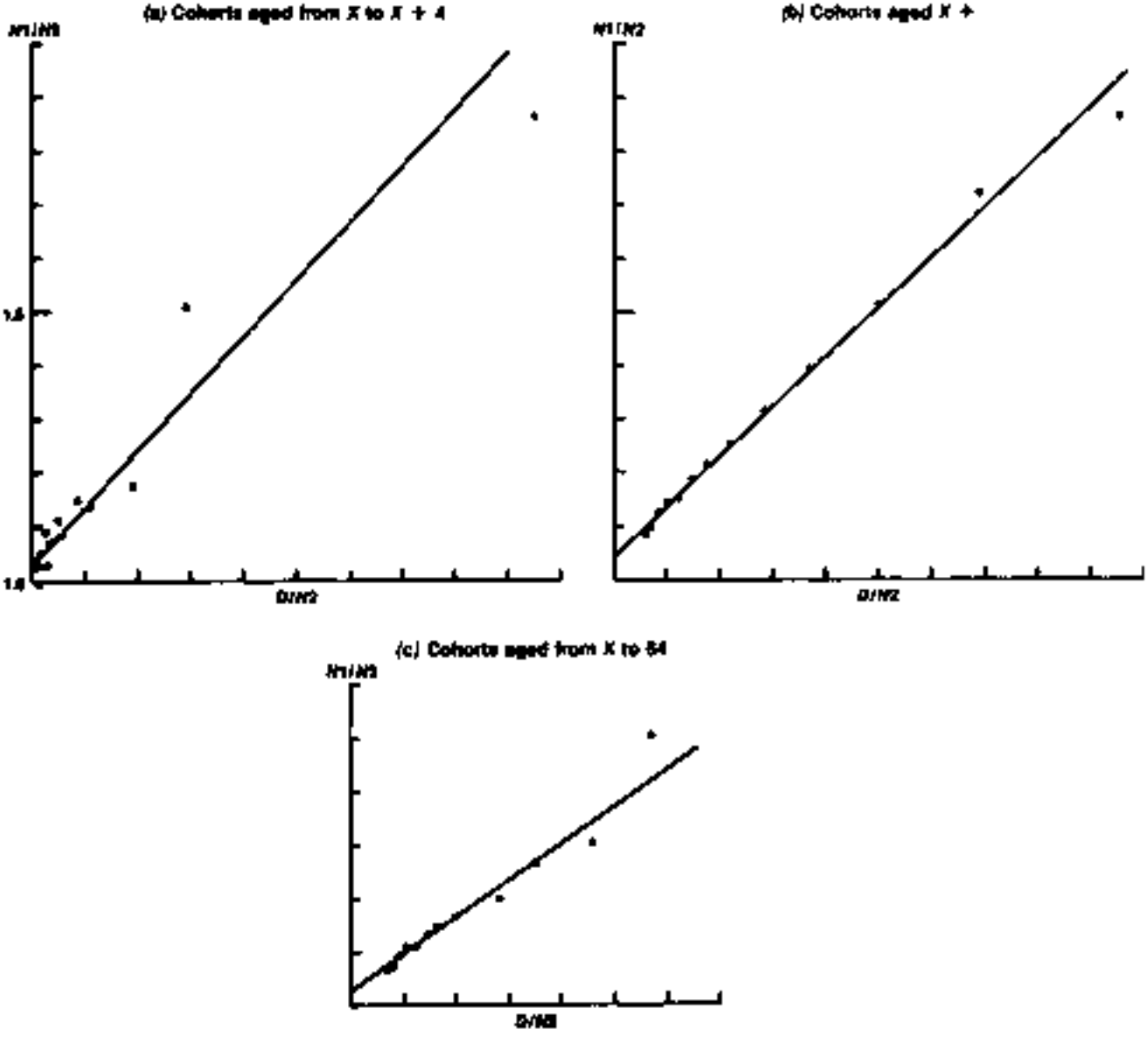
Deaths occurring to the cohort during the period 1965-1970 at ages 70-74 can be estimated from the average annual number of deaths at these ages, 0.5(239+270), and the average exposure to risk, 2.5 years, giving (2.5)(0.5)(239+270)=636.25. Hence, the deaths occurring during the intercensal period to the cohort aged 65 and over at the beginning of it (1960) are

$$\sum_{j=1960}^{1970} D_{65+}^{j} = 4,137.5 + 636.25 + 4,602.5$$

$$= 9,376.25$$

Step 5(a): calculation and plotting of population and death ratios for five-year cohorts. Ratios of the initial to the final cohort size, $N1/N2$, and of cohort deaths to final cohort size, $D/N2$, are calculated for each cohort.

Figure 33. Plots of cohort population ratios, $N1/N2$, against ratios of cohort deaths over population, $D/N2$, for various types of cohorts



Thus, for the cohort aged 5-9 in 1960,

$$ {}_5N1_5/{}_5N2_{15} = 75{,}242/73{,}381 = 1.0254 $$

$$ \sum_{t=1960}^{1970} D^t_{5,+}/{}_5N2_{15} = 902.5/73{,}381 = 0.0123. $$

Values for all cohorts are shown in columns (2) and (3) of table 184 and are plotted in panel (a) of figure 23. The points show a fair amount of variability; and the straight line, fitted by group means, is heavily influenced by the last three points and does not approximate the others very well. The intercept, which estimates the coverage of the first census in relation to that of the second, is 1.0278, suggesting that the 1960 census was some 3 per cent more complete than the 1970 census. The slope, which estimates the coverage of the first census in relation to the completeness of death registration, is 1.0623, suggesting that death registration was some 6 per cent less complete than the enumeration of the 1960 population. These estimates are by no means unreasonable, but the estimate of the slope is sensitive to the fitting procedure used; for instance, if the line is fitted by using least squares (see chapter V, subsection C.4) the value of the intercept becomes 1.046, not very different from that obtained by group means; but the slope is 0.914, a value totally different from that associated with the line fitted by using group means. Furthermore, the former value is rather implausible, because it implies that the coverage of deaths is more complete than that of the population.

TABLE 184. COHORT POPULATION RATIOS, $N1/N2$, AND RATIOS OF COHORT DEATHS OVER POPULATION, $D/N2$, FOR DIFFERENT TYPES OF COHORTS, PANAMA, 1960-1970

Age at census date (1)	Ratios for cohorts aged from x to $x+4$: ${}_5N1_x/{}_5N2_{x+10}$ (2)	Ratios for cohorts aged from x to $x+4$: $\sum D^t_{x,+}/{}_5N2_{x+10}$ (3)	Ratios for cohorts aged x and over: $N1(x+)/N2(x+10+)$ (4)	Ratios for cohorts aged x and over: $\sum D^t_{x,\omega}/N2(x+10+)$ (5)	Ratios for cohorts aged from x to 74: $N1(x,74)/N2(x+10,74)$ (6)	Ratios for cohorts aged from x to 74: $\sum D^t_{x,74}/N2(x+10,74)$ (7)
5	1.0254	0.01230	1.0849	0.06069	1.0651	0.03525
10	0.9920	0.01309	1.0983	0.07163	1.0743	0.04060
15	1.0500	0.01885	1.1239	0.08572	1.0949	0.04749
20	1.0964	0.02189	1.1418	0.10183	1.1063	0.05474
25	1.0286	0.02768	1.1527	0.12112	1.1088	0.06313
30	1.0748	0.03579	1.1862	0.14634	1.1322	0.07347
35	1.1126	0.04505	1.2176	0.17749	1.1501	0.08519
40	1.0815	0.05718	1.2513	0.21998	1.1638	0.09991
45	1.1487	0.08380	1.3158	0.28189	1.2016	0.14276
50	1.1382	0.11035	1.3902	0.37004	1.2330	0.17770
55	1.1716	0.19282	1.5133	0.49687	1.3067	0.22999
60	1.5099	0.28587	1.7209	0.68156	1.5099	0.28587
65	1.8640	0.94969	1.8640	0.94969	-	-
Parameters of straight lines fitted by group means to all points:						
Intercept	1.0278		1.0409		1.0239	
Slope	1.0623		0.9399		1.3916	

Step 5(b): calculation and plotting of population and death ratios for open-ended cohorts. The most convenient way to cumulate is to begin with the higher ages and work towards younger ones, beginning with the cohort aged 65 and over in 1960. In this case, the values of $N1$, $N2$ and D are exactly the same as those obtained in step 4(a):

$$ N1(65+)/N2(75+) = 18{,}403/9{,}873 = 1.8640 $$

$$ \sum_{t=1960}^{1970} D^t_{65,\omega}/N2(75+) = 9{,}376.25/9{,}873 = 0.94969. $$

For the initial cohort aged 60 and over,

$$ N1(60+)/N2(70+) = (18{,}403 + 10{,}101)/(9{,}873 + 6{,}690) = 1.7209 $$

$$ \sum_{t=1960}^{1970} D^t_{60,\omega}/N2(70+) = (9{,}376.25 + 1{,}912.5)/(9{,}873 + 6{,}690) = 0.68156. $$

The calculations continue in the same way, adding a new age group to the previous sums at each step, until the youngest cohort, aged 5-9 in 1960, has been added in. Columns (4) and (5) of table 184 show the ratios obtained, which are plotted in panel (b) of figure 23.

The linearity of the points has been greatly improved by the cumulation, and the straight line fits the observations quite closely. However, the parameters of the line—an intercept of 1.0409 and a slope of 0.9399—imply a similar enumeration coverage differential between the 1960 and 1970 censuses as that obtained in step 5(a), but a quite different coverage of death registration, which is now estimated to overrecord deaths, in relation to the 1960 census, by some 6 per cent.

Step 5(c): calculation and plotting of population and death ratios for truncated cohorts. Cumulations similar to those carried out in step 5(b) can also be made so that the age groups considered are not entirely open-ended, but rather exclude the last one or two age groups. Such truncated cumulation can be useful if there is reason to suppose that special types of errors affect the oldest age groups. Estimates of completeness of death registration will then refer to registered adult deaths excluding both those at young ages and those at old ages.

The cumulations are carried out as before. However, they begin not with the last, open-ended cohort, but with the oldest cohort that one intends to include in the calculations. In this case, the cohort aged 60-64 in 1960 was chosen as the upper limit. For this cohort, the ratios required are the same as those calculated in step 5(a), in which ratios were calculated for individual five-year age cohorts. Thus,

$$N1(60, 64)/N2(70, 74) = 10,101/6,690 = 1.5099$$

$$\sum_{j=1960}^{1970} D_{60, 64}/N2(70, 74) = 1,912.5/6,690 = 0.28587.$$

For the cohort aged 55-64 in 1960,

$$N1(55, 64)/N2(65, 74) = (10,101 + 11,787)/$$

$$(6,690 + 10,061) = 1.3067$$

$$\sum_{j=1960}^{1970} D_{55, 64}/N2(65, 74) = (1,912.5 +$$

$$1,940.0)/(6,690 + 10,061) = 0.22999.$$

The cumulations continue downward with age, adding the next younger age group each time, until age group 5-9, the youngest to be used, has been included. The resulting ratios are shown in columns (6) and (7) of table 184 and are plotted in panel (c) of figure 23.

The plot shows that the degree of linearity of the points is somewhere between that in the fully cumulated case and that in which no cumulation was used. The intercept of the fitted line is similar to those obtained earlier; but the slope is once more quite different, indicating that registered deaths in the approximate age range from 10 to 70 are only about 72 per cent complete with respect to the 1960 census coverage.

D. Estimation of a post-childhood life table from an age distribution and intercensal growth rates

1. *Basis of method and its rationale*

Traditional intercensal survival techniques are greatly complicated by intercensal intervals that are not exact numbers of years in length or are not multiples of five. Furthermore, the application of the method described in subsection B.4, which uses cumulation to reduce the impact of age-reporting errors, is very time-consuming since it involves the projection of an initial population using different mortality levels.

Preston and Bennett[4] propose a different method to estimate adult mortality during the intercensal period from the age distributions produced by two consecutive censuses. The application of the method proposed is simple whatever the length of the intercensal period, and it is not very sensitive to certain types of age-misreporting, particularly heaping. It is also innovative, because it uses the two census age distributions to estimate age-specific growth rates (rather than cohort survivorship probabilities) and then uses these growth rates to transform the observed population age structure into the equivalent of a stationary-population (life-table) ${}_5L_x$ function.

Bennett and Horiuchi[5] show that in any closed population, at a particular time t, the number of persons aged y, $N(y)$, is equal to the number of persons aged x, $N(x)$, multiplied by the probability of surviving from age x to age y, $l(y)/l(x)$ measured at time t, and by an exponential factor involving the integral of the population growth rates also at time t between ages x and y. Thus,

$$N(y) = N(x)[l(y)/l(x)] \exp(-\int_x^y r(u)du), \qquad (D.1)$$

which can be regarded as being equivalent to a stable population relationship, except for the replacement of the exponential of the stable growth rate times the number of years between x and y, $\exp[-(y-x)r]$, by the exponential of the integral of the variable growth rates between x and y, $\exp[-\int_x^y r(u)du]$.

If $N(x)$, $N(y)$ and the set of $r(u)$ values for u between x and y are all known, then the period survivorship probability, $l(y)/l(x)$, can be estimated from equation (D.1). However, in order to introduce a certain amount of smoothing, Preston and Bennett propose the estimation of the expectation of life at each age x, using extensive cumulation both of the reported population and of the observed age-specific growth rates.

In discrete terms, using five-year age groups, the basic equation proposed by Preston and Bennett is

$$e_x = \frac{\sum_{y=x}^{\omega} {}_5N_y \exp[5.0\sum_{x=z}^{y-5} {}_5r_x + 2.5\,{}_5r_y]}{N(x)} \qquad (D.2)$$

where $N(x)$, the number of people aged x, is estimated as

$$N(x) = \frac{{}_5N_{x-5}\exp[-2.5\,{}_5r_{x-5}] + {}_5N_x \exp[2.5\,{}_5r_x]}{10} \qquad (D.3)$$

The advantages of this method are: (*a*) its application is relatively simple even in cases where the intercensal period does not have a convenient length; (*b*) it makes no assumptions concerning stability; (*c*) it introduces an element of cumulation, thus limiting the effects of age errors, and (*d*) the use of growth rates eliminates the effects of age errors for which the pattern is the same at

[4] Samuel H. Preston and Neil G. Bennett, "A census-based method for estimating adult mortality", *Population Studies*, vol. 37, No. 1 (March 1983), pp. 91-104.

[5] Neil G. Bennett and Shiro Horiuchi, "Estimating the completeness of death registration in a closed population", *Population Index*, vol. 47, No. 2 (Summer 1981), pp. 207-221.

both censuses. The estimates yielded by this procedure are probably as reliable as those obtained from any of the intercensal techniques available, and its simplicity of application makes it extremely attractive.

2. *Data required*

The data required for this method are listed below:

(*a*) Two census enumerations with populations classified by the same age groups (and sex), separated by an intercensal interval which should not exceed 20 years;

(*b*) Sufficient information to adjust one census or the other for net intercensal migration and territorial coverage, if necessary.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: adjustment for net intercensal migration and territorial coverage. See step 1 in subsection B.2(b).

Step 2: calculation of age-specific intercensal growth rates. The rate of growth of the population in each five-year age group from the first to the second census is calculated as

$$ {}_5r_x = [\ln({}_5N2_x) - \ln({}_5N1_x)]/t \qquad \text{(D.4)} $$

where ${}_5r_x$ denotes the intercensal growth rate of the population of the age group from x to $x+4$; ${}_5N2_x$ is the population aged from x to $x+4$ at the second census; ${}_5N1_x$ is the population aged from x to $x+4$ at the first census; and t is the length of the intercensal interval in years (with a decimal portion if necessary). Both age distributions must share the same open interval, $A+$. The value of A should be set as high as the two age distributions permit, since age exaggeration is a less severe problem with this method than with the death distribution methods described in chapter V.

Step 3: calculation of average intercensal age distribution. Equation (D.2) requires the use of an average intercensal age distribution, ${}_5N_x$. An adequate approximation to this age distribution can be obtained by simply averaging the initial and final populations of each age group. Thus,

$$ {}_5N_x = 0.5\,({}_5N1_x + {}_5N2_x). \qquad \text{(D.5)} $$

Step 4: cumulation of age-specific growth rates from age 5 upward. The calculation of the growth rate "inflator" appearing in equation (D.2) requires the summation of the age-specific growth rates, ${}_5r_x$, calculated in step 2. It is normally convenient to begin the cumulation process with age 5 and continue upward.

The only difficulty involved in the cumulation is the treatment of the inflation factor associated with the open age interval, $A+$. Although the relative importance of the open interval is much less in this case than in the conceptually similar death distribution techniques discussed in chapter V, because in the calculation of expectations of life the open interval is always present, it is sound practice to minimize the influence of biases due entirely to the weight it may be assigned in the inflation factor. For this reason, a special procedure is suggested to deal with the open interval.

If $R(x)$ is used to denote the inflation factor for the age group from x to $x+4$, then, according to equation (D.2),

$$ R(x) = 2.5\,{}_5r_x + 5.0 \sum_{y=5}^{x-5} {}_5r_y \qquad \text{(D.6)} $$

for $x = 10, 15, \ldots, A-5$. In the case of $x = 5$, (D.6) becomes

$$ R(5) = 2.5\,{}_5r_5 \qquad \text{(D.7)} $$

and for $x = A$, $R(A) = \rho(A) + 5.0 \sum_{y=5}^{A-5} {}_5r_y$ (D.8)

where $\rho(A)$ is calculated by using an equation derived from simulated stable populations and whose form is

$$ \rho(A) = a(A) + b(A)\,r(10+) + c(A)\,\ln(N(45+)/N(10+)) \qquad \text{(D.9)} $$

where $r(10+)$ is the growth rate of the population over age 10, that is,

$$ r(10+) = \ln[N2(10+)/N1(10+)]/t; \qquad \text{(D.10)} $$

$N(10+)$ and $N(45+)$ are the mid-period populations aged 10 and over, and 45 and over, respectively, and estimated as

$$ N(10+) = 0.5(N1(10+) + N2(10+)) $$

and

$$ N(45+) = 0.5(N1(45+) + N2(45+)); \qquad \text{(D.11)} $$

and $a(A)$, $b(A)$ and $c(A)$ are constant coefficients depending upon the actual value of A. Their values are shown in table 185.

TABLE 185. COEFFICIENTS FOR ESTIMATION OF THE EQUIVALENT GROWTH RATE OVER AGE A, $\rho(A)$, FROM THE GROWTH RATE OVER AGE 10 AND THE RATIO OF THE POPULATION OVER AGE 45 TO THE POPULATION OVER AGE 10

Age A (1)	Coefficients $a(A)$ (2)	$b(A)$ (3)	$c(A)$ (4)
45	0.229	20.43	0.258
50	0.205	18.28	0.235
55	0.179	16.02	0.209
60	0.150	13.66	0.176
65	0.119	11.22	0.141
70	0.086	8.77	0.102
75	0.053	6.40	0.063
80	0.025	4.30	0.029
85	0.006	2.68	0.006

Estimation equation:

$$ \rho(A) = a(A) + b(A)\,r(10+) + c(A)\,\ln(N(45+)/N(10+)) $$

Step 5: reduction of age distribution to a stationary form. The average intercensal age distribution obtained in step 3 is converted into a stationary population by multiplying each value ${}_5N_x$ by the exponential of $R(x)$. The results can be regarded as "pseudo" ${}_5L_x{}^*$ values, analogous to the values of ${}_5L_x$ in the usual life table. However, the life-table radix corresponding to the pseudo ${}_5L_x{}^*$ values is not known. Therefore, in general, the pseudo ${}_5L_x{}^*$ values cannot be manipulated as can normal ${}_5L_x$ values.

Recapitulating, the estimation of the ${}_5L_x{}^*$ values is carried out according to the following equations:

$$ {}_5L_x{}^* = {}_5N_x \exp(R(x)) \qquad \text{(D.12)} $$

and

$$ {}_{\omega-A}L_A{}^* = {}_{\omega-A}N_A \exp(R(A)). \qquad \text{(D.13)} $$

It should be noted that the sequence of ${}_5L_x{}^*$ values obtained in this way is likely to be more erratic than one derived from a set of ${}_5q_x$ values calculated on the basis of observed central mortality rates. In some instances, the estimated ${}_5L_x{}^*$ values may even increase with age. Errors in the age distributions used as input are usually the cause of this erratic behaviour. It is in order to minimize the effects of such errors and also to generate a measure that is comparable with those usually found in other sources that the pseudo ${}_5L_x{}^*$ values are converted into expectation of life in the manner described below.

Step 6: calculation of expectation of life. The expectation of life at age x, e_x, is calculated by cumulating the pseudo ${}_5L_x{}^*$ values obtained in the previous step and dividing the sum by an estimate of $l(x)$, the number of survivors to exact age x in the life table. An adequate estimate of $l(x)$ can be obtained as

$$ l^*(x) = ({}_5L_x{}^* + {}_5L_x{}^*)/10.0 \qquad \text{(D.14)} $$

where the * has been added to remind the reader that these are also pseudo $l^*(x)$ values with unknown radix. Then, letting $T_x{}^*$ be the number of person-years lived above age x, its value is calculated as

$$ T_x{}^* = \sum_{y=x}^{\omega-5} {}_5L_y{}^* + {}_{\omega-A}L_A{}^*, \qquad \text{(D.15)} $$

so by combining equations (D.14) and (D.15), e_x can be estimated as

$$ e_x = 10.0(T_x{}^*)/({}_5L_x + {}_5L_x{}^*). \qquad \text{(D.16)} $$

Once the life expectancy figures have been calculated, usually for x ranging from 10 to 50, the levels they imply in a model life-table system can be found, and a final estimate of mortality can be obtained by averaging the most reliable estimates of mortality level (those left after discarding any clearly unsuitable values). In practice, the mortality estimates for values of x up to age 30 or so are reasonably consistent, but after age 30 or 35 they often show progressively lower mortality as age x increases. The best estimate of overall mortality may therefore be an average of the levels associated with e_x for x ranging from 10 to 30, though this conclusion implies that the results will not be a useful basis for the selection of an age pattern of mortality, nor will they be good indicators of the necessity of adjustment when errors in the growth rates arise because of changes in enumeration completeness.

4. *A detailed example: Panama, 1960-1970*

The case of Panama between 1960 and 1970 is used to illustrate the application of this method, so it will be possible to compare its results with those obtained above through the application of the intercensal-survival techniques.

The basic data are shown in columns (2) and (4) of table 173, but for the sake of completeness, they are reproduced in columns (2) and (3) of table 186. Note that when using this method there is no need to adjust for intercensal intervals that are not round numbers of years; therefore, the populations just as enumerated in 1960 and 1970 can be used.

The computational procedure for this example is given below.

Step 1: adjustment for net intercensal migration and territorial coverage. As described in step 1 of subsection B.2(c), no adjustments are carried out in this case.

Step 2: calculation of age-specific intercensal growth rates. The interval between the 1960 and 1970 censuses was 9.41 years (see subsection B.2(c)). The growth rate for each age group is therefore calculated by dividing the difference between the natural logarithms of the final and initial populations of each age group by 9.41. Thus, for the population of age group 5-9,

$$ {}_5r_5 = [\ln({}_5N_{2_5}) - \ln({}_5N_{1_5})]/9.41 $$

$$ = (11.5801 - 11.2463)/9.41 = 0.03547. $$

Results are shown in column (4) of table 186.

Step 3: calculation of average intercensal age distribution. An average age distribution for the intercensal period is obtained simply by calculating the arithmetic means of the initial and final populations of each age group. Thus for age group 5-9,

$$ {}_5N_5 = 0.5({}_5N_{1_5} + {}_5N_{2_5}) = 0.5(76,598 + $$

$$ 106,944) = 91,771. $$

Full results are shown in column (5) of table 186.

Step 4: cumulation of age-specific growth rates from age 5 upward. Cumulated age-specific growth rates are required to estimate ${}_5L_x$ for all values of x from 5 upward. The average population aged 5-9 years, ${}_5N_5$, needs to be inflated by 2.5 years of growth at the age-specific growth rate for the 5-9 age group, namely, ${}_5r_5$. The average population aged 10-14 years, ${}_5N_{10}$, needs to be inflated by five years of growth at the age-specific rate for the 5-9 age group, ${}_5r_5$, plus 2.5 years of growth at the

TABLE 186. ESTIMATION OF INTERCENSAL MORTALITY FOR FEMALES, USING INTERCENSAL GROWTH RATES, PANAMA, 1960-1970

Age group $x, x+5$ (1)	Observed female population: 1960 (2)	Observed female population: 1970 (3)	Intercensal growth rate $_5r_x$ (4)	Average population 1960-1970 (5)	Cumulated growth rate $R(x)$ (6)	Stationary population: In five-year age groups $_5L_x^*$ (7)	Stationary population: At exact age x $l^*(x)$ (8)	Stationary population: Over age x T_x^* (9)	Expectation of life e_x (10)	West mortality level (11)
0	90 071	114 017	0.02505	102 044	-	-	-	-	-	-
5	76 598	106 944	0.03547	91 771	0.08868	100 281	-	-	-	-
10	63 635	85 253	0.03108	74 444	0.25505	96 072	19 635	1 123 232	57.21	16 9
15	54 431	73 381	0.03175	63 906	0.41213	96 500	19 257	1 027 160	53.34	17.4
20	45 634	63 010	0.03429	54 322	0.57723	96 753	19 325	930 660	48.16	16.8
25	37 818	50 924	0.03162	44 371	0.74200	93 185	18 994	833 907	43.90	16.7
30	32 179	40 885	0.02545	36 532	0.88466	88 488	18 167	740 722	40.77	17.8
35	28 724	36 115	0.02433	32 420	1.00913	88 935	17 742	652 234	36.76	18.1
40	23 974	29 409	0.02171	26 692	1.12423	82 154	17 109	563 299	32.92	18.6
45	20 618	25 360	0.02000	22 989	1.23350	78 926	16 108	481 145	29.87	20 3
50	15 068	21 775	0.03913	18 422	1.38633	73 691	15 262	402 219	26.35	21.3
55	11 999	17 632	0.04090	14 816	1.58640	72 393	14 608	328 528	-	-
60	10 243	13 004	0.02495	11 644	1.75103	67 076	13 947	256 135	-	-
65	6 737	10 061	0.04262	8 399	1.91995	57 286	12 436	189 059	-	-
70	5 242	6 690	0.02592	5 966	2.09130	48 297	10 558	131 773	-	-
75+	6 756	9 873	-	8 315	2.30650	83 476	-	83 476	-	-

rate for the 10-14 group, ${}_5r_{10}$. Values of $R(x)$ are therefore found by successive cumulation of the age-specific rates following equations (D.6) and (D.7). For age group 5-9,

$$R(5) = 2.5({}_5r_5) = 0.08868.$$

For age group 10-14,

$$R(10) = 2.5({}_5r_{10}) + 5.0({}_5r_5) = 0.25505.$$

For age group 20-24,

$$R(20) = 2.5({}_5r_{20}) + 5.0({}_5r_5 + {}_5r_{10} + {}_5r_{15}) = 0.57723.$$

To calculate $R(A)$, the inflation factor corresponding to the open-ended interval, equation (D.8) is used in conjunction with equation (D.9) to estimate $\rho(A)$. The latter uses as inputs the values of $r(10+)$, $N(10+)$ and $N(45+)$. The symbol $r(10+)$ is the growth rate of the population 10 and over during the intercensal period, and it is calculated as any other growth rate, as is illustrated below:

$$r(10+) = \ln(N2(10+)/N1(10+))/t$$

$$= \ln(483,372/363,098)/9.41$$

$$= 0.2861/9.41$$

$$= 0.0304.$$

The values of $N(10+)$ and $N(45+)$ are found by cumulating the necessary entries in column (5) of table 186. In the case of $N(45+)$, one begins with ${}_5N_{45}$ and continues until $N(75+)$. For $N(10+)$, the starting-point is ${}_5N_{30}$. The resulting values are $N(10+) = 423,238$ and $N(45+) = 90,551$. Hence,

$$a(75) = a(75) + b(75)r(10+) + c(75)\ln(N(45+)/N(10+))$$

$$= 0.053 + 6.40(0.0304) + 0.063\ln(90,551/423,238)$$

$$= 0.1504$$

and

$$R(75) = \rho(75) + 5.0\sum_{y=5}^{70} {}_5r_y$$

$$= 0.1504 + 2.1561$$

$$= 2.3065.$$

The complete set of $R(x)$ values is shown in column (6) of table 186.

Step 5: reduction of age distribution to a stationary form. Values of ${}_5L_x^*$ are now obtained for each x by multiplying the average population of the age group, ${}_5N_x$, by $\exp[R(x)]$. Thus, for age group 5-9,

$${}_5L_5^* = (91,771)\exp(0.08868) = 100,281$$

and for age group 45-49,

$${}_5L_{45}^* = (22,989)\exp(1.23350) = 78,926.$$

The open interval is treated in just the same way, remembering that the result is an estimate of ${}_{\omega-A}L_A^*$, or T_A^*. Full results are shown in column (7) of table 186.

Step 6: calculation of expectation of life. The expectation of life at age x, e_x, is equal to the person-years lived from age x onward, T_x^*, divided by the number of survivors to age x, $l^*(x)$. The value of $l^*(x)$ can be estimated with sufficient accuracy by averaging adjacent values of person-years lived, that is, ${}_5L_{x-5}^*$ and ${}_5L_x^*$. Thus, for age 10,

$$l^*(10) = ({}_5L_5^* + {}_5L_{10}^*)/10.0$$

$$= (100,281 + 96,072)/10.0 = 19,635.$$

Now that because 5 is the lowest value of x used in the calculations and no value of ${}_5L_0^*$ is obtained, $l^*(10)$ is the youngest population that can be calculated. It may be noted in passing that no value of ${}_5L_0^*$ is obtained because age group 0-4 is generally severely distorted by omission and age-misreporting. Similarly, the highest age for which an $l^*(x)$ value can be estimated is age $A-5$, since no value is available for ${}_5L_A^*$. Column (8) of table 186 shows the range of $l^*(x)$ values.

The $_xT_x^*$ column is calculated by cumulating from age A downward the successive ${}_5L_x^*$ values. Thus, for 75,

$$T_{75}^* = {}_{\omega-75}L_{75}^* = 83,476$$

and for 50,

$$T_{50}^* = T_{55}^* + {}_5L_{50}^* = 328,528 + 73,691 = 402,219.$$

Full results are shown in column (9) of table 186.

The expectation of life at each age x from 10 to 50 is then calculated by dividing each T_x^* by the corresponding $l^*(x)$. Thus, for $x = 10$,

$$e_{10} = T_{10}^* / l^*(10) = 1,123,232/19,635 = 57.21,$$

whereas for $x = 50$,

$$e_{50} = T_{50}^* / l^*(50) = 402,219/15,262 = 26.35.$$

The figures for expectation of life shown in column (10) of table 186 can be compared with those of a model life-table system. Given the assumption made earlier that the West family provides the best fit to the age pattern of mortality in Panama, the West mortality level implied by each female a_x in column (10) has been found by interpolation, the results being shown in column (11).

It will be noticed that the first five values, for the age range from 10 to 30, are more or less consistent, the average level being 17.1. Above age 25, however, the mortality levels rise steadily with age, no doubt as a result of age exaggeration. It is interesting to compare the results obtained by this method with those obtained by the projection technique using cumulated values and described in subsection B.4(d). The similarities between the mortality estimates given in column (11) of table 186 and those in column (16) of table 180 are striking. In the latter case, as in the former, a series of five approximately consistent estimates of mortality level, averaging a level of 17.4 instead of 17.1, is followed by a pronounced upward trend in level with age. The present technique may be preferred on two grounds, the first being its simplicity of application and the second that it does not have the drawback of giving different results according to the direction, forward or backward, in which the projection is performed. It must be pointed out, however, that the estimated adult mortality level of 17.1, which is the best available from table 186, is to some extent distorted by the age exaggeration detected, since it inflates the T_x values at all ages.

ANNEXES

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Annex I

THE SINGULATE MEAN AGE AT MARRIAGE

A. Background of method

The singulate mean age at marriage, *SMAM*, is the mean age at first marriage among those who ever marry (or, in practice, among those who marry by some predefined age-limit). It is computed from the proportions who are single, that is, never married, in each age group.* Since the most frequently considered age groups are five years in length, the process of calculating the singulate mean age at marriage is described for data classified by such age groups. It is assumed here that no first marriages occur after age 50 or before age 15, though the generalization of the procedure described below to the use of other age limits is straightforward.

B. Basic calculations to obtain the singulate mean age at marriage

1. *Data required*

The following data are required for this procedure.

(a) The population aged 15-54 classified by age (five-year age group) and by sex;

(b) The never-married population aged 15-54 classified by age (five-year age group) and sex.

2. *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of proportions single for a given sex. Divide the number of single in each age group by the total population in the same age group. The resulting proportion for the age group from $5i+10$ to $5i+14$ is denoted by $U(i)$, with i usually ranging from 1 to 8.

Step 2: calculation of person-years lived in the single state. Add the proportions single in each age group up to and including that for the age group 45-49 ($i=7$) and multiply the sum by five. The resulting quantity is denoted by RS_1. Let $RS_2 = RS_1 + 15.0$. The quantity 15.0 is the number of person-years lived in the single state from birth to age 15 by the hypothetical cohort of size one being considered. If the lower limit at which marriage takes place is changed to some other age x, the x should be substituted for 15 and the calculation of RS_1 should include all age groups from x to $x+4$ (when x is a multiple of five) to 45-49.

Step 3: estimation of proportion who ever marry. The proportion remaining single at age 50 is estimated as

$$RN = (U(7) + U(8))/2.0 \qquad (B.1)$$

where $U(7)$ is the proportion single (never married) among those aged 45-49; and $U(8)$ is the equivalent proportion among those aged 50-54. Once the proportion remaining single at age 50 is estimated, the proportion ever marrying by that age, RM, is clearly just its complement, that is,

$$RM = 1.0 - RN. \qquad (B.2)$$

Step 4: calculation of number of person-years lived by the proportion ever marrying. Since RN is estimated to be the proportion who have not married by age 50, the total time spent in the single state by this proportion is

$$RS_3 = 50.0RN. \qquad (B.3)$$

Step 5: calculation of singulate mean age at marriage. Lastly, the value of *SMAM* is calculated as follows:

$$SMAM = (RS_2 - RS_3)/RM. \qquad (B.4)$$

That is, the value of *SMAM* as calculated here is the average number of years spent in the single state by those who marry before age 50. It should be noted that although, strictly, this value refers only to the persons who marry by age 50, in most applications it is handled as if it referred to the totality of the ever-married population. Two facts justify this practice: first, in most populations the incidence of first marriages after age 50 is very small; and secondly, in populations with deficient data it is not uncommon to find that after age 50 (and sometimes even after age 45) the proportions single increase as age increases, implying that older cohorts were subject to lower first-marriage rates than were younger groups. Because such a trend is fairly unlikely, the observed increases are usually attributed to reporting errors. Under such conditions, it would be unwise to incorporate the data for older cohorts in the calculation of *SMAM*.

3. *Detailed example*

The data given in table 187 were collected during the National

TABLE 187 POPULATION BY AGE GROUP, SEX AND MARITAL STATUS, PANAMA, 1976

Age Group (1)	Male				Female			
	Single (2)	Ever-married (3)	Unknown (4)	Total (5)	Single (6)	Ever-married (7)	Unknown (8)	Total (9)
15-19	2 678	69	5	2 752	2 171	509	15	2 695
20-24	1 311	585	24	1 940	806	1 251	38	2 095
25-29	356	1 351	62	1 769	288	1 503	37	1 828
30-34	256	1 231	89	1 576	146	1 414	44	1 604
35-39	158	1 108	90	1 356	82	1 223	57	1 362
40-44	96	919	110	1 125	35	1 043	50	1 128
45-49	76	775	120	971	41	832	57	930
50-54	43	644	107	794	22	660	52	734

* John Hajnal, "Age at marriage and proportions marrying", *Population Studies*, vol. VII, No. 2 (November 1953), pp. 111-136.

Demographic Survey in Panama in 1976. They illustrate some of the problems encountered when dealing with real data.

A quick examination of table 187 reveals that, especially at older ages, the marital status of a substantial proportion of the persons interviewed is unknown. It is difficult to imagine a reason for that omission since *a priori* it would seem easy for a person to remember whether he or she had been married.

A possible source of misunderstanding may be that the term "ever married" was used in a broad sense to mean "having belonged to a stable union". Because in many countries, unions not legalized by marriage are socially unacceptable, a respondent in such a situation might tend to avoid acknowledging his or her "marital status". It then seems plausible to assume that most of the "unknowns" were, in fact, "ever married". The validity of this assumption is, however, impossible to establish without recourse to further information. But if the data available are to be used in computing singulate mean ages at marriage for each sex, it is necessary to make some assumption about their true meaning. As suggested, one extreme assumption is to suppose that all the "unknowns" are ever married. Another, less extreme possibility is to assume that both the single and the ever-married persons within each age group have an equal probability of being classified as "unknown" (this assumption would be plausible if most of the unknowns were the result of random errors produced while processing the data). Under this assumption, the proportion single in each age group would remain the same if the unknowns were ignored as if they had declared their marital status properly.

Each of these assumptions leads to a different way of computing the proportion single in each age group and to somewhat different estimates of the desired singulate mean ages at marriage. The computational procedures and results are shown below.

(a) *Calculation of singulate mean age at marriage ignoring those of unknown marital status*

In the first instance, the proportions single are calculated by ignoring the unknowns. The resulting proportions are shown in table 188. Then, using these proportions single, the singulate mean ages at marriage are computed as usual.

TABLE 188. PROPORTIONS SINGLE IGNORING THOSE OF UNKNOWN MARITAL STATUS, PANAMA, 1976

Age group (1)	Proportion single: Males (2)	Females (3)	
15-19	0.9749	0.8101	
20-24	0.6947	0.3918	
25-29	0.3257	0.1608	
30-34	0.1722	0.0936	
35-39	0.1248	0.0628	
40-44	0.0946	0.0325	
45-49	0.0893	0.0470	(0.0324)
50-54	0.0626	0.0323	

(i) *Case 1: males (unknowns ignored)*

The results of the main steps for case 1 are given below:

$$RS_1 = 5.0(2.4762) = 12.381$$

$$RS_2 = RS_1 + 15.0 = 27.381$$

$$RM = (0.0893 + 0.0626)/2.0 = 0.076$$

$$RM = 0.924$$

$$RS_3 = 50.0RM = 3.798$$

$$SMAM = (RS_2 - RS_3)/RM = 25.52.$$

(ii) *Case 2: females (unknowns ignored)*

Note that the proportions single among females do not, as expected, decrease consistently as age increases. The proportion single at ages 45-49, for example, is greater than that at ages 40-44. Inconsistencies of this type are often due to changing marriage patterns, but the fact that the proportion single declines at ages 50-54 suggests that the relatively high value observed at ages 45-49 may be due, at least in part, to misreporting of status rather than to true changes in the marriage patterns of the past.

Therefore, to calculate the singulate mean age of marriage in this case, the reported proportions single were adjusted by replacing the value of 0.0470 for age group 45-49 by the average of the two values adjacent to it, which happen to be also the two smallest proportions single observed. Thus, it was assumed that the proportion single at ages 45-49 was 0.0324. Under this assumption, the singulate mean age at marriage is computed in the usual way. The results of the main steps are shown below:

$$RS_1 = 5.0(1.584) = 7.92$$

$$RS_2 = 7.92 + 15.0 = 22.92$$

$$RM = (0.0324 + 0.0323)/2.0 = 0.0324$$

$$RM = 0.9677$$

$$RS_3 = 50.0(0.0324) = 1.62$$

$$SMAM = (RS_2 - RS_3)/RM = 22.01.$$

It is interesting to note that the value of *SMAM* would have been 21.88 if the observed data, without adjustment, had been used, a value that is very similar to that obtained above.

(b) *Calculation of singulate mean age at marriage assuming that those of unknown marital status are ever married*

The values of *SMAM* are computed next by assuming that the "unknowns" belong, in fact, to the ever-married category. The proportions single obtained under this assumption are shown in table 189. The computations needed to arrive at the final value of *SMAM* are summarized in cases 3 and 4.

(i) *Case 3: males (unknowns considered to be ever married)*

The results for case 3 are shown below.

$$RS_2 = 5.0(2.416) + 15.0 = 27.08$$

$$RS_3 = 50.0[(0.0783 + 0.0542)/2.0] = 50.0(0.0663) = 3.3125$$

$$SMAM = (RS_2 - RS_3)/0.9337 = 25.65.$$

(ii) *Case 4: females (unknowns considered to be ever married)*

Once more, the inconsistency apparent in age group 45-49 was corrected by replacing the reported proportion single, 0.0441, by the average of those adjacent to it (0.0305). An outline of the calculations follows.

$$RS_2 = 5.0(1.5605) + 15.0 = 22.80$$

$$RS_3 = 50.0(0.0302) = 1.51$$

$$SMAM = (RS_2 - RS_3)/0.9698 = 21.95.$$

TABLE 189. PROPORTIONS SINGLE ASSUMING THAT THOSE OF UNKNOWN MARITAL STATUS ARE EVER MARRIED, PANAMA, 1976

Age group (1)	Proportion single: Males (2)	Females (3)	
15-19	0.9731	0.8056	
20-24	0.6861	0.3847	
25-29	0.3143	0.1575	
30-34	0.1624	0.0910	
35-39	0.1165	0.0602	
40-44	0.0853	0.0310	
45-49	0.0783	0.0441	(0.0305)
50-54	0.0542	0.0300	

(c) *Comments on the detailed example*

The *SMAM* estimates obtained by assuming that all those of unknown marital status were, in fact, ever married, are very similar to those obtained earlier by ignoring the respondents with unknown status. This similarity is due to the fact that the numbers of respondents of unknown marital status are a relatively small proportion both of the total population interviewed and of the total population whose marital status was reported. Yet, even though the estimates yielded are similar, the question arises as to which set should be used if further analysis were to be undertaken. If no more information were available about the survey in question, it would be advisable not to make extreme assumptions, so that the *SMAM* estimates computed by ignoring the unknowns would be accepted as representative.

In this case, however, the extreme assumption turns out to be that nearer to the truth. Indeed, since the question in the survey was "On what date did you marry or form a union for the first time?", and it was used to establish both marital status and the time elapsed since first union, all those who were unable to state the date of which their first marriage or union began were classified as "unknown", a term that in this context means "of unknown marital duration" rather than "of unknown marital status".

This example illustrates very clearly the following point: tabulations very often do not provide enough information about the data they display. In order to assess the true meaning of those data, it is important to know how they were obtained and how they were processed before they become part of a given tabulation. Without this additional information, errors in the interpretation of the data are very likely.

C. Need to consider hypothetical cohorts

The singulate mean age at marriage is a measure that logically should refer to the nuptiality experience of a birth cohort. Yet, in practice, data on nuptiality for birth cohorts are very rarely available. Usually, the data at hand refer to the proportions single observed in a population at one or two points in time. Because such proportions refer to a cross-section of the population, it is conceivable that if nuptiality patterns have been changing, the proportion single at age 30, say, may be smaller than observed at age 40, a situation that would never arise among members of the same birth cohort. Therefore, when there exist rapidly changing marriage patterns, the singulate mean age at marriage should not be calculated directly from the distribution by marital status and by age that is observed at a single census or survey. The proportions single in a hypothetical cohort exposed to marriage rates between two surveys or censuses need to be calculated before the value of *SMAM* is computed.

1. *Computational procedure*

The basic steps in the calculations are the same as those described before, except that the distribution of a hypothetical cohort by marital status has to be constructed before calculations of person-years lived are undertaken. Only those steps which are different from the steps described in subsection B.2 are described below.

Step 1: calculation of proportions single for two points in time separated by five or 10 years. Proportions single are calculated for each age group i and time-point j by dividing the single population by the total population in the age group, adjusted, if necessary, for non-response. The resulting proportions are denoted by $U(i, j)$.

Step 2: calculation of proportions single in a hypothetical cohort exposed to intersurvey first-marriage rates. Let the proportion single for age group i of the hypothetical cohort be denoted by $U(i, s)$. Equations (C.1) and (C.2) are used to calculate the values of $U(i, s)$ when the intersurvey interval is five years in length, whereas equations (C.3), (C.4) and (C.5) are used when the intersurvey interval is 10 years in length. For age groups 20-24 and above, $i = 2, \ldots, 8$.

$$U(i, s) = U(i-1, s)[U(i, 2)/U(i-1, 1)]. \qquad \text{(C.1)}$$

For the age group 15-19, for which $i = 1$,

$$U(1, s) = U(1, 2). \qquad \text{(C.2)}$$

Note that in equation (C.1) the values $U(i, 2)$ and $U(i-1, 1)$ refer to the same birth cohort, so that their quotient is a form of "survival rate" in the single state for that cohort. When this survival rate is applied to the previous hypothetical value, $U(i-1, s)$, a transformation in accordance with the observed change in the proportion single during the intersurvey period.

If the intersurvey period is 10 years in length instead of five, it is still possible to calculate the proportions single in a hypothetical cohort. Now

$$U(i, s) = U(i-2, s)[U(i, 2)/U(i-2, 1)]. \qquad \text{(C.3)}$$

for values of i ranging from 3 (age group 25-29) to 8 (age group 50-54). For i equal to 1 and 2, that is, for age groups 15-19 and 20-24, $U(i, s)$ is calculated as the average proportion single at the time of the two censuses or surveys:

$$U(1, s) = 0.5(U(1, 1) + U(1, 2)) \qquad \text{(C.4)}$$

and

$$U(2, s) = 0.5(U(2, 1) + U(2, 2)). \qquad \text{(C.5)}$$

Steps 3-6: calculation of singulate mean age at marriage from the proportions single in a hypothetical cohort. Once the proportions single, $U(i, s)$, have been calculated, the procedure to be followed in calculating the value of *SMAM* is exactly the same as that described in steps 2-5 of subsection B.2.

2. *Fictitious detailed example*

Table 190 shows data for the female population of Japan enumerated in 1955 ($j = 1$) and 1960 ($j = 2$). It illustrates how the data are used to estimate proportions single for a hypothetical cohort. Column (6) shows the resulting set of $U(i, s)$ values. In the steps out-

TABLE 190. FEMALE PROPORTIONS SINGLE, 1955 AND 1960, AND ESTIMATED PROPORTIONS SINGLE IN A HYPOTHETICAL INTERSURVEY COHORT, JAPAN

Age group (i)	Index (j)	Proportion single		Ratio of proportions single at two surveys	Proportion single in hypothetical cohort
		1955	1960	$U(i, 2)/U(i-1, 1)$	$U(i, s)$
	(1)	(2)	(3)	(4)	(5)
10-14	0	(1.000)[a]	(1.000)[a]	-	-
15-19	1	0.983	0.987	0.987	0.987
20-24	2	0.665	0.648	0.659	0.657
25-29	3	0.206	0.212	0.319	0.209
30-34	4	0.079	0.096	0.466	0.097
35-39	5	0.039	0.056	0.709	0.069
40-44	6	0.023	0.031	0.795	0.055
45-49	7	0.017	0.019	0.826	0.045
50-54	8	0.012	0.017	1.000	0.045

[a] Assumed value.

lined below, these values are used to estimate an average value of *SMAM* for the period 1955-1960.

Step 1: calculation of proportions single for each census. Since in this case the values of $U(i, j)$ are already in the appropriate form in columns (3) and (4) of table 190, this step is unnecessary.

Step 2: calculation of proportions single in a hypothetical cohort exposed to intersurvey first-marriage rates. In this example, the intersurvey period is five years, so equations (C.1) and (C.2) are used in calculating $U(i, s)$. For age group 15-19, for which $i = 1$,

$$U(1, s) = U(1, 2) = 0.987.$$

For subsequent age groups, each value of $U(i-1, s)$ is multiplied by the ratio $U(i, 2)/U(i-1, 1)$. Column (5) of table 190 shows the values of these ratios for each value of i. For example, for $i = 2$,

$$U(2, s) = U(1, s)U(2, 2)/U(1, 1)$$

$$= U(1, s)(0.648/0.903)$$

$$= (0.987)(0.659) = 0.650.$$

Full results are shown in column (6) of table 190.

Step 3: calculation of person-years lived in the single state. The sum of the first seven values of $U(i, s)$ is 2.111, so that

$$RS_2 = 5.0(2.111) = 10.555.$$

Note that the value of $U(8, s)$ for ages 50-54 was not included in the sum. Then, the number of person-years desired is

$$RS_3 = 15.0 + RS_2 = 25.555.$$

Step 4: calculation of proportion ever marrying. The proportion remaining single is

$$RN = (0.045 + 0.045)/2.0 = 0.045.$$

Hence, the proportion ever marrying equals

$$RM = 1.0 - 0.045 = 0.955.$$

Step 5: calculation of person-years lived by those remaining single. These person-years are calculated as

$$RS_4 = 50.0RN = 50.0(0.045) = 2.25.$$

Step 6: calculation of mean age at marriage. The value of *SMAM* is calculated as

$$SMAM = (RS_2 - RS_4)/RM = (25.555 - 2.25)/0.955 = 24.40.$$

It is of interest to compare the value of *SMAM* estimated for the intercensal period (24.40 years) with the estimates that would have been obtained by considering each census separately. According to the proportions single presented in columns (3) and (4) of table 190, the value of *SMAM* in 1955 was 24.74 years, while that in 1960 was 24.79 years. Therefore, although the difference is relatively small, the intercensal estimate of *SMAM* is lower than those obtained at the end-points of the period. This outcome implies, in general, that according to the first-marriage rates prevalent during the intercensal period, women would marry at slightly younger ages than they would according to the mixed cohort-period experience observed at a given point in time. However, the difference observed is so small that, at least in this case, what ought to be stressed is the similarity of the different estimates and the fact that, according to them, the singulate mean age at marriage in Japan during the period 1955-1960 remained almost constant.

3. *Second detailed example*

Table 191 shows data for the female population of Tunisia as enumerated in 1966 ($j = 1$) and as estimated for 1976 on the basis of the 1975 census ($j = 2$). It indicates how the data are used to estimate proportions single for a hypothetical cohort. Column (6) shows the resulting set of $U(i, s)$ values. The steps outlined below illustrate how these data are used to estimate an average value of *SMAM* for the period 1966-1976:

Step 1: calculation of proportions single. In this case, the values of $U(i, j)$ are already in the appropriate form in columns (3) and (4) of table 191. Therefore, this step is not necessary.

Step 2: calculation of proportions single in a hypothetical cohort exposed to intersurvey first-marriage rates. Although the true intercensal period in Tunisia was nine years in length, the basic data have been adjusted to represent a 10-year period. Hence, equations (C.3), (C.4) and (C.5) can be used to calculate $U(i, s)$. Thus, for age group 15-19, for which $i = 1$, for example,

$$U(1, s) = 0.5(U(1, 1) + U(1, 2)) = 0.5(0.810 + 0.887) = 0.848;$$

and for age group 20-24, for which $i = 2$,

$$U(2, s) = 0.5(U(2, 1) + U(2, 2)) = 0.5(0.270 + 0.473) = 0.372.$$

For subsequent age groups, each value of $U(i-2, s)$ is multiplied by the ratio $U(i, 2)/U(i-2, 1)$. Column (5) of table 191 shows the value of these ratios for each value of i. For example, for $i = 3$,

$$U(3, s) = U(1, s)U(3, 2)/U(1, 1)$$

$$= U(1, s)(0.158/0.810)$$

$$= (0.848)(0.195) = 0.165.$$

Full results are shown in column (6) of table 191.

TABLE 191. FEMALE PROPORTIONS SINGLE, 1966 AND 1976, AND ESTIMATED PROPORTIONS SINGLE IN A HYPOTHETICAL INTERSURVEY COHORT, TUNISIA

Age group (I)	Index j (2)	Proportion single: 1966 (3)	Proportion single: 1976 (4)	Ratio of proportions single at two surveys $U(i, 2)/U(i-2, 1)$ (5)	Proportion single in hypothetical cohort $U(i, s)$ (6)
15-19	1	0.810	0.887	0.887	0.848
20-24	2	0.270	0.473	0.473	0.372
25-29	3	0.087	0.158	0.195	0.165
30-34	4	0.039	0.054	0.200	0.074
35-39	5	0.024	0.024	0.276	0.046
40-44	6	0.018	0.017	0.436	0.032
45-49	7	0.015	0.016	0.667	0.031
50-54	8	0.015[a]	0.016[a]	0.889	0.028

Sources: For proportions single in 1966, population census of 1966; for proportions single in 1976, estimates from the 1975 population census by M. Bambiet, *Evolution récente de la nuptialité et de la fécondité en Tunisie* (Tunis, Office national du planning familial et de la population, May 1977).

[a] Assumed equal to value for ages 45-49.

Step 3: calculation of person-years lived in the single state. The sum of the first seven values of $U(i, s)$ is 1.568, so that

$$RS_3 = 5.0(1.568) = 7.840.$$

Note that the value of $U(0, s)$ for ages 50-54 was not included in the sum. Then the number of person-years desired is

$$RS_2 = 15.0 + RS_3 = 22.840.$$

Step 4: calculation of proportion ever marrying. An estimate of the proportion remaining single at age 50 is

$$RM = (0.031 + 0.028)/2.0 = 0.0295.$$

Hence, the proportion ever marrying by age 50 equals

$$RM = 1.0 - 0.0295 = 0.9705.$$

Step 5: calculation of person-years lived by those remaining single at age 50. This value is calculated as

$$RS_1 = 50.0RM = 50.0(0.0295) = 1.475.$$

Step 6: calculation of singulate mean age at marriage. The value of *SMAM* is

$$SMAM = (RS_2 - RS_1)/RM = (22.840 - 1.475)/0.9705 = 22.01.$$

If this value of *SMAM* is compared with the estimates that would be obtained by using the data as recorded at the end-points of the period being considered (20.88 years in 1966 and 22.71 years in 1976), it appears that the intercensal value of *SMAM* is higher than the mean of the estimates at the end-points. This outcome suggests that first-marriage rates declined substantially and fairly rapidly in Tunisia during the period 1966-1976.

Annex II

THE EL-BADRY CORRECTION FOR DATA ON CHILDREN EVER BORN

A. Background of method

During the collection or the processing of information on children ever born, an error consisting of the misclassification of women of zero parity as women whose parity is not stated sometimes arises. This error is often due to the ambiguous entries made by interviewers at the time of the survey. For example, ambiguity is clearly present when a dash (-) is used to indicate both that parity information about a given woman was not obtained or that she declared herself as childless. When errors of this type occur on a sufficiently large scale, the quality of the entire set of information on children *ever born* is compromised, especially if this type of information is to be used to estimate fertility indirectly by any of the methods described in chapter II. Whenever a considerable proportion of childless women are classified incorrectly in the category "parity not stated", the exclusion of these women from the denominator when calculating average parities by age would lead to overestimates of average parity, especially at younger ages. If, on the other hand, the women whose parity was not stated are not excluded, the denominator will be too large, producing underestimates of the true average parities.

In 1961, El-Badry[4] proposed a procedure to estimate the proportion of women belonging to the category "parity not stated" who should have been classified as childless. This procedure is based on the observed high correlation existing between the proportions childless and the proportions in the category "parity not stated", especially when only the first four or five five-year age groups are considered.

El-Badry's method is based on a very simple model. Let $Z^*(i)$ be the true proportion childless in age group i and $NS(i)$ be the reported proportion of women classified in the category of parity not stated for the same age group. Then, it is plausible to assume that the following relation holds:

$$NS(i) = \alpha Z^*(i) + \beta \qquad (A.1)$$

where α is the proportion of women who are truly childless and who were erroneously classified as of "parity not stated", and β is the constant, true level not stated. (It is expected that, in every survey, even in the absence of the coding error being discussed, some women will be classified as not having stated their parity. Usually, these are women who were not present at the time of the interview and for whom the selected informant did not know the true number of children ever born). Since the proportion $\alpha Z^*(i)$ of childless women has been misclassified, the reported proportion in age group i, $Z(i)$, must be equal to

$$Z(i) = (1.0 - \alpha) Z^*(i) \qquad (A.2)$$

or

$$Z^*(i) = Z(i)/(1.0 - \alpha). \qquad (A.3)$$

Substituting equation (A.3) in equation (A.1), one finds that

$$NS(i) = [\alpha/(1.0 - \alpha)] Z(i) + \beta \qquad (A.4)$$

or

$$NS(i) = \gamma Z(i) + \beta \qquad (A.5)$$

where $\gamma = \alpha/(1.0 - \alpha)$. Equation (A.5) suggests a method of estimating the true value of $Z^*(i)$: fit a line to the reported points $[Z(i), NS(i)]$ and estimate γ and β. Then the true proportion childless is

$$Z^*(i) = Z(i) + (NS(i) - \beta), \qquad (A.6)$$

and an estimate of the zero-parity error, α, can be obtained by

$$\alpha = \gamma/(1.0 + \gamma) \qquad (A.7)$$

Note that even though algebraically equation (A.6) is equivalent to

$$Z^*(i) = (1.0 + \gamma) Z(i), \qquad (A.8)$$

equations (A.8) and (A.6) may (and usually do) produce slightly different results in practice, since the fit obtained is very rarely exact. For the purpose of adjustment of the zero category, equation (A.6) is recommended.

In most cases, when the required final product is a best estimate of average parities by age group of women, an adjustment of the zero-parity category is not of immediate use, but the true level of non-response is. Once β, an estimate of this level, has been calculated, the female population to be used as denominator in computing average parities by age group can be obtained by multiplying the reported female population in each age group by the factor $(1.0 - \beta)$. Thus, the required output from the El-Badry correction procedure will very often be only β, obtained by fitting equation (A.5).

To conclude, it must be noted that slightly different computational procedures have to be followed according to whether the data on children ever born refer to all women or only to ever-married women. Therefore, two detailed descriptions of the computational procedure are presented in the following sections. It is also useful to note now that in the following discussions, index i is used to denote the age group from $5i + 10$ to $5i + 14$.

B. Data required

The following data are required for this method:

(*a*) The number of women in the zero-parity category (childless women) classified by five-year age group;

(*b*) The number of women in the category "parity not stated" classified by five-year age group;

(*c*) The total number of women classified by five-year age group. However, when the question on parity was asked only of ever-married women, the data required include both the total number of women and the total number of ever-married women in each five-year age group (15-49).

C. Data on children ever born referring to all women

1. *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of proportion with parity not stated. If one denotes by $FP(i)$ the total number of women in age group i and by $FNS(i)$ the number of women whose parity was not stated and who belong to age group i, then the proportion whose parity was not stated is

$$NS(i) = FNS(i)/FP(i) \qquad (C.1)$$

Step 2: calculation of proportion childless. Letting $FZ(i)$ be the reported number of childless women in age group i and $FP(i)$ be as defined above, then the proportion childless is

$$Z(i) = FZ(i)/FP(i). \qquad (C.2)$$

[4] M. A. El-Badry, "Failure of enumerators to make entries of zero errors in recording childless cases in population censuses", *Journal of the American Statistical Association*, vol. 56, No. 296 (December 1961), pp. 909-924.

Step 3: estimation of linear parameters γ *and* β. Once the proportions of women whose parity was not stated and those of childless women have been calculated, the points $[Z(i), NS(i)]$ should be plotted. If they follow approximately a straight line, the use of this method of adjustment is warranted. Otherwise, no adjustment is possible and the user is advised to include all those women classified as "parity not stated" in the denominator when computing average parities. Of course, if the proportion of women with unknown parity in each age group is small, their inclusion or exclusion from the denominator is not likely to introduce serious biases in the average-parity values.

When the plotted points do fall roughly on a straight line, the values of its parameters have to be computed. It is recommended that only the points corresponding to the first four or five age groups be considered. To estimate the parameters of the straight line approximating them, divide the selected points into two groups of equal size according to their abscissa values (repeat the middle point if necessary) and calculate the means of the abscissae and of the ordinates in each group. The parameters of the line passing through these mean points are estimates of the desired γ and β values. The application of this procedure is illustrated in the detailed examples that follow.

When the points are fairly well aligned, the use of the least-squares method to estimate γ and β is, of course, also possible and may yield the best results. See chapter V, subsection C.4 for a description of this method of line-fitting.

Step 4: calculation of true proportions childless. As equation (A.6) indicates, the true proportion childless, $Z^*(i)$, is equal to the sum of the reported proportion childless, $Z(i)$, and the deviation of the reported proportion whose parity was not stated from the estimated constant level, that is, $(NS(i)-\beta)$. Therefore, once β has been computed, the calculation of $Z^*(i)$ is straightforward. Once the estimated $Z^*(i)$ values are available, the true number of childless women, $FZ^*(i)$, can be obtained as the product of each $Z^*(i)$ and the reported female population of age group i, $FP(i)$. The values of $FZ^*(i)$ obtained in this way should be used when fertility is to be estimated by using data on first births (see chapter II, subsection B.3).

Step 5: calculation of denominators for estimation of average parities. Let $FP^*(i)$ denote the number of women whose parity is assumed to be known after the true proportion childless has been estimated. Then

$$FP^*(i) = (1.0-\beta)FP(i) \qquad \text{(C.3)}$$

since, according to the model presented so far, the true level of non-response is the estimated value of β. When data on children ever born are being used to estimate fertility indirectly by the methods described in chapter II or to estimate child mortality by those described in chapter III, the average parity of women of age group i should be calculated by using $FP^*(i)$ as the denominator instead of the reported number $FP(i)$.

2. *Detailed example*

The census carried out in Peru in July 1972 collected information from all women on the number of children they had ever borne, and the necessary information to apply the El-Badry adjustment procedure was published. It is shown in table 192.

TABLE 192. FEMALE POPULATION ACCORDING TO DIFFERENT RESPONSES CONCERNING CHILDREN EVER BORN, PERU, 1972

(Population in thousands)

		Female population			Proportions	
Age group (1)	*Index* i (2)	*Total* $FP(i)$ (3)	*Children* $FZ(i)$ (4)	*Parity not stated* $FNS(i)$ (5)	*Children* $Z(i)$ (6)	*Parity not stated* $NS(i)$ (7)
15-19	1	698.2	242.2	385.7	0.3469	0.5524
20-24	2	578.6	117.3	198.3	0.2027	0.3427
25-29	3	471.5	51.1	93.6	0.1084	0.1985
30-34	4	381.4	27.0	50.4	0.0708	0.1321
35-39	5	373.3	22.7	40.0	0.0608	0.1072
40-44	6	297.8	18.6	29.9	0.0625	0.1004
45-49	7	246.2	16.3	24.3	0.0662	0.0987

A quick examination of the published numbers (those shown in columns (3), (4) and (5)) reveals that a very significant proportion of all women in each age group was classified in the category of "parity not stated". Especially striking is the fact that in age group 15–19 there are more women in that category than in the childless group. All these observations appear to be consistent with the existence of a zero-parity error of the type described in section A. Adjustment by means of El-Badry's procedure should be attempted.

Step 1: calculation of proportion with parity not stated. According to equation (C.1), the proportion of women whose parity was not stated in age group i, denoted by $NS(i)$, is just the quotient of the number of women in that category in that age group and the total female population of the same age group. Therefore, $NS(i)$ is obtained by dividing each of the entries in column (5) (women with parity not stated) by the corresponding entry in column (3) (total female population) of table 192. For example, for age group 20-24,

$$NS(2) = 198.3/578.6 = 0.3427.$$

The complete set of $NS(i)$ values is shown in column (7) of table 192.

Step 2: calculation of proportions childless. According to equation (C.2), the proportion childless in each age group, $Z(i)$, is obtained by dividing the reported number of childless women (column (4) of table 192) by the total number of women in each age group (column (3) of the same table). For instance,

$$Z(3) = 51.1/471.5 = 0.1084.$$

The full set of $Z(i)$ values is shown in column (6) of table 192.

Step 3: estimation of linear parameters, β *and* γ. Figure 24 shows a plot of the first five points $[Z(i), NS(i)]$ calculated in the previous steps. The $Z(i)$ values are plotted on the x-axis, while those for $NS(i)$ appear on the y-axis. Even in the largely expanded scale used in plotting these points, they display a fairly linear trend. The point farthest right (corresponding to age group 15-19) is the one that deviates most markedly from the trend defined by the others. However, given its importance (the proportion childless is highest for that age group), it is not eliminated from the calculations leading to the selection of a best fitting straight line.

Two methods of fitting are illustrated. The first method is based on group means, according to which the points $[Z(i), NS(i)]$ are divided into two groups, each containing three points. Table 193 shows this division and the means in each group. Using these mean values, γ is estimated as follows:

$$\gamma_1 = (\overline{NS}_2 - \overline{NS}_1)/(\overline{Z}_2 - \overline{Z}_1)$$

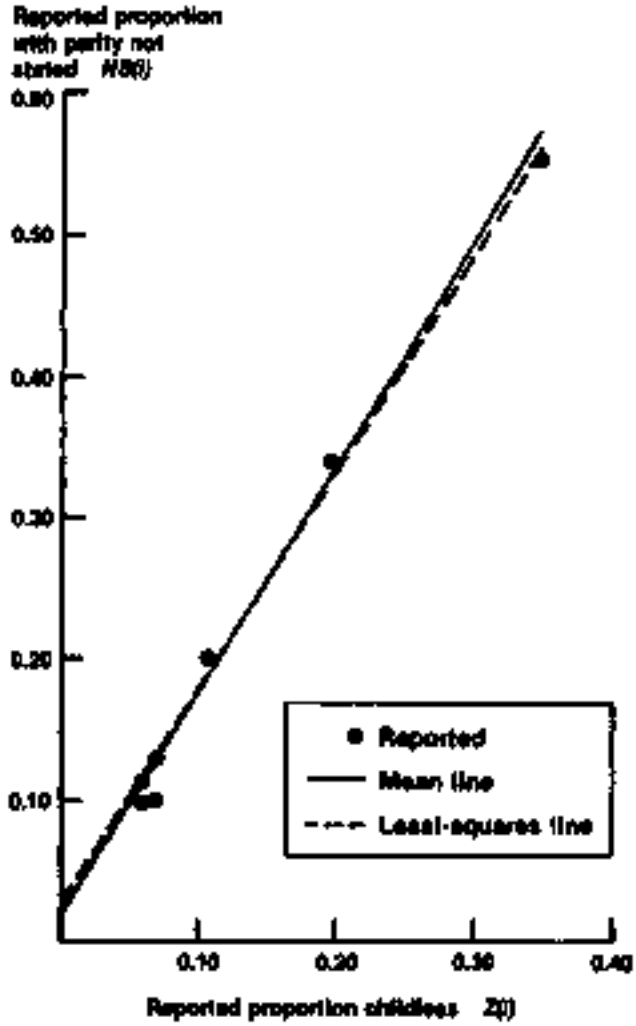
$$= (0.1459 - 0.3685)/(0.0600 - 0.2193) = 1.5693$$

The value of β is then calculated as

$$\beta_1 = 0.1459 - (1.5693)(0.0600) = 0.0204.$$

The line whose parameters are γ_1 and β_1 is represented as the mean line in figure 24.

Figure 24. Estimation of the true level of non-response on the subject of children ever born, Peru, 1972



Source: Based on data given in table 192.

For the sake of illustration, another line is fitted using the method of least squares. According to this method, the values of γ and β are obtained using the following equations:

$$\gamma_2 = \left[\sum_{i=1}^{5}(Z(i)-\bar{Z})(NS(i)-\overline{NS})\right] / \left[\sum_{i=1}^{5}(Z(i)-\bar{Z})^2\right] \qquad (C.4)$$

and

$$\beta_2 = \overline{NS} - \gamma_2 \bar{Z} \qquad (C.5)$$

where $\bar{Z}$ and $\overline{NS}$ are the means of the reported proportions of childless women and of those whose parity was not stated, respectively.

The calculations necessary to estimate γ_2 and β_2 are not carried out in detail. Only the most important intermediate results are given. Note that only the first five age groups are used in this case:

$$\bar{Z} = 0.1579 \text{ and } \overline{NS} = 0.2666$$

$$\sum_{i=1}^{5}(Z(i)-\bar{Z})(NS(i)-\overline{NS}) = 0.0880$$

$$\sum_{i=1}^{5}(Z(i)-\bar{Z})^2 = 0.0572.$$

TABLE 193. FITTING OF A STRAIGHT LINE BY USING GROUP MEANS: PERU, 1972

Age group (i)	Reported proportion childless Z(i)	Reported proportion with parity not stated NS(i)
	(a) *First group*	
15-19	0.3469	0.5524
20-24	0.2023	0.3427
25-29	0.1084	0.1985
Mean	$\bar{Z}_1 = 0.2193$	$\overline{NS}_1 = 0.3645$
	(b) *Second group*	
25-29	0.1084	0.1985
30-34	0.0708	0.1321
35-39	0.0608	0.1072
Mean	$\bar{Z}_2 = 0.0800$	$\overline{NS}_2 = 0.1459$

so that

$$\gamma_2 = 0.0880/0.0572 = 1.5385$$

and

$$\beta_2 = 0.2666 - (1.5385)(0.1579) = 0.0236.$$

The least-squares line defined by these parameters is also shown in figure 24. Note that the two fitted lines almost coincide. Therefore either of the two can be chosen as representative of the reported points. In carrying out the next steps, the least-squares line is used as representative.

Step 4: calculation of true proportions childless. Equation (A.6) is used to calculate the true proportion childless, $Z^*(i)$, from the reported $Z(i)$, $NS(i)$ and the estimated β. $Z^*(3)$, for example, is

$$Z^*(3) = Z(3) + NS(3) - \beta_2$$

$$= 0.1084 + 0.1985 - 0.0236 = 0.2833.$$

The complete set of $Z^*(i)$ values is shown in column (3) of table 194. Note that according to these estimates of the true proportions childless, approximately 14 per cent of all women in age group 40-49 had zero parity in 1972. This value appears rather high for a country like Peru, where marriage is probably nearly universal and where there is no reason to expect a higher-than-normal incidence of sterility. These high values of $Z^*(i)$ at older ages may indicate that this type of adjustment is not adequate for the data at hand. Hence, before adopting these results for future use, it would be advisable to consider the observed proportions single at each age so as to assess the plausibility of the estimated proportions childless. The search for any other reasons that might explain the high proportions of women with parity not stated yielded by the census is also recommended.

When El-Badry's adjustment procedure is applied to data that will later serve to estimate fertility by using information on first births (see chapter II, subsection 8.3), the actual number of ever-fertile women (not childless) is required. This number can be estimated by subtracting the estimated proportions childless from one and then multiplying the resulting number by the total female population. For example, the ever-fertile women in age group 20-24, $FF(2)$, would be

$$FF(2) = (1.0 - 0.5218)(578.6) = 276.7$$

or equivalently,

$$FF(2) = FP(2) - FZ^*(2) = 578.6 - 301.9 = 276.7$$

Step 5: calculation of denominators for estimation of average parities. The value of FP^* is calculated according to equation (C.3). Thus

$$FP^*(2) = (1.0 - \beta_2)FP(2) = (1.0 - 0.0236)(578.6) = 564.9.$$

Other values of $FP^*(i)$ are shown in column (6) of table 194.

Note that even though in the first three steps of this procedure attention is focused mainly on the first five age groups, all seven

TABLE 194. ESTIMATED PROPORTIONS CHILDLESS AND ESTIMATED TOTAL FEMALE POPULATION WITH KNOWN PARITY, PERU, 1972

(Population in thousands)

Age group (i)	Index i (2)	Estimated true proportion childless $Z^*(i)$ (3)	True number of childless women $FZ^*(i)$ (4)	Number of ever-married women $EMP(i)$ (5)	Estimated number of women with known parity $FP^*(i)$ (6)
15-19	1	0.8757	611.4	86.8	681.7
20-24	2	0.5218	301.9	276.0	564.9
25-29	3	0.2833	133.6	337.9	460.4
30-34	4	0.1793	68.4	343.0	372.4
35-39	5	0.1444	53.9	319.4	364.5
40-44	6	0.1393	41.5	256.1	290.8
45-49	7	0.1413	34.8	211.4	290.4

groups are required for the purpose of fertility estimation. Table 194 refers therefore to all of them.

As pointed out before, if one is satisfied with the consistency of the estimated values obtained in this and in previous steps, future fertility or child mortality estimation should make use of the estimated rather than the reported values. Therefore, the $FP^*(i)$ estimates should be used as denominators when calculating average parities.

D. Data on children ever born referring to ever-married women

In many countries, questions on fertility are asked only of ever-married women, on the assumption that all single women are childless. To the extent to which this assumption is true, fertility for the total female population can be estimated from data referring exclusively to ever-married women; but, especially when El-Badry's adjustment procedure is applied, due allowance must be made for the fact that the parity of single women is not reported. For this reason, the computational procedure one must follow when fertility information is available only for ever-married women is described in some detail.

1. *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of proportions with parity not stated. If one denotes by $EMP(i)$ the number of ever-married women in the age group from $5i+10$ to $5i+14$, and by $FNS_0(i)$ the number of ever-married women of the same age group whose parity was not stated, then the proportion with parity not stated is

$$NS_0(i) = FNS_0(i)/EMP(i). \tag{D.1}$$

Step 2: calculation of proportions childless. Denoting by $FZ_0(i)$ the proportion of childless women among those ever-married in age group i and letting $EMP(i)$ be as defined above, the proportion childless is

$$Z_0(i) = FZ_0(i)/EMP(i) \tag{D.2}$$

Step 3: estimation of linear parameters γ and β. As in the case where data for all women are available, a graph of the points $[Z_0(i), NS_0(i)]$ should be made. If they fall approximately in a straight line, its parameters, γ and β, should be estimated by using either group means or the least-squares method (see chapter V, subsection C.4). In general, only the first four or five age groups should be considered when fitting the line.

Step 4: calculation of true proportions childless among ever-married women. The true proportion childless among ever-married women of age group i, $Z_0^*(i)$, is obtained by adding to the reported proportion $Z_0(i)$ the deviation of the proportion whose parity is not stated from its estimated constant level, $(NS_0(i)-\beta)$; algebraically,

$$Z_0^*(i) = Z_0(i) + NS_0(i) - \beta. \tag{D.3}$$

Then, by multiplying this estimated true proportion childless by the corresponding number of ever-married women, an estimated number of childless ever-married women, FZ_0^*, can be obtained as

$$FZ_0^*(i) = Z_0^*(i)EMP(i). \tag{D.4}$$

However, this number refers only to zero-parity women among those who have ever been married. The total number of childless women in age group i, denoted by $FZ^*(i)$, can be obtained only by making some assumption about the fertility of single women. If one accepts that all single women are childless, then

$$FZ^*(i) = FZ_0^*(i) + (FP(i) - EMP(i)) \tag{D.5}$$

where $FP(i)$ is the total number of women in age group i. Since only very rarely does one have some basis for making other assumptions about the childbearing experience of never-married women, it is recommended that equation (D.5) be used to estimate the number of childless women among all women in age group i. This number, $FZ^*(i)$, should be used if fertility estimation is to be attempted by using the first-birth method described in chapter II, subsection B.3.

Step 5: calculation of denominators for estimation of average parities. Let $FP^*(i)$ denote the number of women whose parity can be assumed to be known after the true number of childless women has been estimated. Then

$$FP^*(i) = FP(i) - \beta EMP(i); \tag{D.6}$$

that is, the number of women of known parity equals the total number of women, $FP(i)$, minus those among ever-married women whose parity is unknown. The value of $FP^*(i)$ defined by equation (D.6) should be used as a denominator whenever average parities are required to estimate fertility or child mortality.

2. *Detailed example*

The case of Thailand in 1970 is interesting because during the census carried out in that year the question on children ever born was asked only of ever-married women. Table 195 shows the basic data collected during the 1970 census.

Steps 1 and 2: calculation of proportions with parity not stated and proportions childless. Column (4) of table 196 shows the proportions of ever-married women who did not state their parity, $NS_0(i)$; and column (3) gives the proportions childless, $Z_0(i)$, also among ever-married women. These proportions were computed using equations (D.1) and (D.2). As an example, $NS_0(3)$ and $Z_0(3)$ are calculated here:

$$NS_0(3) = 56.5/963.6 = 0.0586$$

$$Z_0(3) = 25.7/963.6 = 0.0267.$$

Step 3: estimation of linear parameters γ and β. Figure 25 shows a plot of the points $(Z_0(i), NS_0(i))$ listed in table 196. Note that only four points were plotted (those with i ranging from 2 to 5). The first point, corresponding to age group 15-19, falls outside the graph and is not aligned with the others. Therefore, for the purpose of estimating γ and β, this point is ignored. Using the least-squares method in fitting a line to the other four points, the following γ and β estimates are obtained:

$$\gamma = 1.4797$$

and

$$\beta = 0.0147.$$

TABLE 195. FEMALE POPULATION ACCORDING TO DIFFERENT RESPONSES CONCERNING CHILDREN EVER BORN, THAILAND, 1970

(Thousands)

Age group (1)	Index i (2)	Total number of women P(i) (3)	Ever-married women EM(i) (4)	Women with own parity PZ(i) (5)	Women with parity not stated NS(i) (6)
15-19	1	1 885.4	357.6	148.7	23.6
20-24	2	1 361.7	843.6	59.2	90.2
25-29	3	1 143.4	963.6	25.7	56.5
30-34	4	1 077.1	989.6	15.4	37.0
35-39	5	957.6	906.6	11.6	28.4

Step 4: calculation of true proportions childless. Column (3) of table 197 shows the proportion childless estimated according to equation (D.3). For example, for age group 20-24,

$$Z_0^*(2) = 0.0702 + 0.1176 - 0.0147 = 0.1731.$$

Once the estimated true proportions childless are computed, the estimated number of childless women among the ever-married $FZ_0^*(i)$, can be obtained by using equation (D.4). Thus, for age group 20-24,

$$FZ_0^*(2) = (0.1731)(843.6) = 146.0$$

Then, the estimated number of childless women among all of the women is computed by adding the number of single women to FZ_0^*.

Therefore,

$$FZ^*(2) = 146.0 + 1361.7 - 843.6 = 664.1.$$

All FZ^* values are shown in column (5) of table 197. These values should be used in estimating the true proportion childless among all women when the first-birth method of fertility estimation is used (see chapter II, subsection B.3).

Step 5: calculation of denominators for estimation of average parities. Using equation (D.6), FP^* is estimated from the total number of women and the ever-married women of age group i. For example, for age group 20-24,

TABLE 196. REPORTED PROPORTIONS CHILDLESS AND OF PARITY NOT STATED AMONG EVER-MARRIED WOMEN, THAILAND, 1970

Age group (1)	Index i (2)	Reported proportion childless $Z_0(i)$ (3)	Reported proportion with parity not stated $NS(i)$ (4)
15-19	1	0.4158	0.0660
20-24	2	0.0702	0.1176
25-29	3	0.0267	0.0586
30-34	4	0.0160	0.0374
35-39	5	0.0128	0.0313

$$FP^*(2) = 1361.7 - 0.0147(843.6) = 1349.3.$$

The complete set of FP^* values is shown in column (6) of table 197. These values should be used as denominators when calculating average parities.

Figure 25. Estimation of the true level of non-response on the subject of children ever born, Thailand, 1970

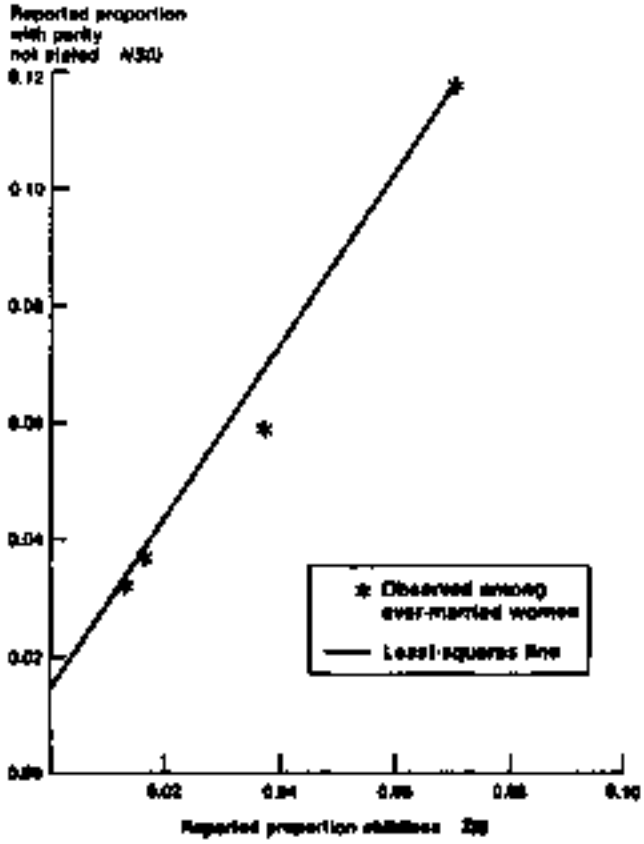


TABLE 197. ADJUSTED DATA USING PROPORTIONS AMONG EVER-MARRIED WOMEN, THAILAND, 1970

(Population in thousands)

Age group (1)	Index i (2)	True proportion childless $Z_0(i)$ (3)	True number of childless ever-married women $FZ_0^*(i)$ (4)	Estimated number of children ever born $FE^*(i)$ (5)	Number of women with known parity $FP^*(i)$ (6)
15-19	1	0.4671	167.0	1 494.8	1 880.1
20-24	2	0.1731	146.0	664.1	1 349.3
25-29	3	0.0706	68.0	247.8	1 129.2
30-34	4	0.0387	38.3	125.8	1 062.6
35-39	5	0.0294	26.7	77.7	944.3

3. *Comments on detailed example*

To conclude, it should be pointed out that it is extremely important to know how the data on fertility were collected in order to choose a suitable adjustment procedure. Lack of information on the sources of the data being used may cause serious estimation errors. An example of one possible error is illustrated in table 198, which shows the adjusted proportions of childless women, their actual numbers and the adjusted number of women of known parity for Thailand in 1970. These values were estimated by assuming, incorrectly, that the fertility information referred to all women. Under this assumption, the computational procedure for data on all women (section C) was used to arrive at the estimates given in table 198. A line was fitted by using least squares to the four points ranging from age groups 2 to 5. The parameters γ and β estimated in this way have the values:

$$\gamma = 1.3539$$

and

$$\beta = 0.0152.$$

A comparison between the estimates of FZ^* and FP^* shown in table 198 and the correct values given in table 197 reveals that the two FP^* values are relatively similar, although the differences between the correct value and that from "all women" are not trivial. However, a great difference is evident between the two types of FZ^* estimates (the true number childless). This difference is easy to explain: the FZ^* values obtained when proportions calculated with respect to ever-married women are used as input incorporate the single women, while those obtained using proportions among all women do not incorporate them. Of course, if one knows that single women were assumed to be childless, the appropriate adjustment can be made. But knowledge of this assumption implies knowledge of the data sources, and such knowledge should allow the investigator to choose the appropriate adjustment procedure in the first place so that no such errors would arise.

TABLE 198. ADJUSTED DATA USING PROPORTIONS AMONG ALL WOMEN, BASED ON AN INCORRECT ASSUMPTION, THAILAND, 1970

(Population in thousands)

Age group (i)	Index (i)	Adjusted proportion childless $P(i)$ (3)	Adjusted proportion with parity one or more $M(i)$ (4)	True proportion childless $P^*(i)$ (5)	Estimated number childless $PZ^*(i)$ (6)	Number of women with known parity $P^*(i)$ (7)
15-19	1	0.0789	0.0125	0.0782	143.7	1 836.7
20-24	2	0.0435	0.0729	0.1012	137.9	1 341.0
25-29	3	0.0215	0.0494	0.0547	64.8	1 126.0
30-34	4	0.0147	0.0344	0.0339	36.5	1 060.7
35-39	5	0.0121	0.0297	0.0266	25.5	943.0

Annex III

THE MEAN AGE OF CHILDBEARING

A. Direct method for calculation of mean age of childbearing

1. *Basis of method and its rationale*

The mean age of childbearing is the average age at which women bear their children, in the absence of mortality and age-distribution effects. It is a measure of the timing of childbearing for a mortality-free cohort of women passing through their reproductive life, or, alternatively, for a set of period age-specific fertility rates.

If one denotes by $f(i)$ the fertility rate experienced by women in the age group from $5i + 10$ to $5i + 14$, the mean age of childbearing, μ, is defined by

$$\mu = \sum_{i=1}^{I} a(i)\, f(i) / \sum_{i=1}^{I} f(i) \qquad \text{(A.1)}$$

where $a(i)$ is the central age-point in the ith age interval, and I is the age group containing the upper age-limit of the childbearing span.

2. *Data required*

As equation (A.1) indicates, knowledge of the fertility schedule to which the population of interest is subject is sufficient to provide an estimate of μ. Since fertility rates are usually obtained by dividing the number of births occurring during a given time period by the person-years lived by the population subject to the risk of childbearing during that period, it follows that the data required are

(a) The total number of births occurring during a given year, classified by age of mother;

(b) The number of women at the mid-point of the year in question, classified by age.

Knowledge of these data would permit the calculation of period fertility rates from which a period mean age of childbearing can be calculated making use of equation (A.1).

When cohort fertility rates specific by age are available, a cohort mean age of childbearing can be computed. In some instances, the fertility rates available correspond neither to a well-defined time period nor to a single cohort. In such cases, even though a mean age of childbearing can still be computed, its use as a parameter indicating the timing of childbearing is valid only under the assumption of unchanging fertility. Bearing this in mind, the way in which μ is computed is discussed in the next section.

3. *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of fertility rates. The fertility rate for group i, denoted by $f(i)$, is calculated by dividing the number of births occurring to women in age group i by the total number of women in that age group.

Step 2: calculation of mean age of childbearing. The denominator of equation (A.1) is calculated by adding the fertility rates obtained in the previous step. Usually, the age groups considered range from that beginning at age 15 to that ending at age 49. Then the central age for each of the age groups considered is calculated and multiplied by the corresponding fertility rate. Addition of all these products (again using as limits of summation the age groups containing ages 15 and 49) yields the numerator of equation (A.1). Therefore, its division by the sum of the $f(i)$ values yields the value of μ.

B. Other methods for estimating mean age of childbearing

1. *Necessity of these methods*

Frequently, in countries lacking adequate data on fertility or mortality, the data required for the calculation of μ are either not available or are seriously flawed. Two methods for estimating the mean age of childbearing, μ, have been proposed for such cases. It is worth noting here that while equation (A.1) is exact (except for grouping errors) in the sense that it defines μ, the methods and equations presented below are only estimators of μ.

2. *Estimation method 1*[a]

Let $P(2)$ be the average parity per woman (number of children ever born divided by the number of women) reported by women aged 20-24 and $P(3)$ be the average parity of women aged 25-29. Then, by using linear regression, it was established that a good predictor of μ for populations where there is little practice of birth control is the ratio of $P(3)$ over $P(2)$ in accordance with the following equation:

$$\mu = 2.25(P(3)/P(2)) + 23.95. \qquad \text{(B.1)}$$

This estimate of μ is relatively accurate and very simple to obtain. Besides, it requires as input data only the values of $P(2)$ and $P(3)$, which are very frequently available.

3. *Estimation method 2*

When only the proportions of women married in each age group, $m(i)$, are known, a mean age of childbearing can be calculated by assuming that marital fertility has the same shape as natural fertility. This assumption is valid only for populations where there is very little practice of birth control within marriage. Furthermore, for the estimated mean age of childbearing to be representative of the entire population, illegitimacy must be rare.

The standard natural fertility schedule for married women is shown in table 199. Note that the value of the marital fertility rate at ages 15-19 depends upon the proportion of women married during those ages, $m(1)$. All other rates are fixed.

TABLE 199. STANDARD NATURAL FERTILITY SCHEDULE FOR MARRIED WOMEN

Age group (i)	Index i	Marital fertility rate $\gamma(i)$
15-19	1	$1.2 - 0.7\,m(1)$[b]
20-24	2	1.000
25-29	3	0.935
30-34	4	0.853
35-39	5	0.685
40-44	6	0.349
45-49	7	0.051

[b] Proportion married in age group 15-19.

If one denotes these model fertility rates by $\gamma(i)$, the rates for the population in question would be computed as the products of the proportions married and the marital fertility rates, that is, as

$$f(i) = \gamma(i)\, m(i);$$

and once f is obtained, the calculation of μ is carried out as described above in subsection A.3.

All these methods of estimation are used in the following example.

[a] Based on *Manual IV, Methods of Estimating Basic Demographic Measures from Incomplete Data* (United Nations publication, Sales No. E.67.XIII.2).

TABLE 200. PROPORTIONS OF WOMEN BY MARITAL STATUS, NUMBER OF WOMEN AND CHILDREN BORN IN 1961

Age group (i)	Index i (2)	Proportion single s(i) (3)	Proportion widowed and divorced w(i) (4)	Number of women (thousands) (5)	Number of children born in 1961 (6)
10-14	0	0.700	0.003	1 832	0
15-19	1	0.257	0.011	1 585	325
20-24	2	0.054	0.021	1 363	361
25-29	3	0.019	0.037	1 166	326
30-34	4	0.010	0.066	995	195
35-39	5	0.008	0.121	838	114
40-44	6	0.007	0.205	705	21
45-49	7	0.006	0.302	625	1

C. A DETAILED EXAMPLE

1. *Data used*

The data shown in table 200 correspond to a population enumerated in 1961. The mean age of childbearing is to be estimated for this population.

2. *Direct calculation of mean age of childbearing*

Step 1: calculation of fertility rates. Table 200 shows the number of women enumerated in 1961 and the number of children born in that year (obtained from vital registration data) classified by age of mother. The fertility rates, $f(i)$, for each age group can therefore be calculated simply by dividing the number of births in 1961 by the corresponding female population (this population was enumerated near the middle of the year, so that adjustment is not necessary). Values of $f(i)$ are listed in column (3) of table 201. For $i = 3$, $f(3)$ is calculated explicitly here:

$$f(3) = 326/1.166 = 0.2796$$

TABLE 201. CALCULATION OF MEAN AGE OF CHILDBEARING BY THE DIRECT METHOD

Age group (1)	Index i (2)	Fertility rate f(i) (3)	Central age a(i) (4)	Age-weighted fertility a(i)f(i) (5)
15-19	1	0.2050	17.5	3.59
20-24	2	0.2646	22.5	5.95
25-29	3	0.2796	27.5	7.69
30-34	4	0.1960	32.5	6.37
35-39	5	0.1360	37.5	5.10
40-44	6	0.0298	42.5	1.27
45-49	7	0.0036	47.5	0.08
TOTAL		1.1124		30.05

Step 2: calculation of mean age of childbearing. Column (4) of table 201 shows the central age values corresponding to each of the age groups considered and column (5) shows the products of these central ages by the observed $f(i)$ values. The sums of the latter and of the products just calculated are shown on the line marked "Total", that is,

$$\sum_{i=1}^{7} f(i) = 1.1124$$

and

$$\sum_{i=1}^{7} a(i) f(i) = 30.05.$$

So that, by equation (A.1),

$$\mu = 30.05/1.1124 = 27.01.$$

It is important to point out that only when the assumption of constant fertility holds true is this value of μ equivalent to that of a cohort. If the fertility schedules followed by different cohorts in the population are different, the value of μ calculated above is that of a composite parameter that does not refer to any cohort, but rather to the experience observed during a certain period (1961 in this case). In the latter case, inferences drawn from it must be treated with caution.

Furthermore, note that the central ages, $a(i)$, used in computing the mean age of childbearing, μ, are highly dependent upon the way in which information on births in a year is collected. When the number of births occurring in a given year is obtained by means of a sample survey or a census and the age of mothers is ascertained as of the time of the survey and not as of the date on which the most recent birth took place, it is clear that most mothers were younger at the time of this birth than at the time of the survey or census. If it can be assumed that births are uniformly distributed in time, women were, on the average, six months younger at the time their births in the past year took place. Therefore, the true age groups considered when using survey data are shifted by six months to the younger ages, becoming 14.5-19.5, 19.5-24.5 and so on, instead of the usual 15-19, 20-24 etc., and, in consequence, their "central ages", $a(i)$, are also six months smaller (17, 22, 27, 32, 37 etc.). In general, these central ages should be used when calculating μ from survey data.

3. *Estimation of mean age of childbearing using method 1*

Next to be estimated is the mean age of childbearing, μ, by means of the equation (B.1). For the population under consideration, it is known that $P(2) = 2.389$ and $P(3) = 3.812$, so that the calculation of μ is straightforward:

$$\mu = (2.25)(3.812/2.389) + 23.95 = 27.54.$$

Once more, this value of μ is equivalent to that of a cohort only if fertility has remained constant during the past 30 or 40 years. When fertility has been changing, μ calculated as given above represents, at best, a rough "average mean" for younger cohorts (those under age 30). In such a case, its value should be interpreted with caution.

Note that the estimate of μ obtained here is slightly greater than that obtained by the direct method, probably because equation (B.1) was obtained from data pertaining to populations that practise little birth control, and it is therefore likely to overestimate the mean age of childbearing in populations where some limitation of family size is common.

4. *Estimation of mean age of childbearing using method 2*

According to this method, marital fertility is assumed to have the same pattern as natural fertility (that is, it is assumed that no parity-dependent birth control is practised by the population in question). Under this assumption, a value of μ can be obtained from knowledge of the proportion of married women in each age group.

The proportion of married women in each age group is equal to one minus the proportion of women who are single, widowed or divorced. Thus,

$$m(i) = 1 - s(i) - w(i)$$

where $m(i)$ is the proportion married in age group i, $s(i)$ is the proportion single, and $w(i)$ is the proportion widowed or divorced in the same age group.

Column (3) of table 202 shows the values of $m(i)$. Column (4)

TABLE 202. ESTIMATION OF MEAN AGE OF CHILDBEARING, μ, BY USING METHOD 2

Age group (1)	Index i (2)	Proportion of women married $m(i)$ (3)	Standard natural fertility $\gamma(i)$ (4)	Calculated fertility $f(i)$ (5)	Central age $a(i)$ (6)	Age-weighted fertility $a(i)f(i)$ (7)
15-19	1	0.732	0.688	0.503	17.5	8.80
20-24	2	0.925	1.000	0.925	22.5	20.81
25-29	3	0.944	0.935	0.883	27.5	24.28
30-34	4	0.924	0.853	0.788	32.5	25.61
35-39	5	0.871	0.685	0.597	37.5	22.39
40-44	6	0.788	0.349	0.275	42.5	11.69
45-49	7	0.692	0.051	0.035	47.5	1.65
TOTAL				4.006		115.24

shows the values of $\gamma(i)$, the standard natural fertility schedule, copied from table 199. Note that $\gamma(1)$ was obtained as

$$\gamma(1) = 1.2 - 0.7(m(1)) = 1.2 - 0.7(0.732) = 0.688.$$

Column (5) shows the values of $f(i)$ as estimated by multiplying the entries for $m(i)$ in column (3) by $\gamma(i)$ in column (4). Using these values of $f(i)$ and the central ages, $a(i)$, listed in column (6), the products $a(i)f(i)$ are obtained. Then the value of μ is computed by dividing the sum of these products (shown as "Total" under column (7)) by the sum of the $f(i)$ values (shown as "Total" under column (5)):

$$\mu = 115.24/4.006 = 28.77.$$

As might be expected, this value of μ is higher than either of the two estimates obtained previously, because natural fertility, embodying unrestricted fertility at older ages, represents an upper bound to the fertility experience of a human population. In this sense, the value of μ just calculated is probably also an upper limit for the true value of μ.

In conclusion, by using the three possible methods of estimating the mean age of childbearing, μ, an equal number of different values for this parameter has been obtained. The estimate derived directly from observed data and the definition of μ is preferred, but were those data unavailable, method 1 would produce a relatively good estimate of the desired parameter. The value of μ obtained by assuming that marital fertility is closely approximated by natural fertility is, on the other hand, unacceptable. In general, method 2 is to be avoided if at all possible.

Annex IV

LINEAR INTERPOLATION

A. Computational procedure

Interpolation is used frequently in the *Manual*; and because simplicity is the main criterion when choosing the type of interpolation that is to be performed, the linear type is very often preferred.

It is well known that any two points define a line uniquely. Therefore, if three points are to lie on a straight line, but only one of the co-ordinates of the third is known, the other one is uniquely determined and can be calculated. This calculation is described below.

Consider the line defined by the points (x_1, y_1) and (x_2, y_2). A third point (x, y) would lie on this line only if the following relation holds:

$$(y_2-y_1)/(x_2-x_1)=(y-y_1)/(x-x_1). \qquad (A.1)$$

that is, only if the slope of the line defined by (x_1, y_1) and (x_2, y_2) is exactly the same as the slope of the one defined by (x_1, y_1) and (x, y).

Now suppose that the value of x is known, but not that of y. Solving for y in equation (A.1), the following expression is obtained:

$$y=(y_2-y_1)(x-x_1)/(x_2-x_1)+y_1. \qquad (A.2)$$

which allows the calculation of y if (x_1, y_1), (x_2, y_2) and x are known. However, equation (A.2) is not the simplest equation that one can use in calculating y. A better expression is obtained by rearranging its terms in the following way:

$$y=((x-x_1)/(x_2-x_1))y_2+$$

$$(1.0-(x-x_1)/(x_2-x_1))y_1. \qquad (A.3)$$

an expression that can be rewritten as

$$y=\Theta y_2+(1.0-\Theta)y_1 \qquad (A.4)$$

where

$$\Theta=(x-x_1)/(x_2-x_1) \qquad (A.5)$$

Equations (A.4) and (A.5) suggest a way in which linear interpolation can be performed in two simple steps. First, calculate the value of Θ, the "interpolation factor", solely by using the values of the observed abscissae (x-values). Then, use equation (A.4) to calculate the desired interpolated ordinate, y.

These steps are simple enough, but in following them it is important to match the indices properly. Note that although Θ is obtained by pivoting on x_1 (this value is the only one appearing in equation (A.5)), its value (that of Θ) is "applied to" or multiplied by y_2 in equation (A.4), while y_1 is multiplied by $(1.0-\Theta)$.

Some of the cases where linear interpolation arises in this *Manual* are illustrated in the following examples.

B. First example

The birth rate of a West model stable population at mortality level 19 and growth rate 0.0150 is 0.0264. If the growth rate were 0.0200, the value of the birth rate would be 0.0304. The question then is what the birth rate value is for a population with the same mortality level and a growth rate of 0.0169.

To answer this question by using linear interpolation, the two points defining a line must be identified first. The co-ordinates of each point are the pair of (growth rate, birth rate) values; thus,

$$(x_1, y_1)=(0.0150, 0.0264)$$

and

$$(x_2, y_2)=(0.0200, 0.0304).$$

The third point, that for which the ordinate is unknown, is (0.0169, y). According to this identification,

$$\Theta=(0.0169-0.0150)/(0.0200-0.0150)$$

$$=0.0019/0.0050=0.38$$

and therefore

$$y=(0.38)(0.0304)+(0.62)(0.0264)=0.0279.$$

Note that in this example the abscissae of the points defining the line were ordered by increasing value, that is, $x_1<x_2$. Furthermore, $x_1<x<x_2$ so that the value of Θ can be said to equal

$$\Theta=(\text{central}-\text{smallest})/(\text{largest}-\text{smallest}). \qquad (B.1)$$

Using this mnemonic notation, equation (A.4) can be transformed to

$$y=\Theta y(\text{largest})+(1.0-\Theta)y(\text{smallest}) \qquad (B.2)$$

where y(smallest) does not mean the smallest y value, but rather, the y value associated with the smallest x, and similarly, y(largest) is the value associated with the largest x value used.

These mnemonic expressions do not improve the clarity of equations (A.4) and (A.5), but they are included here in the hope that they may allow the user to perform linear interpolation correctly and in as mechanical a way as possible. They are used in the next example.

C. Second example

Among the West model life tables, the $q(2)$ value 0.0858 is associated with mortality level 17 for females, while at mortality level 15 the value of $q(2)$ for females is 0.1164. The question then is what the female value of $q(2)$ would be at level 15.36.

Since three different values for the level are cited, and only two are given for $q(2)$, the levels are taken as abscissae and the $q(2)$ values as ordinates. Then, according to mnemonic equation (B.1),

$$\Theta=(15.36-15)/(17-15)=0.36/2=0.18.$$

and by equation (B.2),

$$y=(0.18)(0.0858)+(0.82)(0.1164)=0.1109$$

Note that in the application of equation (B.2), $\Theta=0.18$ is multiplied by the value of $q(2)$ associated with the largest level used, namely 17, and not by the largest $q(2)$ value. Because in this case the largest level is associated with the smallest value of $q(2)$, the distinction is essential.

D. Third example

This example examines a case of extrapolation, rather than one of interpolation. Extrapolation takes place when the value of x does not fall between the values x_1 and x_2, but rather, is outside the interval defined by them. When this is the case, Θ is either negative or greater than one. Yet, equations (A.4) and (A.5) still hold true and may be used to calculate the missing value y.

Consider then the following problem. According to the West model life tables, at level 21 the expectation of life at birth for males is 66.02, while at level 23 it is 71.19. The question is what the value of e_0 would be at level 24.

Once more, the levels are taken to represent the abscissae values; therefore,

$$\Theta = (24-21)/(25-21) = 3/2 = 1.5,$$

a value, as expected, greater than one. In the second step, y is calculated as usual:

$$y = (1.5)(71.19) + (-0.5)(66.02) = 73.78.$$

Note that since Θ is greater than one, $(1.0-\Theta)$ is negative. Furthermore, because the values of life expectancy at birth increase as the level of mortality increases (that is, the slope of the line associating level with e_0 values is positive), it is expected that e_0 at level 24 would be greater than that at level 25. This common-sense check would immediately permit one to discard the value

$$y^* = (1.5)(66.02) + (-0.5)(71.19) = 63.44$$

that would be obtained if, by mistake the values Θ and $1.0 - \Theta$ were not matched to the right indices.

As a final comment, it should be pointed out that linear extrapolation is not recommended in most demographic applications, since the assumption of linearity outside the bounds of printed or otherwise known quantities is difficult to justify. Therefore, in choosing the values x_1 and x_2 that will be used as anchors for linear interpolation, one should select, if at all possible, those which enclose the reported value of x.

Annex V

SMOOTHING OF AN AGE DISTRIBUTION

A. Background of methods

Throughout this *Manual*, stress has been laid on the fact that age distributions are frequently distorted by age-misreporting. Although it is not recommended that age distributions be smoothed[a] or adjusted prior to applying any of the analytical procedures described in the main body of this *Manual*, on the grounds that their results may be distorted by the smoothing or adjustment process, an age distribution that is smooth and as close to correct as possible is still useful, particularly as a basis for population projections. This annex describes and illustrates some fairly simple procedures for adjusting distorted age distributions: the techniques described in section B can be applied to a single age distribution, whereas the procedure described in section C requires a series of two or more age distributions.

B. Smoothing and adjustment of a single age distribution

1. *Fitting of a stable population*

The most drastic form of adjustment for an age distribution is to fit a stable population to the population being studied, following the procedures described in chapter VII, and to adopt the age distribution of the stable population as a representation of the true age distribution. The adopted age distribution will be free of all irregularities, whether these are true or the result of error, and will also be free of any features that the actual age distribution does not share with the models. Thus, the extent of adjustment may be more severe than is actually required, but the result will at least be internally consistent.

This procedure is not described in detail here, nor is it illustrated, since the methods for fitting a stable population are covered thoroughly in chapter VII.

2. *Reduction of effects of age heaping*

(a) *Basis of method and its rationale*

Smoothing and adjustment procedures are usually applied to cumulated age distributions (that is, to the number of persons or the proportion of persons under given ages) since the process of cumulation removes the effects of errors that do not result in a net transfer of people across each of the age boundaries used. If heaping tends to transfer people whose true ages are both under and over an attractive age to this age, the use of attractive ages as boundaries for cumulation will not minimize the effects of heaping on the cumulated age distribution. In such circumstances, the use of age boundaries half-way between attractive ages would probably minimize net transfers across boundaries.[b] Therefore, a simple way of smoothing age distributions would be to fit a succession of polynomials to the reported population, expressed either as numbers of persons or as the proportions of persons under ages $a+3$, $a+8$, $a+13$ and $a+18$, where a is a multiple of five, and to compute the values of these polynomials over their central ranges in order to obtain a smoothed single-year age distribution.

However, the problem of smoothing is somewhat complicated by the fact that ages ending in zero are generally more attractive than ages ending in five, so that the cumulated population under ages ending in eight will tend to be too small, whereas the cumulated population under ages ending in three will tend to be too large. This tendency can be allowed for by fitting two polynomials, one to cumulated populations under ages ending in eight and one to cumulated populations under ages ending in three. The two polynomials would then be evaluated over a common central range; and best estimates of the true cumulated population, relatively free of heaping effects, would be found by averaging.

(b) *Data required*

The information needed consists of the population classified by sex and single year of age, preferably up to age 85 or beyond.

(c) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: cumulation of population figures. For each sex, the population under each age ending in three (3, 13, 23 and so on) and in eight (8, 18, 28 and so on) is required. The single-year age distribution is used to obtain the required numbers; the population under 3 years is the sum of persons aged 0, 1 and 2, the population under age 8 can be found by adding the numbers aged 5, 6 and 7 years to those aged 0-4, and so on. Note that there is no need to use proportions, it is simpler to use the actual numbers.

Step 2: fitting of polynomials to the cumulated age distribution. A third-degree polynomial is fitted to the population under ages $a+3$, $a+13$, $a+23$ and $a+33$, where a is an age ending in zero, and evaluated between $a+13$ and $a+23$; such a polynomial will probably overestimate slightly the population under each age because of the heaping bias. A similar polynomial is fitted to the population under ages $a+8$, $a+18$, $a+28$ and $a+38$, and again evaluated between the ages of $a+13$ and $a+23$; such a polynomial will probably underestimate the population under each age. Therefore, to reduce those biases, the estimated population under each age within the range $a+13$ and $a+23$ can be calculated as the average of the estimates produced by each polynomial: as long as the gain of each term is a loss of a neighbouring five the resulting cumulated age distribution should be almost free of heaping effects and can be broken up into conventional five-year age groups. The process continues by increasing a by 10. Note that ages under 13, which fortunately do not often exhibit the typical heaping patterns, require special treatment; and that, in general, the technique can only be applied up to age 72 (assuming that a single-year age distribution is available up to age 85).

The population under age 13 can be smoothed by fitting a third-degree polynomial to the estimated population under ages 0, 10, 20 and 30. The populations under ages 20 and 30 will already have been obtained as described above. The population under age 10 can be estimated from both of the first two polynomials, that beginning from age 3 and that beginning from age 8, and an average used as an estimate. The population under age 0 is, of course, zero. The polynomial fitted to these four estimated values can then be calculated between ages 0 and 13. Note that this procedure is likely to produce some fairly minor discontinuities in the smoothed single-year age distribution, though the distribution by five-year age groups should be unaffected.

In order to obtain an age distribution in conventional five-year groups adjusted for the effects of heaping bias, a short-cut can be used that avoids calculating the coefficients for every polynomial and instead estimates directly the population under two exact ages. If one

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[a] The term "to smooth" is used in this *Manual* in its most general sense to mean the elimination or minimization of irregularities often present in reported data or in preliminary estimates obtained from them. In this sense, the set of possible "smoothing techniques" encompasses a wide variety of procedures, ranging from the fitting of models to simple averaging. The traditional smoothing techniques applied to age distributions and to observed age-specific mortality rates are part of this set, but they do not exhaust it. The somewhat rougher procedures described in this *Manual* are necessary because the base data available are both deficient and incomplete.

[b] Ken Hill, Hania Zlotnik and Janet Dutch, *Procedures for Reducing the Effects of Age Errors on Indirect Demographic Estimation Techniques*, Laboratories for Population Statistics Scientific Report Series No. 35 (Chapel Hill, North Carolina, Carolina Population Center, 1982).

denotes by $N(x-)$ the population under age x, the population under age $a+12$ is given by

$$\hat{N}((a+12)-) = -0.048N(a-) + 0.864N((a+10)-) \quad (B.1)$$
$$+0.216N((a+20)-) - 0.032N((a+30)-),$$

and the population under age $a+17$ is given by

$$\hat{N}((a+17)-) = -0.0455N(a-) + 0.3315N((a+10)-) \quad (B.2)$$
$$+0.7735N((a+20)-) - 0.0595N((a+30)-).$$

Thus, if a is equal to 3, the population under age 15 will be given by equation (B.1) and the population under 20 by equation (B.2). If a is equal to 8, on the other hand, equation (B.1) will give the population under age 20 and equation (B.2) the population under 25.

For the special cases at the beginning of the age range, the following equation can be used to estimate the population under age 10 from the populations under ages 3, 13, 23 and 33, and the population under age 15 from the populations under ages 8, 18, 28 and 38:

$$\hat{N}((a+7)-) = 0.1495N(a-) + 1.0465N((a+10)-) \quad (B.3)$$
$$-0.2415N((a+20)-) + 0.0455N((a+30)-).$$

The estimate of the population under age 10 from the populations under ages 8, 18, 28 and 38 can be obtained from

$$\hat{N}(10-) = 0.672N(8-) + 0.504N(18-) \quad (B.4)$$
$$-0.224N(28-) + 0.048N(38-).$$

The estimate of the population under age 5 from the adjusted population under ages 10, 20 and 30 can be obtained using the equation

$$\hat{N}(5-) = 0.9375\hat{N}(10-) - 0.3125\hat{N}(20-) + 0.0625\hat{N}(30-). \quad (B.5)$$

Then, to reduce even more the possible biases caused by age-heaping, "best estimates" of the population under each age x, multiple of five, are obtained by averaging the $\hat{N}(x-)$ estimates obtained above whenever two such estimates are available for a given x. Thus, the final estimates of the population under age x, $N^*(x-)$, are computed as

$$N^*(x-) = (\hat{N}_3(x-) + \hat{N}_8(x-))/2.0$$

where the subindices 3 and 8 indicate that the intermediate $\hat{N}(x-)$ estimates are obtained from ages ending in three and eight, respectively.

Lastly, for ages x at the beginning and at the end of the age range (age 5, for example) for which only one $\hat{N}(x-)$ estimate is available, $N^*(x-)$ is set equal to it.

(d) *A detailed example*

The procedure described above is illustrated by applying it to the age distribution of the male population of Sri Lanka, as reported by the fertility survey in Sri Lanka in 1975.[1] Table 203 shows the basic data.

The computational procedure is described below.

TABLE 203 MALE POPULATION, BY SINGLE YEAR OF AGE, SRI LANKA, 1975

Age (x)	Population (x)	Age (x)	Population (x)	Age (x)	Population (x)	Age (x)	Population (x)
0	572	27	350	54	120	81	3
1	506	28	366	55	268	82	8
2	550	29	267	56	110	83	5
3	602	30	383	57	108	84	4
4	596	31	265	58	157	85	32
5	637	32	294	59	81	86	3
6	600	33	270	60	261	87	9
7	643	34	190	61	65	88	3
8	607	35	443	62	85	89	1
9	562	36	220	63	92	90	8
10	624	37	183	64	67	91	0
11	546	38	288	65	225	92	0
12	667	39	192	66	31	93	3
13	620	40	342	67	66	94	0
14	535	41	147	68	103	95	7
15	650	42	219	69	38	96	2
16	545	43	195	70	160	97	1
17	517	44	125	71	26	98	0
18	529	45	391	72	41	99	9
19	519	46	132	73	50	Not stated	0
20	550	47	150	74	38		
21	417	48	247	75	130		
22	484	49	126	76	23	Total	23 918
23	467	50	291	77	21		
24	416	51	139	78	24		
25	460	52	162	79	8		
26	367	53	169	80	71		

Step 1: cumulation of population counts. The numbers of males under ages 3, 8, 13, 18 and so on are found by cumulating the numbers in successive single-year age groups up to, but not including, the upper boundary age group. Thus, if $N(y)$ is the population of age y and $N(x-)$ is the population under age x,

$$N(x-) = \sum_{y=0}^{x-1} N(y).$$

Thus

$$N(3-) = N(0) + N(1) + N(2) = 572 + 506 + 550 = 1,628$$

and

$$N(8-) = N(3-) + N(3) + N(4) + N(5) + N(6) + N(7)$$
$$= 1,628 + 602 + 596 + 637 + 600 + 643 = 4,706.$$

Full results are shown in table 204; the calculations are continued up to the population under age 88.

[1] Ministry of Plan Implementation, Department of Census and Statistics, *World Fertility Survey—Sri Lanka 1975: First Report* (Colombo, 1978).

TABLE 204. ADJUSTMENT, FOR HEAPING, OF THE MALE AGE DISTRIBUTION, SRI LANKA, 1975

Age x (1)	Male population under age x (2)	Age b (3)	Adjusted population under age b based on ages ending in: Three (4)	Eight (5)	Average population under age b (6)	Population in the age group from b to b+4: Adjusted (7)	Reported (8)
-	-	0	-	-	-	2 928	2 826
3	1 628	5	-	-	(2 928)	3 016	3 049
8	4 706	10	5 912	5 975	5 944	2 974	3 012
13	7 712	15	8 879	8 956	8 918	2 706	2 760
18	10 599	20	11 618	11 629	11 624	2 325	2 334
23	13 098	25	13 946	13 951	13 949	1 865	1 810
28	15 158	30	15 790	15 837	15 814	1 485	1 402
33	16 733	35	17 304	17 293	17 299	1 244	1 326
38	18 039	40	18 563	18 523	18 543	1 100	1 028
43	19 227	45	19 661	19 624	19 643	989	1 046
48	20 220	50	20 654	20 609	20 632	872	881
53	21 185	55	21 514	21 494	21 504	732	724
58	21 960	60	22 241	22 231	22 236	579	570
63	22 609	65	22 821	22 809	22 815	440	459
68	23 090	70	23 255	23 254	23 255	323	315
73	23 454	75	.	23 578	23 578	341*	377*
78	23 716	.	.	.	.	.	.
83	23 830	.	.	.	.	.	.
88	23 885	.	.	.	.	.	.

* Population aged 75 and over.

Step 2: fitting of polynomials to the cumulated age distribution. Estimates of the population under ages 15, 20 and so on, up to age 75 can be obtained from polynomials fitted to the numbers under ages ending in three or eight by the use of equations (B.1) and (B.2). Thus, the population under ages 15 and 20 can be found as follows using the populations under ages ending in three:

$$\hat{N}_3(15-) = -0.048N(3-) + 0.864N(13-) + 0.216N(23-) - 0.032N(33-)$$

$$= (-0.048)(1,628) + (0.864)(7,712) + (0.216)(13,098) + (-0.032)(16,733)$$

$$= 8,878.7$$

and

$$\hat{N}_3(20-) = -0.0455N(3-) + 0.3315N(13-) + 0.7735N(23-) - 0.0595N(33-)$$

$$= (-0.0455)(1,628) + (0.3315)(7,712) + (0.7735)(13,098) + (-0.0595)(16,733)$$

$$= 11,618.1;$$

and the populations under ages 20 and 25 can be estimated from the populations under ages ending in eight:

$$\hat{N}_8(20-) = -0.048N(8-) + 0.864N(18-) + 0.216N(28-) - 0.032N(38-)$$

$$= (-0.048)(4,706) + (0.864)(10,599) + (0.216)(15,158) + (-0.032)(18,039)$$

$$= 11,628.5$$

and

$$\hat{N}_8(25-) = (-0.0455)(4,706) + (0.3315)(10,599) + (0.7735)(15,158) + (-0.0595)(18,039)$$

$$= 13,950.8.$$

Equations (B.1) and (B.2) are then applied to the populations under ages 10 years older and the process is repeated up to age 75. The full set of estimates based on ages ending in three is shown in column (4) of table 204, while column (5) shows the estimates obtained from ages ending in eight. Estimates of the population under age 15 based on populations under ages ending in eight and of the population under 10 based on populations under ages ending in three are obtained by applying equation (B.3):

$$\hat{N}_3(15-) = 0.1495N(8-) + 1.0465N(18-) - 0.2415N(28-) + 0.0455N(38-)$$

$$= 8,955.5$$

and

$$\hat{N}_3(10-) = 0.1495N(3-) + 1.0465N(13-) - 0.2415N(23-) + 0.0455N(33-)$$

$$= 5,912.2.$$

The estimate of the population under age 10 from populations under ages 8, 18, 28 and 38 is obtained by applying equation (B.4).

Once two estimates are available for each of the populations under ages ending in zero or five, from 10 to 70, a best estimate at each age is obtained by averaging the two values corresponding to it. Thus, for example,

$$N^*(20-) = (\hat{N}_3(20-) + \hat{N}_8(20-))/2.0$$

$$= (11,618 + 11,629)/2.0 = 11,624.$$

Full results are shown in column (6) of table 204.

The population under age 5 is then found by substituting best estimates of the populations under ages 10, 20 and 30 into equation (B.5):

$$N^*(5-) = 0.9375N^*(10-) - 0.3125N^*(20-) + 0.0625N^*(30-)$$

$$= 2,928.4.$$

The cumulated population estimates can then be successively subtracted from each other to provide an age distribution in five-year age groups, the population aged from x to $x+4$ being obtained by subtracting $N^*(x-)$ from $N^*(x+5)-)$. Thus, for age group 20-24,

$${}_5N_{20} = N^*(25-) - N^*(20-) = 13,949 - 11,624 = 2,325.$$

Full results are shown in column (7) of table 204 and may be compared with the reported age distribution shown in column (3).

3. *Comparison with a standard age distribution*

(a) *Basis of method and its rationale*

In some circumstances, when the quality of the age data is poor or when ages seem to be affected by systematic misreporting other than heaping, some adjustment procedure more drastic than that described in subsection B.2 may be necessary. One approach to such adjustment involves the comparison of the reported age distribution with some standard age distribution, followed by the adjustment of the standard age distribution to reflect the main features of the reported one: the result should be an age distribution that retains some of the broad characteristics of the reported distribution while at the same time being free of obvious bias.

In practice, a stable population derived from a suitable model life table and having a reasonable rate of growth for the application in hand can be used as the standard age distribution, particularly if there is reason to believe that the actual population is stable or quasi-stable. The comparison of the reported and model distributions is made easier by the use of a transformation that linearizes the relationship between age and the cumulated proportion of the population under each age. In growing populations, such as those of most developing countries, a suitable transformation is

$$Y(x) = \ln[(1.0 + C(x))/(1.0 - C(x))] \qquad (B.6)$$

where $C(x)$ is the proportion of the population under age x. From here on, the transformation defined by equation (B.6) is referred to as the Y-transformation. Values of $Y(x)$ for the population in question can be plotted against standard values, $YS(x)$, for a suitable standard stable population; and deviations from an approximately parabolic relationship can be noted (in practice, when the data are not extremely distorted, this relationship deviates only very slightly from that represented by a straight line). An adjusted age distribution can be obtained by fitting a curve to the points that are regarded as least biased and then reversing the transformation process to obtain a smooth cumulated age distribution. Tests with model populations have suggested that adequate results can be obtained by fitting a second-degree polynomial (a parabola) to the selected points, a process that is simplified because one can require the fitted parabola to pass through the origin (since the population proportion under age zero has to be zero) and through the means of both the first half and the second half of the selected points.

The following data are required for this method:

(b) *Data required*

(a) The population classified by sex and five-year age group;

(b) A standard age distribution by five-year age group. A stable population subject to a growth rate and mortality risks similar to those of the population being studied is usually a suitable choice.

(c) *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of proportions under ages ending in zero or five. The proportion of the population under each age x, denoted by $C(x)$, is found by summing the populations of each five-year age group below the age in question and dividing the resulting sum by the total population of known age, that is, excluding those of unstated age.

Step 2: calculation of Y-transformation of the reported proportions under ages five years apart. The Y-transformation of each of the $C(x)$ proportions calculated in the previous step is obtained by using equation (B.6). For the sake of completeness, this equation is repeated here:

$$Y(x) = \ln[(1.0 + C(x))/(1.0 - C(x))] \qquad (B.6)$$

Note that the Y-transformation of any proportion $C(x)$ assumes values ranging from zero to infinity.

Step 3: selection of a standard age distribution. The standard population age distribution, against which the reported population is to be compared, should be selected from that of stable populations whose mortality level and pattern approximate as closely as possible those observed in the population being studied. Among these stable populations, the one having the same growth rate as the reported population is preferable.

Step 4: the Y-transformation of the standard age distribution. Steps 1 and 2 are repeated for the selected standard age distribution.

Step 5: relation of the Y-transformations of the reported and standard age distributions. A second-degree polynomial is assumed to represent the relationship between the Y-transformations of the reported and the standard age distributions. To simplify the fitting procedure, the polynomial selected is that passing through the origin, the mean of a first group of points and the mean of a second group of points, where the groups used are of equal size but exclude any extreme values that fall well away from a likely parabola. Therefore, the fitted equation has the form

$$Y(x) = \alpha(YS(x))^2 + \beta YS(x), \qquad (B.7)$$

where α and β can be estimated as

$$\alpha = \left(\frac{1.0}{\overline{YS}_1 - \overline{YS}_2}\right)\left[\frac{\overline{Y}_1}{\overline{YS}_1} - \frac{\overline{Y}_2}{\overline{YS}_2}\right] \qquad (B.8)$$

and

$$\beta = \frac{\overline{Y}_1}{\overline{YS}_1} - \alpha\overline{YS}_1 \qquad (B.9)$$

where $(\overline{YS}_1, \overline{Y}_1)$ and $(\overline{YS}_2, \overline{Y}_2)$ are the mean points for the first and the second groups, respectively. As usual, YS denotes the Y-transformation of the standard.

Step 6: calculation of estimated age distribution. Having estimated α and β, equation (B.7) can be used to obtain adjusted values of the Y-transformation, $Y^*(x)$, which can, in turn, be inverted to produce estimates of an adjusted age distribution. For the sake of completeness, equation (B.10) indicates how the adjusted values $Y^*(x)$ are obtained and equation (B.11) shows how an estimate of the adjusted $C^*(x)$, the estimated proportion under age x, is calculated:

$$Y^*(x) = \alpha(YS(x))^2 + \beta YS(x). \qquad (B.10)$$

$$C^*(x) = (\exp[Y^*(x)] - 1.0)/(\exp[Y^*(x)] + 1.0). \qquad (B.11)$$

These $C^*(x)$ proportions can be multiplied by the total reported population to find the estimated number of persons under age x, and estimates of the population in five-year age groups can then be obtained by subtraction.

(d) *A detailed example*

Once more, the age distribution of the male population of Sri Lanka, as recorded by the fertility survey in 1975, is used to illustrate the method (see table 203).

Step 1: calculation of proportions under ages ending in zero or five. Column (2) of table 206 shows the proportions under each age x, multiple of five, denoted by $C(x)$, also including in this case that under age 1.

Step 2: calculation of Y-transformation of the reported proportions under ages x five years apart. The proportions $C(x)$ calculated in the previous step are transformed, using equation (B.6), into $Y(x)$ values. Thus, for age 15,

$$Y(15) = \ln[(1.0 + C(15))/(1.0 - C(15))]$$

$$= \ln[(1.0 + 0.3716)/(1.0 - 0.3716)] = 0.7806.$$

The full set of $Y(x)$ values is shown in column (4) of table 205.

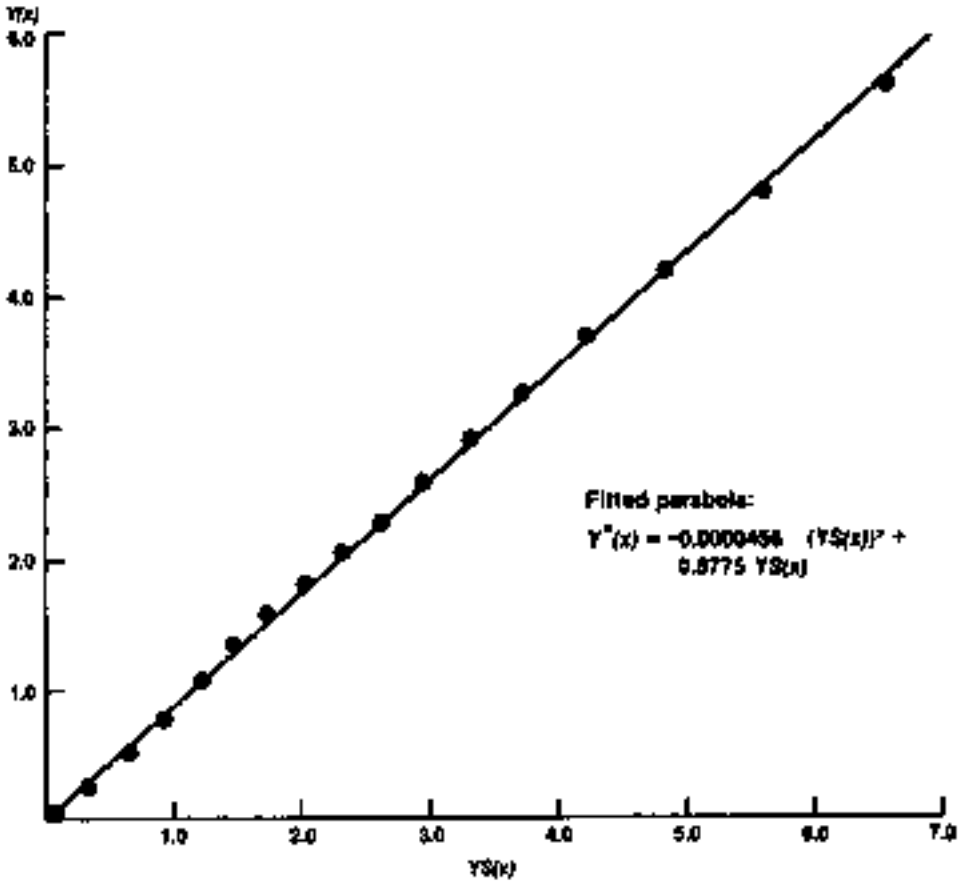
Step 3: selection of a standard age distribution. Procedures for fitting a stable population to the one observed are presented in chapter VII and need not be described again here. For Sri Lanka, a West model stable population of mortality level 16, with a growth rate of 0.030, was selected. The proportions under each age x for this population are shown in column (3) of table 206. They are referred to below as "standard proportions".

TABLE 205. ESTIMATION OF AN ADJUSTED MALE AGE DISTRIBUTION BY USING Y-TRANSFORMATIONS AND THE WEST MODEL STABLE POPULATION AS STANDARD, SRI LANKA, 1975

Age x (l_x)	Proportion under age x: Reported (l_x)	Proportion under age x: Standard[1] (s_x)	Y-transformation of the proportion under age x: Reported ($Y(x)$)	Y-transformation of the proportion under age x: Standard ($YS(x)$)	Estimated Y-transformation ($Y^*(x)$)	Estimated proportion under age x (l_x^*)
1	0.0239	0.0372	0.0478	0.0744	0.0653	0.0326
5	0.1182	0.1702	0.2375	0.3437	0.3016	0.1497
10	0.2456	0.3133	0.5014	0.6484	0.5690	0.2771
15	0.3716	0.4353	0.7806	0.9328	0.8185	0.3878
20	0.4870	0.5393	1.0642	1.2063	1.0585	0.4848
25	0.5845	0.6276	1.3383	1.4749	1.2941	0.5697
30	0.6602	0.7022	1.5861	1.7433	1.5296	0.6439
35	0.7188	0.7653	1.8103	2.0178	1.7704	0.7090
40	0.7643	0.8183	2.0620	2.3093	2.0269	0.7659
45	0.8173	0.8626	2.2913	2.6068	2.2872	0.8156
50	0.8610	0.8992	2.5944	2.9360	2.5759	0.8586
55	0.8978	0.9291	2.9215	3.3035	2.8983	0.8953
60	0.9281	0.9527	3.2890	3.7205	3.2641	0.9264
65	0.9519	0.9708	3.7033	4.2120	3.6952	0.9515
70	0.9711	0.9838	4.2225	4.8078	4.2178	0.9710
75	0.9843	0.9923	4.8394	5.5558	4.8738	0.9848
80	0.9929	0.9971	5.6373	6.5347	5.7323	0.9933

[1]Coale-Demeny West model stable population, level 12, $r = 0.01$.

Figure 26. Plot of the Y-transformation of the proportions under age x in the reported population against the Y-transformation of the equivalent proportions in the standard, Sri Lanka, 1975



Step 4: the Y-transformation of the standard age distribution. Step 2 is applied to the standard proportions under age x, with the results shown in column (5) of table 205.

Step 5: relation of the Y-transformations of the reported and standard age distributions. The $[YS(x), Y(x)]$ points should be examined before attempting to fit a parabola to them, so that outliers can be discarded. Figure 26 displays a plot of the points, which show no obvious outliers. Therefore, a parabola passing through the origin, the mean of the first nine points and the mean of the second nine points (the ninth point being included in both groups) is selected as an acceptable fit. Thus,

$$\bar{Y}_1 = (0.0478 + 0.2375 + \cdots + 2.0620)/9.0 = 1.0476,$$

$$\bar{Y}_2 = (2.0620 + 2.2973 + \cdots + 5.6373)/9.0 = 3.5074,$$

$$\overline{YS}_1 = (0.0744 + 0.3437 + \cdots + 2.3033)/9.0 = 1.1939,$$

and

$$\overline{YS}_2 = (2.3033 + 2.6068 + \cdots + 6.5347)/9.0 = 3.9978.$$

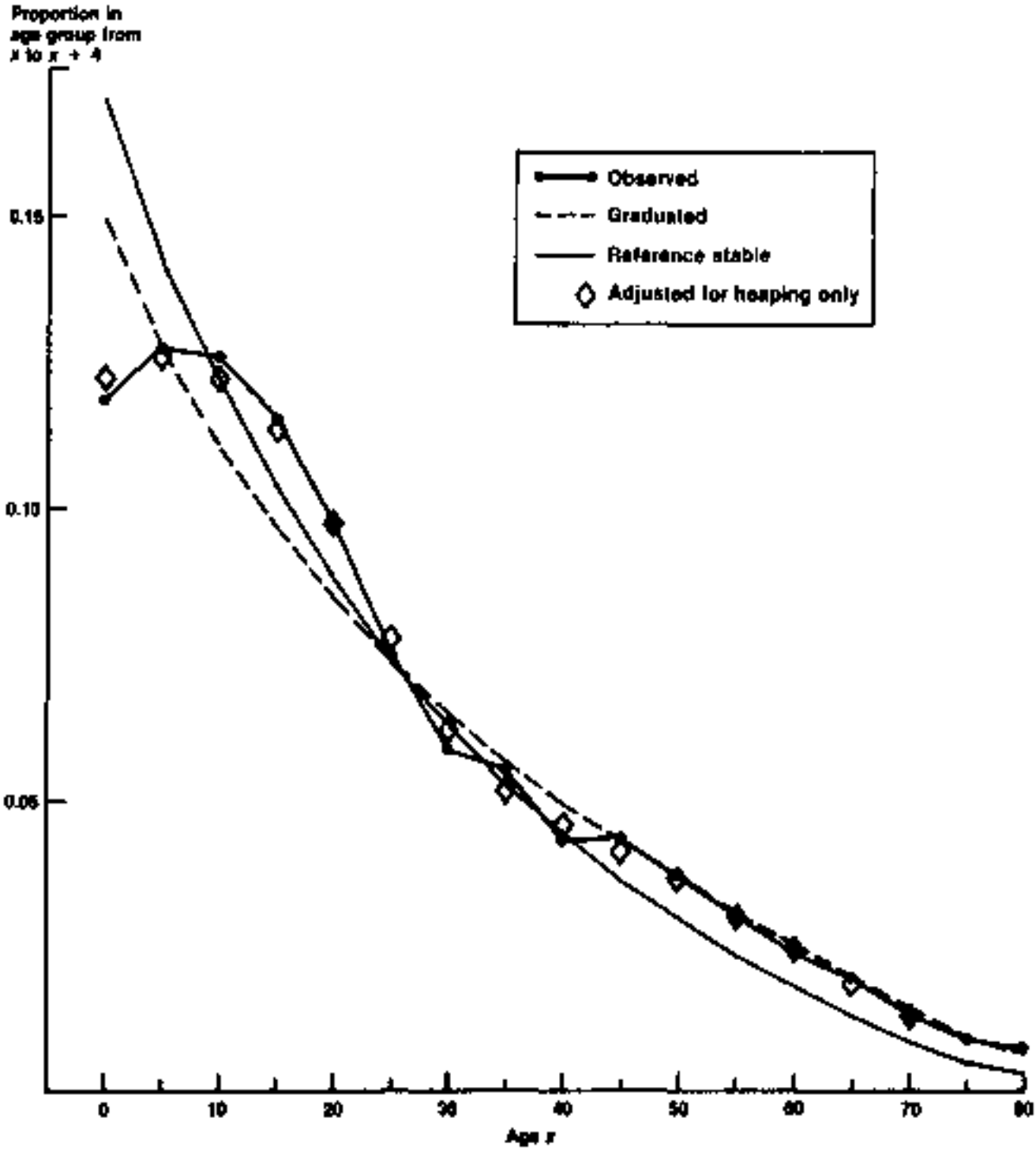
Equations (B.8) and (B.9) can now be used to estimate the parameters α and β in equation (B.10):

$$\alpha = \left(\frac{1.0}{1.1939 - 3.9978}\right)\left(\frac{1.0476}{1.1939} - \frac{3.5074}{3.9978}\right) = -0.0000456$$

and

$$\beta = \frac{3.5074}{3.9978} - (-0.0000456)(3.9978) = 0.8775.$$

Figure 27. A comparison of the results of different adjustment procedures for male age distribution, Sri Lanka, 1975



Step 6: calculation of estimated age distribution. Substituting the values of α and β just obtained in equation (B.10), we obtained adjusted values of the Y-transformation. Thus, for age 15,

$$Y^*(15) = -0.0000456(YS(15))^2 + 0.8775\,YS(15)$$

$$= -0.0000456(0.9328)^2 + (0.8775)(0.9328) = 0.8185.$$

Column (6) of table 205 shows all the $Y^*(x)$ values. Then, the estimated proportions under each age x are obtained by reversing the Y-transformation by means of equation (B.11). Again using age 15 as an example,

$$C^*(15) = (\exp[2Y^*(15)] - 1.0)/(\exp[2Y^*(15)] + 1.0) = 0.3878.$$

Column (7) of table 205 shows all the estimated $C^*(x)$ values. Adjusted proportions in each five-year age group can now be obtained by subtracting from the proportion under age x the proportion under each age $x-5$. Thus, the adjusted proportion in age group 15-19, ${}_5C_{15}$, is found as

$${}_5C_{15}^* = C^*(20) - C^*(15) = 0.4848 - 0.3878 = 0.0970.$$

Figure 27 shows a plot of the adjusted proportions in five-year age groups, the corresponding proportions in the observed population, the standard proportions and the proportions obtained by applying the smoothing procedure described in subsection B.2. It is clear that the male population of Sri Lanka has been drastically destabilized by declining fertility, so that the procedure described in this section cannot be expected to work well at younger ages, above age 25. However, its results seem acceptable, and they provide an obviously better approximation to the observed population than the original standard age distribution on which the fitted one is based. The age distribution obtained when the effects of heaping are removed as described in subsection B.2 is much closer to the reported distribution, damping only slightly (and probably, at least in the right direction) the effects of destabilization at younger ages and retaining the trough in the reported age distribution between ages 30 and 45, though smoothing out its irregularities. In this particular application, where age-reporting does not seem too poor and where the reported age distribution is clearly not stable, the smoothed age distribution obtained by the procedure that merely reduces the effects of age-heaping (subsection B.2) is probably the best approximation available to the true age distribution.

C. Demeny and Shorter adjustment technique for two age distributions

1. *Basis of method and its rationale*

Demeny and Shorter[4] proposed a method based on intercensal probabilities of survival to adjust the age distributions produced by two consecutive censuses. This method assumes that the probabilities of intercensal survival are known, that both censuses achieved the same level of coverage and that both suffered from similar proportionate age-reporting errors, that is, that the recorded population of age group x is equal to the true population of the age group multiplied by an adjustment factor typical only of the age group and not changing from census to census. Given two censuses five years apart, the population aged 0-4 at the time of the first census is assumed to be correct and is projected forward to the second census. An adjustment factor for age group 5-9 is then obtained as the ratio of the projected to the reported population aged 5-9 at the second census, and this factor is then used to adjust the reported population aged 5-9 at the first census. The adjusted population aged 5-9 at the first census is then projected forward to the second census, and an adjustment factor for age group 10-14 is found as the ratio of the projected to the actual population aged 10-14 at the second census. The chaining process continues from age group to age group until the entire age range has been covered. Two adjusted age distributions are then available, that for the first census being made up of the adjusted populations of each age group and that for the second census consisting of the projections of the adjusted population from the first census. The final step is to scale the two adjusted age distributions to agree with the reported total population.

The method depends upon the availability of a suitable mortality schedule in order to carry out the projection, and a mistake in the specification of mortality will introduce a systematic bias in the results. The other two assumptions involved, that of no change in enumeration completeness and that of a constant proportional error for a given age group at both censuses, may also be frequently unfeasible. A change in enumeration completeness may result in large distortions of the entire estimated age distribution if this method is applied, though such large errors are likely to be obvious. Because of these potential problems, the estimates yielded by this procedure, though useful, will have to be carefully scrutinized before being adopted for future use.

2. *Data required*

The data required are listed below:

(*a*) The distribution of the population by age group and sex from two consecutive censuses. The method is most readily applied when the intercensal interval is five years, but it can also be applied to a 10-year interval. However, it is difficult and even inadvisable to use data corresponding to intervals that are not multiples of five, since in such cases the differential effects of heaping are likely to distort the results obtained;

(*b*) A suitable set of survivorship probabilities referring to the intercensal period.

3. *Computational procedure*

The procedure for this method is described below.

Step 1: synthesizing of the population corresponding to a five-year interval. If the two censuses were held 10 years apart, the first at time t_1 and the second at time t_2, the method is best applied by inventing an intervening age distribution, thus creating two five-year intervals, and applying the method twice. The age distribution for the mid-point of the 10-year interval can be created by finding the annual growth rate of each age group from x to $x+4$, as

$$r(x) = (\ln {}_5N_{2,x} - \ln {}_5N_{1,x})/(t_2 - t_1),$$

and then applying the growth rate for five years to the initial population:

$${}_5N_x = {}_5N_{1,x} \exp[5\,r(x)],$$

Here, ${}_5N_{1,x}$ and ${}_5N_{2,x}$ denote, respectively, the reported populations in the age group from x to $x+4$ in the first and second censuses.

Step 2: projection of initial age distribution. The population of age group 0-4 at the first census is projected forward to age group 5-9 at the second census, and an adjustment factor for age group 5-9 is obtained as the ratio of the projected to the reported population aged 5-9 at the time of the second census. Thus, if $k(5)$ is the adjustment factor for the age group from x to $x+4$,

$$k(5) = ({}_5N_0 {}_5L_5/{}_5L_0)/{}_5N_{2,5}. \qquad (C.1)$$

The projected population aged 5-9 is taken as the adjusted value, ${}_5N_{2,5}$, at the second census. Thus,

$${}_5\hat{N}_{2,5} = ({}_5N_0 {}_5L_5/{}_5L_0). \qquad (C.2)$$

The reported population aged 5-9 at the first census is then adjusted by $k(5)$ to obtain the adjusted population aged 5-9 at the first census, ${}_5\hat{N}_{1,5}$, and this population is then projected forward to age group 10-14 at the second census; thus,

$${}_5\hat{N}_{1,5} = ({}_5N_{1,5})k(5), \qquad (C.3)$$

and

$$k(10) = ({}_5\hat{N}_{1,5})({}_5L_{10}/{}_5L_5)/{}_5N_{2,10}.$$

[4] Paul Demeny and Frederic C. Shorter, *Estimating Turkish Mortality, Fertility and Age Structure*, Statistical Institute Paper No. 2 (Istanbul, Istanbul University, 1968).

where the first two terms are simply the adjusted population aged 10-14 at the second census, that is,

$$ {}_5\hat{N}2_{10} = ({}_5\hat{N}1_5)({}_5L_{10}/{}_5L_5). $$

The procedure continues until the open-ended interval (which requires a slightly modified treatment) is reached. Since it is assumed that the adjustment factor required is the same for both censuses, that for the open-ended interval, $k(A+)$, is equal to the ratio of the adjusted to the reported population aged A and over for both age distributions. That is,

$$ k(A+) = \frac{\hat{N}2(A+)}{N2(A+)} = \frac{\hat{N}1(A+)}{N1(A+)}. \qquad (C.4) $$

However, the adjusted population aged A and over at the second census is also equal to the sum of two elements, the first being the projection to the time of the second census of the adjusted population aged from $A-5$ to $A-1$ at the first census, and the second being the survivors to the second census of the adjusted population (not known) aged A and over at the first census. Thus,

$$ \hat{N}2(A+) = ({}_5\hat{N}1_{A-5})({}_5L_A/{}_5L_{A-5}) + \hat{N}1(A+)(T_{A+5}/T_A) \qquad (C.5) $$

where T_{A+5}/T_A is the probability that the population aged A and over has of surviving five years. By combining equations (C.4) and (C.5), and rearranging terms, an expression for $k(A+)$ is obtained

$$ k(A+) = \frac{({}_5\hat{N}1_{A-5})({}_5L_A/{}_5L_{A-5})}{N2(A+) - (T_{A+5}/T_A)N1(A+)}. \qquad (C.6) $$

Since all the variables in equation (C.6) are known, the value of $k(A+)$ may be readily obtained.

Step 3: scaling of adjusted age distributions. The two age distributions derived in step 2 are based on the provisional assumption that the youngest age group is accurately recorded by both censuses. If this assumption is not valid, as will frequently be the case, both the adjusted age distributions will have been scaled according to the ratios of the reported to the true populations aged 0-4, while being unaffected in age pattern. The assumption that age group 0-4 is correctly reported is not necessary if one scales the adjusted population age distributions by the ratio of the total reported population to the total adjusted population at each point in time. The scaling factors are unlikely to be exactly the same for both age distributions, but they should be very similar.

4. *A detailed example*

A part of the example given by Demeny and Shorter is presented here to illustrate the calculations. The female population of Turkey as recorded by the 1955 and 1960 censuses is adjusted below. The basic data are shown in columns (2) and (3) of table 206 (the 1955 population data have been adjusted in order to allow for the effects of migration). Column (4) shows survivorship ratios taken from a South model life table of level 16, with an expectation of life at birth of 58.25 years.

Step 1: synthesizing of the population for a five-year interval. The actual intercensal interval is five years, so this step is not required.

Step 2: projection of initial age distribution. The adjusted population aged 5-9 in 1960 is taken to be the 1955 population aged 0-4 projected five years. Thus, if one denotes by ${}_5S_0$ the ratio ${}_5L_{5}/{}_5L_0$,

$$ {}_5\hat{N}2_5 = ({}_5\hat{N}1_0)({}_5S_0) = (1.869)(0.9599) = 1.813. $$

The adjustment factor for age group 5-9, $k(5)$, is then calculated as the ratio of the adjusted to the reported population aged 5-9 in 1960. Thus,

$$ k(5) = {}_5\hat{N}2_5/{}_5N2_5 = 1,813/1,928 = 0.940. $$

The adjustment factor, $k(5)$, is then applied to the 1955 population aged 5-9:

$$ {}_5\hat{N}1_5 = k(5)\ {}_5N1_5 = (0.940)(1,570) = 1,476. $$

Then, the adjusted 1955 population aged 5-9 is projected forward to 1960 to obtain the adjusted 1960 population aged 10-14:

$$ {}_5\hat{N}2_{10} = ({}_5\hat{N}1_5)({}_5S_5) = (1,476)(0.9903) = 1,462. $$

The procedure continues until one reaches the open interval, in this case 75 and over. The last normal step completed is the calculation of ${}_5\hat{N}1_{70}$. The adjustment factor for the population aged 75 and over is found by applying equation (C.6):

$$ k(75+) = ({}_5\hat{N}1_{70})({}_5S_{70})/[N2(75+) - ({}_5S_{75+})(N1(75+))] $$

$$ = (148.0)(0.6936)/[182.1 - (0.4414)(159.3)] $$

$$ = 0.918 $$

TABLE 206. ADJUSTMENT OF TWO CONSECUTIVE ENUMERATIONS OF THE FEMALE POPULATION USING COHORT SURVIVAL. TURKEY, 1955 AND 1960

(Population in thousands)

Age group (1)	1955 population (2)	1960 population (3)	Survivorship probability (4)	Raw adjustment of 1960 population (5)	Initial adjustment factor (6)	Raw adjustment of 1955 population (7)	Final adjusted population: 1955 (8)	Final adjusted population: 1960 (9)
0-4	1 889.0	2 079.0	0.9599	(2 079.0)	-	(1 859.0)	1 962.0	2 160.0
5-9	1 570.0	1 928.0	0.9903	1 813.0	0.940	1 476.0	1 533.0	1 884.0
10-14	1 097.0	1 439.0	0.9907	1 462.0	0.982	1 077.0	1 118.0	1 519.0
15-19	1 094.0	1 059.0	0.9873	1 065.0	1.008	1 103.0	1 145.0	1 109.0
20-24	1 117.0	1 128.0	0.9846	1 089.0	0.965	1 078.0	1 119.0	1 132.0
25-29	1 027.0	1 177.0	0.9830	1 061.0	0.902	926.4	962.0	1 102.0
30-34	735.1	985.8	0.9813	910.7	0.924	679.2	705.3	946.3
35-39	528.5	693.8	0.9783	666.5	0.961	507.9	527.4	692.6
40-44	624.7	548.4	0.9741	496.9	0.906	566.0	587.7	516.3
45-49	475.6	500.0	0.9660	551.0	1.103	524.6	544.8	572.9
50-54	531.0	581.2	0.9523	506.8	0.872	463.0	480.8	526.6
55-59	310.4	369.4	0.9272	440.9	1.194	370.6	384.8	458.1
60-64	369.3	461.4	0.8829	343.6	0.745	290.0	301.1	357.0
65-69	173.7	213.8	0.8065	256.1	1.198	208.1	216.1	266.1
70-74	172.1	195.7	0.6436	168.3	0.860	148.0	153.7	174.9
75+	159.3	182.1	0.4414	167.2	0.918	146.2	151.8	173.6
TOTAL	11 893.7[a]	13 591.6[a]		13 079.3		11 453.0	11 892.5[a]	13 590.4[a]

[a] The slight differences observed are due to rounding.

where ${}_5T_{75+}$ is equal to T_{80}/T_{75}. The adjusted populations in the open interval are then obtained by multiplying the recorded populations aged 75+ by $k(75+)$, so

$$\hat{N}1(75+) = (159.3)(0.918) = 146.2$$

and

$$\hat{N}2(75+) = (182.1)(0.918) = 167.2.$$

The first adjustment of the 1960 age distribution is shown in column (5) in table 206, the adjustment factors are shown in column (6), and the first adjustment of the 1955 age distribution is shown in column (7).

Step 3: scaling of adjusted age distributions. It will be seen that the sum of column (7) in table 206, representing the total adjusted 1955 population, is not equal to the sum of column (2), representing the total reported 1955 population. This difference is an indication of the actual error in the population aged 0–4 in 1975 in relation to the overall level of enumeration. Therefore, the assumption concerning the completeness of enumeration of the population aged 0–4 can be dropped simply by multiplying the adjusted population of each age group by the ratio of the total reported population to the total adjusted population. Thus, for example, the final adjusted population aged 0–4 in 1955 is equal to the first adjusted value given in column (7) of table 206 multiplied by 11,893.7/11,453.0. Columns (8) and (9) show the final adjusted populations for 1955 and 1960, respectively.

Annex VI

ADULT SURVIVORSHIP RATIOS FOR COALE-DEMENY MODEL LIFE TABLES FOR USE WITH ADULT MORTALITY ESTIMATES FROM ORPHANHOOD

TABLE 207. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 25, $_{l}(N)/_{l}(25)$, FOR MATERNAL ORPHANHOOD ANALYSIS. NORTH MODEL

	Values of age N									
Level	35	40	45	50	55	60	65	70	75	80
1	0.81612	0.71871	0.62830	0.54437	0.45942	0.36315	0.25809	0.15340	0.06707	0.01864
2	0.83111	0.74044	0.65504	0.57466	0.49188	0.39652	0.29056	0.18104	0.08593	0.02756
3	0.84472	0.76036	0.67979	0.60300	0.52267	0.42926	0.32265	0.20936	0.10641	0.03819
4	0.85714	0.77870	0.70283	0.62961	0.55191	0.46055	0.35425	0.23817	0.12828	0.05052
5	0.86855	0.79569	0.72451	0.65481	0.57976	0.49076	0.38536	0.26730	0.15137	0.06442
6	0.87910	0.81147	0.74487	0.67844	0.60635	0.51999	0.41594	0.29668	0.17557	0.07981
7	0.88887	0.82625	0.76346	0.70096	0.63178	0.54825	0.44601	0.32618	0.20069	0.09663
8	0.89800	0.84012	0.78139	0.72237	0.65617	0.57560	0.47552	0.35573	0.22663	0.11477
9	0.90653	0.85315	0.79836	0.74276	0.67954	0.60210	0.50447	0.38528	0.25327	0.13414
10	0.91455	0.86546	0.81446	0.76223	0.70203	0.62779	0.53286	0.41475	0.28051	0.15464
11	0.92207	0.87709	0.82973	0.78084	0.72363	0.65271	0.56070	0.44409	0.30822	0.17620
12	0.92917	0.88810	0.84431	0.79862	0.74440	0.67682	0.58802	0.47333	0.33663	0.19919
13	0.93527	0.89769	0.85694	0.81407	0.76244	0.69801	0.61211	0.49920	0.36171	0.21938
14	0.94166	0.90754	0.86987	0.82966	0.78043	0.71864	0.63522	0.52386	0.38567	0.23892
15	0.94781	0.91713	0.88255	0.84509	0.79842	0.73951	0.65892	0.54957	0.41115	0.26023
16	0.95375	0.92641	0.89493	0.86032	0.81635	0.76054	0.68310	0.57623	0.43814	0.28335
17	0.95946	0.93535	0.90699	0.87525	0.83412	0.78162	0.70764	0.60372	0.46653	0.30829
18	0.96490	0.94394	0.91863	0.88983	0.85160	0.80259	0.73238	0.63188	0.49617	0.33499
19	0.97009	0.95216	0.92986	0.90398	0.86876	0.82334	0.75718	0.66051	0.52691	0.36334
20	0.97502	0.96000	0.94064	0.91768	0.88549	0.84378	0.78184	0.68941	0.55851	0.39317
21	0.97968	0.96744	0.95095	0.93084	0.90170	0.86375	0.80623	0.71836	0.59074	0.42428
22	0.98420	0.97468	0.96101	0.94371	0.91768	0.88342	0.82969	0.74631	0.62208	0.45512
23	0.98914	0.98223	0.97177	0.95764	0.93548	0.90570	0.85443	0.78241	0.66514	0.50029
24	0.99283	0.98819	0.98067	0.96993	0.95216	0.92794	0.88818	0.82177	0.71471	0.55547

TABLE 208. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 25, $_{l}(N)/_{l}(25)$, FOR MATERNAL ORPHANHOOD ANALYSIS. SOUTH MODEL

	Values of age N									
Level	35	40	45	50	55	60	65	70	75	80
1	0.82910	0.74967	0.67704	0.60808	0.52828	0.43504	0.31330	0.18642	0.07832	0.01831
2	0.84344	0.76962	0.70119	0.63539	0.55839	0.46719	0.34618	0.21582	0.09859	0.02718
3	0.85653	0.78791	0.72357	0.66093	0.58682	0.49797	0.37832	0.24556	0.12034	0.03768
4	0.86850	0.80483	0.74440	0.68488	0.61370	0.52745	0.40971	0.27550	0.14337	0.04978
5	0.87960	0.82057	0.76391	0.70743	0.63929	0.55579	0.44042	0.30560	0.16753	0.06341
6	0.88986	0.83526	0.78221	0.72877	0.66366	0.58306	0.47045	0.33572	0.19265	0.07850
7	0.89943	0.84903	0.79948	0.74898	0.68690	0.60933	0.49980	0.36582	0.21859	0.09492
8	0.90838	0.86199	0.81582	0.76821	0.70915	0.63469	0.52851	0.39583	0.24528	0.11262
9	0.91680	0.87420	0.83132	0.78655	0.73050	0.65918	0.55659	0.42570	0.27259	0.13151
10	0.92429	0.88516	0.84531	0.80322	0.75029	0.68222	0.58364	0.45507	0.29991	0.15108
11	0.93208	0.89646	0.85952	0.81992	0.76957	0.70427	0.60894	0.48233	0.32554	0.16966
12	0.93941	0.90715	0.87309	0.83602	0.78831	0.72597	0.63418	0.51005	0.35225	0.18969
13	0.94630	0.91727	0.88605	0.85152	0.80653	0.74726	0.65930	0.53815	0.38020	0.21121
14	0.95279	0.92687	0.89843	0.86642	0.82422	0.76814	0.68424	0.56652	0.40868	0.23416
15	0.95890	0.93595	0.91022	0.88074	0.84136	0.78855	0.70895	0.59506	0.43820	0.25850
16	0.96464	0.94456	0.92149	0.89449	0.85794	0.80848	0.73334	0.62370	0.46844	0.28417
17	0.97007	0.95273	0.93221	0.90769	0.87398	0.82792	0.75738	0.65232	0.49929	0.31107
18	0.97520	0.96055	0.94237	0.92048	0.88943	0.84673	0.78056	0.68022	0.53016	0.33879
19	0.98016	0.96789	0.95214	0.93270	0.90451	0.86538	0.80422	0.70957	0.56385	0.37067
20	0.98427	0.97429	0.96118	0.94415	0.91918	0.88416	0.82888	0.74129	0.60176	0.40831
21	0.98793	0.98005	0.96928	0.95483	0.93315	0.90240	0.85332	0.77345	0.64172	0.44988
22	0.99114	0.98514	0.97658	0.96467	0.94631	0.91994	0.87737	0.80622	0.68377	0.49584
23	0.99383	0.98951	0.98298	0.97353	0.95849	0.93658	0.90078	0.83903	0.72782	0.54670
24	0.99604	0.99312	0.98843	0.98132	0.96954	0.95213	0.92327	0.87164	0.77371	0.60306

TABLE 209. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 25, $l(N)/l(25)$, FOR MATERNAL ORPHANHOOD ANALYSIS, EAST MODEL

Level	Values of age N									
	35	40	45	50	55	60	65	70	75	80
1	0.83270	0.74996	0.67263	0.59814	0.51336	0.40987	0.28933	0.16797	0.07318	0.01991
2	0.84395	0.76869	0.69563	0.62424	0.54207	0.44032	0.31950	0.19388	0.09051	0.02770
3	0.85815	0.78606	0.71711	0.64883	0.56925	0.46930	0.34923	0.22028	0.10909	0.03678
4	0.86942	0.80223	0.73726	0.67207	0.59527	0.49812	0.37857	0.24710	0.12894	0.04712
5	0.87987	0.81736	0.75623	0.69411	0.62016	0.52563	0.40748	0.27423	0.14960	0.05864
6	0.88964	0.83154	0.77415	0.71508	0.64399	0.55231	0.43596	0.30161	0.17127	0.07131
7	0.89878	0.84492	0.79115	0.73507	0.66690	0.57817	0.46398	0.32913	0.19377	0.08510
8	0.90739	0.85757	0.80730	0.75418	0.68894	0.60329	0.49155	0.35676	0.21699	0.09991
9	0.91548	0.86954	0.82268	0.77245	0.71016	0.62770	0.51867	0.38442	0.24085	0.11570
10	0.92314	0.88091	0.83735	0.78996	0.73063	0.65142	0.54535	0.41210	0.26529	0.13243
11	0.92986	0.89103	0.85052	0.80575	0.74911	0.67302	0.56987	0.43781	0.28830	0.14857
12	0.93693	0.90141	0.86372	0.82132	0.76708	0.69367	0.59302	0.46193	0.30989	0.16377
13	0.94367	0.91138	0.87655	0.83659	0.78489	0.71439	0.61658	0.48693	0.33277	0.18031
14	0.95007	0.92095	0.88896	0.85151	0.80248	0.73510	0.64047	0.51274	0.35693	0.19830
15	0.95618	0.93012	0.90096	0.86609	0.81983	0.75575	0.66464	0.53929	0.38236	0.21772
16	0.96196	0.93891	0.91255	0.88027	0.83688	0.77630	0.68900	0.56652	0.40898	0.23864
17	0.96746	0.94730	0.92372	0.89403	0.85363	0.79665	0.71345	0.59434	0.43672	0.26102
18	0.97268	0.95532	0.93447	0.90745	0.86998	0.81674	0.73787	0.62250	0.46544	0.28478
19	0.97762	0.96297	0.94478	0.92053	0.88591	0.83648	0.76215	0.65096	0.49499	0.30984
20	0.98232	0.97027	0.95468	0.93283	0.90140	0.85584	0.78619	0.67953	0.52520	0.33609
21	0.98679	0.97718	0.96415	0.94512	0.91709	0.87599	0.81200	0.71142	0.56060	0.36845
22	0.99050	0.98325	0.97296	0.95711	0.93317	0.89753	0.84085	0.74877	0.60424	0.41078
23	0.99368	0.98857	0.98088	0.96827	0.94861	0.91890	0.87039	0.78848	0.65792	0.46070
24	0.99623	0.99296	0.98766	0.97829	0.96308	0.93962	0.90016	0.83034	0.70712	0.52007

TABLE 210. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 25, $l(N)/l(25)$, FOR MATERNAL ORPHANHOOD ANALYSIS, WEST MODEL

Level	Values of age N									
	35	40	45	50	55	60	65	70	75	80
1	0.79384	0.69402	0.60153	0.51694	0.42457	0.33044	0.22690	0.13609	0.06459	0.02138
2	0.81082	0.71760	0.62994	0.54835	0.45786	0.36375	0.25783	0.16222	0.08138	0.02955
3	0.82626	0.73928	0.65638	0.57791	0.48967	0.39686	0.28872	0.18823	0.09959	0.03907
4	0.84043	0.75936	0.68111	0.60588	0.52015	0.42969	0.31945	0.21490	0.11900	0.04984
5	0.85352	0.77805	0.70434	0.63240	0.54940	0.46239	0.34997	0.24207	0.13954	0.06181
6	0.86563	0.79552	0.72623	0.65762	0.57751	0.49487	0.38024	0.26964	0.16106	0.07492
7	0.87690	0.81191	0.74692	0.68165	0.60455	0.52736	0.41020	0.29752	0.18346	0.08913
8	0.88745	0.82734	0.76655	0.70460	0.63061	0.55970	0.43983	0.32560	0.20668	0.10432
9	0.89735	0.84190	0.78520	0.72656	0.65577	0.59191	0.46909	0.35383	0.23048	0.12048
10	0.90666	0.85569	0.80297	0.74762	0.68005	0.62393	0.49800	0.38215	0.25486	0.13750
11	0.91544	0.86875	0.81991	0.76781	0.70351	0.65575	0.52648	0.41049	0.27978	0.15534
12	0.92375	0.88113	0.83611	0.78721	0.72621	0.68732	0.55458	0.43881	0.30515	0.17393
13	0.93169	0.89282	0.85156	0.80560	0.74785	0.71857	0.58183	0.46663	0.33049	0.19291
14	0.93926	0.90383	0.86644	0.82343	0.76868	0.74945	0.60823	0.49414	0.35584	0.21234
15	0.94649	0.91421	0.88090	0.84078	0.78885	0.77990	0.63399	0.52089	0.38150	0.23225
16	0.95344	0.92409	0.89511	0.85777	0.80850	0.74969	0.65968	0.54764	0.40757	0.25277
17	0.95946	0.93301	0.90674	0.87219	0.82677	0.76661	0.68281	0.57428	0.43299	0.27384
18	0.96590	0.94486	0.91948	0.88836	0.84625	0.78948	0.70924	0.59971	0.45568	0.29097
19	0.97197	0.95421	0.93250	0.90426	0.86540	0.81216	0.73583	0.62573	0.48451	0.31497
20	0.97771	0.96311	0.94458	0.91943	0.88409	0.83455	0.76241	0.65823	0.51432	0.34034
21	0.98342	0.97202	0.95668	0.93473	0.90304	0.85751	0.78990	0.68927	0.54638	0.36826
22	0.98853	0.98006	0.96806	0.94985	0.92278	0.88265	0.82172	0.72730	0.58822	0.40722
23	0.99266	0.98689	0.97815	0.96389	0.94190	0.90800	0.85511	0.76895	0.63615	0.45408
24	0.99593	0.99245	0.98673	0.97644	0.95983	0.93291	0.88949	0.81411	0.69119	0.51140

TABLE 211. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 32.5, $l(N)/l(32.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS, NORTH MODEL

Level	Values of age N								
	40	45	50	55	60	65	70	75	80
1	0.82948	0.70925	0.58760	0.46849	0.34853	0.23527	0.13069	0.05058	0.01068
2	0.84321	0.73109	0.61389	0.50103	0.38221	0.26486	0.15625	0.06666	0.01717
3	0.85562	0.75098	0.64203	0.53129	0.41459	0.29612	0.18259	0.08441	0.02522
4	0.86686	0.76926	0.66640	0.55988	0.44576	0.32699	0.20954	0.10360	0.03486
5	0.87717	0.78618	0.68921	0.58696	0.47584	0.35739	0.23693	0.12452	0.04601
6	0.88666	0.80191	0.71060	0.61274	0.50486	0.38735	0.26462	0.14578	0.05860
7	0.89545	0.81657	0.73078	0.63727	0.53289	0.41682	0.29253	0.16844	0.07261
8	0.90363	0.83032	0.74982	0.66068	0.55999	0.44575	0.32065	0.19198	0.08791
9	0.91124	0.84322	0.76788	0.68305	0.58619	0.47414	0.34863	0.21632	0.10443
10	0.91838	0.85538	0.78500	0.70449	0.61155	0.50202	0.37669	0.24130	0.12218
11	0.92506	0.86686	0.80129	0.72502	0.63609	0.52934	0.40468	0.26684	0.14096
12	0.93136	0.87771	0.81634	0.74479	0.66033	0.55679	0.43331	0.29371	0.16151
13	0.93662	0.88696	0.83032	0.76206	0.68116	0.58026	0.45775	0.31666	0.17917
14	0.94181	0.89586	0.84289	0.77786	0.70016	0.60165	0.48018	0.33805	0.19597
15	0.94693	0.90470	0.85548	0.79388	0.71959	0.62370	0.50380	0.36100	0.21448
16	0.95198	0.91347	0.86806	0.81004	0.73942	0.64675	0.52857	0.38554	0.23479
17	0.95693	0.92209	0.88055	0.82624	0.75951	0.67026	0.55436	0.41161	0.25691
18	0.96171	0.93054	0.89288	0.84238	0.77973	0.69423	0.58104	0.43910	0.28085
19	0.96631	0.93875	0.90495	0.85835	0.79993	0.71846	0.60842	0.46786	0.30651
20	0.97082	0.94669	0.91671	0.87405	0.81999	0.74277	0.63630	0.49768	0.33376
21	0.97509	0.95433	0.92810	0.88938	0.83975	0.76702	0.66445	0.52834	0.36244
22	0.97927	0.96177	0.93874	0.90343	0.85754	0.78883	0.69009	0.55677	0.38987
23	0.98403	0.97002	0.95106	0.92065	0.88050	0.81840	0.72651	0.59915	0.43289
24	0.98840	0.97800	0.96343	0.93865	0.90526	0.85142	0.76876	0.65043	0.48761

TABLE 212. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 32.5, $l(N)/l(32.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS, SOUTH MODEL

Level	Values of age N								
	40	45	50	55	60	65	70	75	80
1	0.86716	0.77288	0.67577	0.57120	0.45506	0.32438	0.19543	0.08695	0.02362
2	0.87767	0.78957	0.69760	0.59599	0.48333	0.35315	0.22101	0.10483	0.03185
3	0.88715	0.80481	0.71769	0.62005	0.51007	0.38089	0.24640	0.12342	0.04105
4	0.89585	0.81883	0.73636	0.64261	0.53546	0.40770	0.27155	0.14257	0.05120
5	0.90383	0.83182	0.75377	0.66383	0.55964	0.43363	0.29644	0.16223	0.06215
6	0.91120	0.84390	0.77008	0.68388	0.58268	0.45870	0.32103	0.18224	0.07390
7	0.91805	0.85519	0.78542	0.70287	0.60471	0.48298	0.34531	0.20256	0.08631
8	0.92442	0.86577	0.79987	0.72093	0.62581	0.50653	0.36924	0.22310	0.09935
9	0.93042	0.87573	0.81356	0.73810	0.64609	0.52938	0.39180	0.24384	0.11296
10	0.93546	0.88419	0.82521	0.75298	0.66384	0.54963	0.41394	0.26271	0.12571
11	0.94036	0.89234	0.83652	0.76727	0.68078	0.56898	0.43431	0.28106	0.13828
12	0.94557	0.90102	0.84843	0.78217	0.69841	0.58911	0.45563	0.30063	0.15198
13	0.95065	0.90939	0.86000	0.79680	0.71589	0.60931	0.47735	0.32098	0.16663
14	0.95528	0.91743	0.87123	0.81112	0.73316	0.62951	0.49940	0.34206	0.18217
15	0.95979	0.92511	0.88203	0.82509	0.75018	0.64968	0.52172	0.36379	0.19862
16	0.96408	0.93250	0.89255	0.83870	0.76693	0.66972	0.54421	0.38607	0.21592
17	0.96816	0.93956	0.90264	0.85193	0.78336	0.68958	0.56678	0.40886	0.23401
18	0.97208	0.94637	0.91245	0.86486	0.79956	0.70937	0.58955	0.43223	0.25300
19	0.97687	0.95479	0.92454	0.88076	0.81912	0.73360	0.61794	0.46226	0.27834
20	0.98077	0.96180	0.93505	0.89523	0.83824	0.75777	0.64724	0.49447	0.30671
21	0.98437	0.96843	0.94520	0.90954	0.85737	0.78276	0.67824	0.52955	0.33892
22	0.98770	0.97469	0.95500	0.92368	0.87673	0.80862	0.71119	0.56404	0.37583
23	0.99074	0.98050	0.96434	0.93755	0.89624	0.83537	0.74621	0.61046	0.41846
24	0.99344	0.98580	0.97310	0.95098	0.91575	0.86291	0.78342	0.65734	0.46815

TABLE 213. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 32.5, $l(N)/l(32.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS. EAST MODEL

	Values of age N								
Level	40	45	50	55	60	65	70	75	80
1	0.86251	0.75843	0.64935	0.53757	0.42175	0.30226	0.18412	0.08791	0.02798
2	0.87299	0.77559	0.67196	0.56368	0.44918	0.32859	0.20651	0.10329	0.03548
3	0.88263	0.79147	0.69303	0.58828	0.47540	0.35426	0.22886	0.11922	0.04373
4	0.89144	0.80618	0.71278	0.61158	0.50057	0.37925	0.25113	0.13559	0.05281
5	0.89961	0.81990	0.73137	0.63369	0.52472	0.40359	0.27327	0.15233	0.06267
6	0.90720	0.83275	0.74890	0.65479	0.54797	0.42735	0.29525	0.16938	0.07311
7	0.91432	0.84483	0.76551	0.67491	0.57038	0.45051	0.31706	0.18670	0.08266
8	0.92097	0.85622	0.78127	0.69414	0.59200	0.47312	0.33866	0.20423	0.09386
9	0.92722	0.86698	0.79627	0.71259	0.61288	0.49516	0.36003	0.22191	0.10506
10	0.93312	0.87719	0.81055	0.73029	0.63306	0.51670	0.38119	0.23973	0.11686
11	0.93853	0.88663	0.82383	0.74675	0.65193	0.53683	0.40099	0.25643	0.12806
12	0.94337	0.89497	0.83580	0.76093	0.66801	0.55394	0.41790	0.27087	0.13787
13	0.94810	0.90318	0.84693	0.77521	0.68436	0.57154	0.43552	0.28618	0.14847
14	0.95272	0.91126	0.85838	0.78958	0.70095	0.58960	0.45385	0.30235	0.15991
15	0.95723	0.91921	0.86973	0.80388	0.71773	0.60808	0.47285	0.31939	0.17222
16	0.96161	0.92694	0.88093	0.81819	0.73463	0.62692	0.49246	0.33726	0.18540
17	0.96587	0.93456	0.89195	0.83241	0.75160	0.64602	0.51263	0.35593	0.19944
18	0.96997	0.94193	0.90276	0.84645	0.76853	0.66529	0.53322	0.37530	0.21431
19	0.97393	0.94907	0.91329	0.86027	0.78535	0.68462	0.55414	0.39527	0.22993
20	0.97772	0.95596	0.92352	0.87381	0.80195	0.70391	0.57523	0.41573	0.24625
21	0.98138	0.96265	0.93379	0.88787	0.81982	0.72541	0.59972	0.44052	0.26694
22	0.98527	0.97002	0.94542	0.90444	0.84165	0.75251	0.63148	0.47365	0.29360
23	0.98897	0.97712	0.95699	0.92147	0.86483	0.78223	0.66744	0.51256	0.33076
24	0.99237	0.98379	0.96823	0.93875	0.88934	0.81488	0.70848	0.55892	0.37482

TABLE 214 MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 32.5, $l(N)/l(32.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS. WEST MODEL

	Values of age N								
Level	40	45	50	55	60	65	70	75	80
1	0.80605	0.67240	0.54608	0.41998	0.30638	0.19863	0.11122	0.04920	0.01436
2	0.82220	0.69724	0.57666	0.45353	0.33924	0.22762	0.13323	0.06271	0.02026
3	0.83682	0.72006	0.60519	0.48545	0.37127	0.25662	0.15610	0.07747	0.02717
4	0.85017	0.74116	0.63195	0.51590	0.40239	0.28557	0.17964	0.09332	0.03513
5	0.86242	0.76080	0.65716	0.54503	0.43270	0.31438	0.20375	0.11016	0.04404
6	0.87374	0.77909	0.68096	0.57293	0.46217	0.34298	0.22828	0.12788	0.05385
7	0.88423	0.79625	0.70351	0.59969	0.49088	0.37134	0.25317	0.14640	0.06455
8	0.89402	0.81237	0.72491	0.62541	0.51879	0.39939	0.27831	0.16560	0.07608
9	0.90317	0.82758	0.74529	0.65015	0.54599	0.42713	0.30365	0.18542	0.08837
10	0.91176	0.84197	0.76472	0.67398	0.57248	0.45452	0.32909	0.20577	0.10138
11	0.91983	0.85561	0.78329	0.69694	0.59826	0.48154	0.35460	0.22660	0.11506
12	0.92743	0.86855	0.80103	0.71909	0.62338	0.50818	0.38010	0.24782	0.12936
13	0.93453	0.88079	0.81806	0.74052	0.64784	0.53441	0.40628	0.26988	0.14454
14	0.94138	0.89241	0.83392	0.76048	0.67065	0.55874	0.42912	0.28919	0.15791
15	0.94757	0.90297	0.84843	0.77868	0.69149	0.58122	0.45116	0.30810	0.17132
16	0.95359	0.91333	0.86287	0.79701	0.71276	0.60449	0.47436	0.32849	0.18601
17	0.95942	0.92348	0.87718	0.81541	0.73438	0.62848	0.49863	0.35014	0.20199
18	0.96505	0.93336	0.89127	0.83375	0.75620	0.65303	0.52388	0.37305	0.21927
19	0.97047	0.94295	0.90507	0.85193	0.77812	0.67803	0.54997	0.39715	0.23784
20	0.97565	0.95222	0.91855	0.86988	0.79999	0.70331	0.57674	0.42230	0.25763
21	0.98043	0.96070	0.93108	0.88673	0.82107	0.72832	0.60399	0.44869	0.27905
22	0.98520	0.96965	0.94495	0.90646	0.84687	0.76030	0.64034	0.48554	0.31049
23	0.98965	0.97817	0.95870	0.92673	0.87442	0.79575	0.68224	0.52974	0.34996
24	0.99355	0.98588	0.97169	0.94680	0.90310	0.83445	0.73022	0.58304	0.40035

TABLE 215 MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 37.5, $_{l}(N)/_{l}(37.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS, NORTH MODEL

Level	Values of age N							
	45	50	55	60	65	70	75	80
1	0.79981	0.66263	0.52876	0.39304	0.26305	0.14737	0.05704	0.01205
2	0.81591	0.68735	0.55916	0.42655	0.29559	0.17438	0.07440	0.01917
3	0.83038	0.70991	0.58747	0.45842	0.32743	0.20190	0.09334	0.02789
4	0.84355	0.73075	0.61394	0.48881	0.35856	0.22978	0.11360	0.03822
5	0.85563	0.75009	0.63881	0.51788	0.38896	0.25785	0.13508	0.05007
6	0.86676	0.76807	0.66229	0.54569	0.41867	0.28602	0.15757	0.06334
7	0.87707	0.78491	0.68448	0.57236	0.44769	0.31420	0.18092	0.07798
8	0.88664	0.80069	0.70550	0.59797	0.47599	0.34230	0.20501	0.09388
9	0.89558	0.81546	0.72547	0.62259	0.50358	0.37028	0.22975	0.11096
10	0.90396	0.82958	0.74450	0.64628	0.53053	0.39808	0.25500	0.12912
11	0.91183	0.84286	0.76264	0.66909	0.55680	0.42567	0.28068	0.14827
12	0.91922	0.85542	0.78001	0.69155	0.58312	0.45381	0.30760	0.16915
13	0.92555	0.86644	0.79521	0.71080	0.60550	0.47766	0.33044	0.18697
14	0.93150	0.87642	0.80880	0.72801	0.62558	0.49928	0.35149	0.20377
15	0.93739	0.88639	0.82256	0.74560	0.64624	0.52200	0.37404	0.22223
16	0.94322	0.89634	0.83643	0.76350	0.66781	0.54578	0.39810	0.24241
17	0.94894	0.90619	0.85029	0.78162	0.68977	0.57050	0.42360	0.26439
18	0.95453	0.91590	0.86409	0.79983	0.71212	0.59602	0.45043	0.28809
19	0.95995	0.92539	0.87773	0.81800	0.73468	0.62216	0.47843	0.31343
20	0.96517	0.93461	0.89112	0.83600	0.75727	0.64873	0.50739	0.34027
21	0.97019	0.94353	0.90416	0.85371	0.77977	0.67549	0.53712	0.36846
22	0.97504	0.95169	0.91590	0.86937	0.79972	0.69962	0.56443	0.39525
23	0.98024	0.96108	0.93035	0.88978	0.82702	0.73417	0.60246	0.43745
24	0.98546	0.97078	0.94581	0.91216	0.85791	0.77462	0.65539	0.49133

TABLE 216. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 37.5, $_{l}(N)/_{l}(37.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS, SOUTH MODEL

Level	Values of age N							
	45	50	55	60	65	70	75	80
1	0.84796	0.74142	0.62559	0.49927	0.35589	0.21441	0.09540	0.02591
2	0.85935	0.75942	0.64881	0.52617	0.38445	0.24060	0.11412	0.03467
3	0.87008	0.77589	0.67033	0.55143	0.41178	0.26638	0.13343	0.04438
4	0.87965	0.79106	0.69035	0.57524	0.43798	0.29172	0.15317	0.05500
5	0.88848	0.80512	0.70904	0.59776	0.46316	0.31663	0.17328	0.06639
6	0.89664	0.81821	0.72662	0.61909	0.48737	0.34110	0.19363	0.07852
7	0.90422	0.83045	0.74317	0.63938	0.51067	0.36511	0.21417	0.09126
8	0.91129	0.84193	0.75883	0.65872	0.53316	0.38865	0.23483	0.10457
9	0.91790	0.85273	0.77364	0.67720	0.55488	0.41172	0.25558	0.11840
10	0.92350	0.86194	0.78646	0.69336	0.57407	0.43234	0.27439	0.13130
11	0.92883	0.87074	0.79885	0.70863	0.59225	0.45208	0.29256	0.14394
12	0.93449	0.87994	0.81122	0.72435	0.61099	0.47255	0.31180	0.15763
13	0.93993	0.88889	0.82356	0.73993	0.62977	0.49338	0.33176	0.17220
14	0.94515	0.89756	0.83564	0.75533	0.64855	0.51450	0.35240	0.18767
15	0.95015	0.90594	0.84742	0.77049	0.66717	0.53584	0.37361	0.20399
16	0.95495	0.91404	0.85889	0.78539	0.68564	0.55731	0.39537	0.22111
17	0.95952	0.92182	0.87003	0.80001	0.70423	0.57882	0.41753	0.23896
18	0.96393	0.92937	0.88091	0.81439	0.72253	0.60049	0.44025	0.25770
19	0.96937	0.93866	0.89421	0.83184	0.74450	0.62738	0.46932	0.28249
20	0.97395	0.94686	0.90654	0.84883	0.76714	0.65341	0.50071	0.31038
21	0.97833	0.95486	0.91884	0.86614	0.79076	0.68518	0.53496	0.34239
22	0.98249	0.96264	0.93108	0.88375	0.81510	0.71689	0.57219	0.37884
23	0.98638	0.97012	0.94317	0.90162	0.84038	0.75069	0.61412	0.42097
24	0.98996	0.97721	0.95500	0.91962	0.86636	0.78673	0.66012	0.47013

TABLE 217. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 37.5, $_{l}(N)_{x}/_{l}(37.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS. EAST MODEL

	Values of age x							
Level	45	50	55	60	65	70	75	80
1	0.83337	0.71351	0.59069	0.46342	0.33212	0.20232	0.09660	0.03074
2	0.84576	0.73276	0.61468	0.48982	0.35832	0.22319	0.11263	0.03869
3	0.85713	0.75053	0.63708	0.51484	0.38366	0.24785	0.12911	0.04736
4	0.86759	0.76707	0.65817	0.53870	0.40813	0.27026	0.14592	0.05662
5	0.87727	0.78254	0.67803	0.56144	0.43183	0.29239	0.16298	0.06642
6	0.88628	0.79704	0.69688	0.58320	0.45483	0.31425	0.18027	0.07675
7	0.89469	0.81068	0.71474	0.60404	0.47710	0.33577	0.19771	0.08754
8	0.90258	0.82357	0.73173	0.62405	0.49874	0.35700	0.21529	0.09873
9	0.90999	0.83577	0.74795	0.64329	0.51973	0.37789	0.23292	0.11028
10	0.91700	0.84734	0.76343	0.66179	0.54015	0.39848	0.25061	0.12216
11	0.92346	0.85806	0.77777	0.67901	0.55913	0.41765	0.26709	0.13338
12	0.92909	0.86725	0.78994	0.69348	0.57506	0.43383	0.28120	0.14312
13	0.93461	0.87641	0.80218	0.70817	0.59143	0.45068	0.29614	0.15364
14	0.94004	0.88548	0.81448	0.72308	0.60822	0.46818	0.31190	0.16496
15	0.94537	0.89449	0.82676	0.73816	0.62539	0.48631	0.32849	0.17713
16	0.95057	0.90335	0.83902	0.75332	0.64287	0.50500	0.34585	0.19012
17	0.95564	0.91207	0.85118	0.76856	0.66059	0.52419	0.36395	0.20394
18	0.96056	0.92061	0.86319	0.78372	0.67844	0.54377	0.38272	0.21855
19	0.96531	0.92892	0.87499	0.79879	0.69635	0.56362	0.40203	0.23387
20	0.96989	0.93698	0.88654	0.81364	0.71417	0.58361	0.42179	0.24984
21	0.97433	0.94512	0.89864	0.82976	0.73421	0.60699	0.44586	0.27018
22	0.97929	0.95448	0.91309	0.84970	0.75970	0.63751	0.47817	0.29843
23	0.98408	0.96382	0.92804	0.87100	0.78781	0.67220	0.51622	0.33312
24	0.98862	0.97299	0.94336	0.89371	0.81889	0.71196	0.56166	0.37666

TABLE 218. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 37.5, $_{l}(N)_{x}/_{l}(37.5)$, FOR PATERNAL ORPHANHOOD ANALYSIS. WEST MODEL

	Values of age x							
Level	45	50	55	60	65	70	75	80
1	0.77179	0.62680	0.48206	0.35166	0.22799	0.12786	0.05647	0.01648
2	0.79036	0.65367	0.51410	0.38455	0.25801	0.15103	0.07109	0.02296
3	0.80717	0.67840	0.54417	0.41618	0.28767	0.17498	0.08685	0.03045
4	0.82252	0.70132	0.57254	0.44656	0.31692	0.19936	0.10357	0.03899
5	0.83667	0.72269	0.59939	0.47585	0.34573	0.22407	0.12114	0.04843
6	0.84970	0.74267	0.62485	0.50406	0.37407	0.24897	0.13947	0.05873
7	0.86182	0.76144	0.64908	0.53131	0.40192	0.27402	0.15846	0.06987
8	0.87310	0.77910	0.67217	0.55758	0.42925	0.29912	0.17799	0.08177
9	0.88367	0.79580	0.69422	0.58300	0.45608	0.32423	0.19798	0.09436
10	0.89360	0.81161	0.71531	0.60758	0.48239	0.34927	0.21839	0.10759
11	0.90294	0.82662	0.73550	0.63136	0.50818	0.37421	0.23914	0.12143
12	0.91176	0.84088	0.75486	0.65439	0.53346	0.39901	0.26015	0.13580
13	0.92006	0.85455	0.77337	0.67736	0.55898	0.42441	0.28192	0.15099
14	0.92782	0.86701	0.79066	0.69727	0.58092	0.44615	0.30067	0.16418
15	0.93483	0.87837	0.80616	0.71589	0.60173	0.46708	0.31905	0.17737
16	0.94170	0.88968	0.82177	0.73490	0.62327	0.48910	0.33869	0.19179
17	0.94842	0.90087	0.83743	0.75421	0.64543	0.51210	0.35959	0.20744
18	0.95496	0.91189	0.85304	0.77369	0.66814	0.53599	0.38168	0.22434
19	0.96128	0.92267	0.86849	0.79324	0.69121	0.56066	0.40487	0.24246
20	0.96738	0.93318	0.88373	0.81273	0.71452	0.58592	0.42903	0.26173
21	0.97292	0.94293	0.89802	0.83132	0.73759	0.61168	0.45480	0.28260
22	0.97891	0.95399	0.91513	0.85497	0.76757	0.64647	0.49018	0.31346
23	0.98467	0.96507	0.93289	0.88024	0.80104	0.68677	0.53326	0.35229
24	0.98993	0.97568	0.95070	0.90682	0.83788	0.73323	0.58544	0.40200

Annex VII

ADULT SURVIVORSHIP RATIOS FROM COALE-DEMENY MODEL LIFE TABLES FOR USE WITH ADULT MORTALITY ESTIMATES FROM WIDOWHOOD

TABLE 219. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $l(N)/l(20)$, FOR ANALYSIS BASED ON SURVIVAL OF FIRST WIFE. NORTH MODEL

	Values of age N							
Level	25	30	35	40	45	50	55	60
1	0.92360	0.84150	0.75384	0.66386	0.58035	0.50282	0.42406	0.33544
2	0.93018	0.85452	0.77309	0.68875	0.60931	0.53454	0.45754	0.36912
3	0.93598	0.86622	0.79065	0.71169	0.63628	0.56440	0.48922	0.40178
4	0.94124	0.87690	0.80678	0.73295	0.66152	0.59262	0.51945	0.43350
5	0.94604	0.88669	0.82168	0.75275	0.68523	0.61938	0.54848	0.46428
6	0.95046	0.89574	0.83555	0.77128	0.70759	0.64483	0.57631	0.49423
7	0.95452	0.90411	0.84845	0.78868	0.72874	0.66909	0.60305	0.52332
8	0.95830	0.91193	0.86056	0.80509	0.74881	0.69225	0.62881	0.55160
9	0.96182	0.91922	0.87192	0.82058	0.76788	0.71441	0.65360	0.57911
10	0.96511	0.92608	0.88264	0.83527	0.78605	0.73564	0.67753	0.60589
11	0.96819	0.93253	0.89274	0.84919	0.80336	0.75601	0.70061	0.63194
12	0.97108	0.93859	0.90230	0.86242	0.81990	0.77553	0.72288	0.65725
13	0.97350	0.94372	0.91049	0.87390	0.83424	0.79250	0.74224	0.67951
14	0.97615	0.94925	0.91921	0.88590	0.84913	0.80988	0.76182	0.70150
15	0.97870	0.95457	0.92762	0.89758	0.86375	0.82709	0.78142	0.72376
16	0.98113	0.95970	0.93577	0.90893	0.87801	0.84409	0.80096	0.74620
17	0.98346	0.96463	0.94360	0.91989	0.89199	0.86078	0.82033	0.76869
18	0.98569	0.96933	0.95110	0.93043	0.90549	0.87710	0.83942	0.79111
19	0.98777	0.97379	0.95823	0.94052	0.91850	0.89293	0.85815	0.81328
20	0.98975	0.97801	0.96503	0.95016	0.93100	0.90827	0.87641	0.83513
21	0.99163	0.98203	0.97148	0.95935	0.94299	0.92306	0.89416	0.85653
22	0.99342	0.98589	0.97773	0.96827	0.95469	0.93750	0.91164	0.87731
23	0.99512	0.99058	0.98491	0.97804	0.96762	0.95355	0.93149	0.90183
24	0.99719	0.99378	0.99005	0.98562	0.97792	0.96722	0.94949	0.92534

TABLE 220 FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $l(N)/l(20)$, FOR ANALYSIS BASED ON SURVIVAL OF FIRST WIFE. SOUTH MODEL

	Values of age N							
Level	25	30	35	40	45	50	55	60
1	0.91423	0.83778	0.76130	0.68837	0.62168	0.55836	0.48509	0.39947
2	0.92251	0.85153	0.78061	0.71229	0.64896	0.58806	0.51640	0.43239
3	0.93208	0.86407	0.79835	0.73440	0.67443	0.61604	0.54696	0.46415
4	0.93808	0.87557	0.81473	0.75500	0.69831	0.64247	0.57571	0.49480
5	0.94356	0.88617	0.82996	0.77426	0.72080	0.66751	0.60321	0.52443
6	0.94861	0.89599	0.84414	0.79234	0.74202	0.69132	0.62955	0.55310
7	0.95332	0.90516	0.85745	0.80940	0.76216	0.71402	0.65484	0.58089
8	0.95768	0.91373	0.86995	0.82551	0.78130	0.73571	0.67915	0.60783
9	0.96176	0.92178	0.88173	0.84077	0.79953	0.75647	0.70256	0.63397
10	0.96535	0.92883	0.89227	0.85450	0.81602	0.77540	0.72430	0.65858
11	0.96922	0.93647	0.90339	0.86885	0.83307	0.79469	0.74588	0.68259
12	0.97278	0.94362	0.91384	0.88246	0.84933	0.81328	0.76686	0.70621
13	0.97611	0.95028	0.92370	0.89536	0.86489	0.83118	0.78727	0.72942
14	0.97920	0.95652	0.93298	0.90759	0.87975	0.84840	0.80708	0.75217
15	0.98209	0.96238	0.94173	0.91920	0.89392	0.86497	0.82629	0.77443
16	0.98479	0.96786	0.94998	0.93020	0.90748	0.88089	0.84490	0.79619
17	0.98733	0.97304	0.95779	0.94066	0.92043	0.89620	0.86291	0.81743
18	0.98972	0.97794	0.96524	0.95068	0.93288	0.91102	0.88029	0.83802
19	0.99202	0.98281	0.97235	0.96018	0.94475	0.92526	0.89730	0.85848
20	0.99381	0.98657	0.97819	0.96827	0.95524	0.93831	0.91350	0.87870
21	0.99537	0.98987	0.98337	0.97552	0.96480	0.95042	0.92884	0.89822
22	0.99668	0.99269	0.98785	0.98188	0.97335	0.96147	0.94317	0.91689
23	0.99777	0.99504	0.99162	0.98730	0.98080	0.97136	0.95635	0.93450
24	0.99861	0.99688	0.99466	0.99174	0.98708	0.97997	0.96820	0.95081

TABLE 221. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $_{n}l_{x}/l_{20}$, FOR ANALYSES BASED ON SURVIVAL OF FIRST WIFE, EAST MODEL

	Values of age n							
Level	25	30	35	40	45	50	55	60
1	0.92947	0.85217	0.77397	0.69707	0.62519	0.55596	0.47715	0.38096
2	0.93531	0.86399	0.79123	0.71897	0.65063	0.58386	0.50695	0.41883
3	0.94061	0.87480	0.80719	0.73938	0.67452	0.61030	0.53545	0.44880
4	0.94549	0.88482	0.82203	0.75850	0.69708	0.63544	0.56283	0.47097
5	0.95001	0.89412	0.83589	0.77654	0.71843	0.65942	0.58916	0.49936
6	0.95420	0.90278	0.84890	0.79346	0.73870	0.68233	0.61450	0.52702
7	0.95811	0.91090	0.86113	0.80953	0.75801	0.70429	0.63897	0.55396
8	0.96175	0.91851	0.87269	0.82477	0.77643	0.72533	0.66260	0.58022
9	0.96518	0.92569	0.88360	0.83927	0.79404	0.74555	0.68544	0.60584
10	0.96840	0.93247	0.89397	0.85308	0.81089	0.76500	0.70754	0.63083
11	0.97108	0.93823	0.90297	0.86526	0.82593	0.78245	0.72745	0.65356
12	0.97414	0.94464	0.91271	0.87810	0.84139	0.80008	0.74725	0.67574
13	0.97702	0.95072	0.92199	0.89045	0.85641	0.81737	0.76686	0.69798
14	0.97973	0.95646	0.93082	0.90229	0.87095	0.83426	0.78622	0.72021
15	0.98229	0.96191	0.93925	0.91366	0.88501	0.85075	0.80532	0.74237
16	0.98470	0.96706	0.94725	0.92455	0.89859	0.86681	0.82408	0.76442
17	0.98698	0.97192	0.95486	0.93499	0.91169	0.88243	0.84251	0.78628
18	0.98912	0.97653	0.96210	0.94493	0.92431	0.89757	0.86052	0.80786
19	0.99113	0.98087	0.96895	0.95444	0.93641	0.91224	0.87806	0.82907
20	0.99304	0.98501	0.97549	0.96352	0.94804	0.92635	0.89513	0.84989
21	0.99503	0.98914	0.98189	0.97233	0.95937	0.94043	0.91254	0.87164
22	0.99649	0.99231	0.98703	0.97981	0.96955	0.95375	0.92990	0.89439
23	0.99771	0.99497	0.99141	0.98632	0.97864	0.96606	0.94645	0.91680
24	0.99867	0.99707	0.99491	0.99164	0.98636	0.97700	0.96130	0.93838

TABLE 222. FEMALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $_{n}l_{x}/l_{20}$, FOR ANALYSES BASED ON SURVIVAL OF FIRST WIFE, WEST MODEL

	Values of age n							
Level	25	30	35	40	45	50	55	60
1	0.90820	0.81509	0.72097	0.63031	0.54631	0.46949	0.38360	0.30010
2	0.91623	0.83048	0.74290	0.65748	0.57717	0.50241	0.41950	0.33328
3	0.92349	0.84449	0.76304	0.68271	0.60616	0.53370	0.45221	0.36585
4	0.93009	0.85731	0.78168	0.70627	0.63349	0.56352	0.48379	0.39779
5	0.93611	0.86912	0.79899	0.72834	0.65934	0.59200	0.51430	0.42911
6	0.94164	0.88007	0.81512	0.74910	0.68384	0.61925	0.54381	0.45978
7	0.94678	0.89026	0.83024	0.76870	0.70717	0.64537	0.57238	0.48983
8	0.95155	0.89977	0.84446	0.78725	0.72941	0.67047	0.60006	0.51926
9	0.95598	0.90867	0.85785	0.80485	0.75064	0.69458	0.62690	0.54808
10	0.96014	0.91707	0.87052	0.82159	0.77097	0.71782	0.65295	0.57630
11	0.96404	0.92496	0.88253	0.83751	0.79043	0.74020	0.67822	0.60393
12	0.96772	0.93244	0.89394	0.85274	0.80912	0.76180	0.70277	0.63098
13	0.97123	0.93942	0.90462	0.86707	0.82680	0.78236	0.72627	0.65705
14	0.97388	0.94500	0.91336	0.87905	0.84186	0.79999	0.74665	0.67982
15	0.97729	0.95198	0.92398	0.89316	0.85904	0.81974	0.76899	0.70443
16	0.98050	0.95857	0.93407	0.90667	0.87510	0.83909	0.79117	0.72919
17	0.98330	0.96476	0.94363	0.91959	0.89079	0.85800	0.81309	0.75397
18	0.98629	0.97056	0.95266	0.93191	0.90727	0.87638	0.83465	0.77865
19	0.98890	0.97601	0.96118	0.94363	0.92215	0.89422	0.85580	0.80315
20	0.99135	0.98113	0.96925	0.95478	0.93642	0.91148	0.87645	0.82734
21	0.99375	0.98622	0.97729	0.96596	0.95071	0.92889	0.89741	0.85216
22	0.99588	0.99078	0.98446	0.97602	0.96407	0.94593	0.91898	0.87902
23	0.99746	0.99426	0.99014	0.98439	0.97567	0.96144	0.93951	0.90569
24	0.99866	0.99692	0.99460	0.99113	0.98541	0.97514	0.95855	0.93167

TABLE 223. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $_{l}(N')_{x}/{}_{l}(20)$, FOR ANALYSIS BASED ON SURVIVAL OF FIRST HUSBAND, NORTH MODEL

Level	Values of age x							
	25	30	35	40	45	50	55	60
1	0.90632	0.81625	0.72975	0.64120	0.54825	0.45422	0.36245	0.26942
2	0.91394	0.83055	0.74973	0.66626	0.57166	0.48664	0.39588	0.30200
3	0.92039	0.84351	0.76799	0.68942	0.60510	0.51732	0.42809	0.33405
4	0.92607	0.85531	0.78479	0.71087	0.63084	0.54648	0.45913	0.36555
5	0.93261	0.86615	0.80032	0.73089	0.65508	0.57421	0.48908	0.39649
6	0.93783	0.87618	0.81478	0.74966	0.67800	0.60080	0.51806	0.42683
7	0.94260	0.88548	0.82825	0.76729	0.69970	0.62618	0.54606	0.45661
8	0.94705	0.89414	0.84091	0.78397	0.72032	0.65049	0.57316	0.48580
9	0.95119	0.90225	0.85278	0.79964	0.73994	0.67383	0.59939	0.51439
10	0.95503	0.90987	0.86398	0.81453	0.75866	0.69623	0.62483	0.54240
11	0.95869	0.91704	0.87458	0.82868	0.77655	0.71781	0.64949	0.56982
12	0.96208	0.92378	0.88458	0.84212	0.79369	0.73853	0.67343	0.59705
13	0.96490	0.92993	0.89292	0.85340	0.80815	0.75655	0.69435	0.62084
14	0.96778	0.93513	0.90147	0.86487	0.82267	0.77403	0.71431	0.64296
15	0.97062	0.94081	0.90995	0.87627	0.83720	0.79165	0.73464	0.66590
16	0.97341	0.94640	0.91830	0.88759	0.85168	0.80935	0.75525	0.68940
17	0.97611	0.95186	0.92650	0.89872	0.86601	0.82700	0.77599	0.71332
18	0.97874	0.95716	0.93450	0.90961	0.88013	0.84451	0.79675	0.73749
19	0.98123	0.96228	0.94224	0.92022	0.89394	0.86175	0.81738	0.76175
20	0.98370	0.96720	0.94972	0.93049	0.90737	0.87863	0.83775	0.78593
21	0.98602	0.97190	0.95688	0.94037	0.92036	0.89506	0.85771	0.80985
22	0.98828	0.97649	0.96388	0.95008	0.93310	0.91076	0.87650	0.83198
23	0.99140	0.98271	0.97313	0.96231	0.94861	0.93006	0.90033	0.86106
24	0.99378	0.98753	0.98056	0.97263	0.96240	0.94801	0.92367	0.89082

TABLE 224. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $_{l}(N')_{x}/{}_{l}(20)$, FOR ANALYSIS BASED ON SURVIVAL OF FIRST HUSBAND, SOUTH MODEL

Level	Values of age x							
	25	30	35	40	45	50	55	60
1	0.91403	0.83532	0.76452	0.69366	0.61824	0.54056	0.45611	0.36401
2	0.92132	0.84875	0.78266	0.71593	0.64406	0.56903	0.48615	0.39426
3	0.92792	0.86099	0.79928	0.73646	0.66811	0.59578	0.51473	0.42343
4	0.93391	0.87221	0.81461	0.75557	0.69061	0.62106	0.54199	0.45162
5	0.93943	0.88256	0.82886	0.77342	0.71180	0.64502	0.56805	0.47889
6	0.94451	0.89217	0.84215	0.79016	0.73179	0.66778	0.59304	0.50528
7	0.94922	0.90109	0.85458	0.80590	0.75072	0.68947	0.61701	0.53084
8	0.95359	0.90946	0.86626	0.82078	0.76869	0.71018	0.64009	0.55564
9	0.95770	0.91730	0.87727	0.83485	0.78579	0.73000	0.66229	0.57973
10	0.96154	0.92397	0.88664	0.84689	0.80047	0.74711	0.68168	0.60098
11	0.96479	0.93110	0.89647	0.85929	0.81549	0.76441	0.70112	0.62289
12	0.96869	0.93812	0.90638	0.87206	0.83097	0.78247	0.72135	0.64611
13	0.97195	0.94471	0.91579	0.88426	0.84596	0.80002	0.74122	0.66996
14	0.97516	0.95092	0.92471	0.89588	0.86037	0.81705	0.76068	0.69257
15	0.97820	0.95681	0.93319	0.90700	0.87423	0.83356	0.77971	0.71493
16	0.98103	0.96235	0.94122	0.91760	0.88755	0.84952	0.79827	0.73695
17	0.98372	0.96758	0.94883	0.92771	0.90030	0.86493	0.81633	0.75863
18	0.98624	0.97256	0.95615	0.93743	0.91264	0.87993	0.83404	0.77986
19	0.98934	0.97825	0.96472	0.94903	0.92757	0.89818	0.85565	0.79597
20	0.99155	0.98239	0.97141	0.95821	0.93968	0.91334	0.87464	0.81896
21	0.99349	0.98646	0.97741	0.96659	0.95094	0.92813	0.89311	0.84189
22	0.99518	0.98988	0.98280	0.97421	0.96138	0.94196	0.91107	0.86476
23	0.99661	0.99282	0.98754	0.98101	0.97088	0.95487	0.92834	0.88744
24	0.99780	0.99526	0.99136	0.98691	0.97932	0.96670	0.94473	0.90973

TABLE 225 MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $l(N)/l(20)$, FOR ANALYSIS BASED ON SURVIVAL OF FIRST HUSBAND. EAST MODEL

	Values of age N							
Level	25	30	35	40	45	50	55	60
1	0.92932	0.86200	0.79193	0.72327	0.62720	0.53699	0.44456	0.34878
2	0.93476	0.87228	0.80692	0.73297	0.65118	0.56418	0.47327	0.37713
3	0.93969	0.88171	0.82070	0.75128	0.67370	0.58992	0.50075	0.40667
4	0.94427	0.89041	0.83351	0.76839	0.69490	0.61439	0.52716	0.43148
5	0.94848	0.89850	0.84549	0.78446	0.71495	0.63775	0.55258	0.45756
6	0.95236	0.90603	0.85667	0.79956	0.73394	0.66004	0.57710	0.48296
7	0.95600	0.91307	0.86718	0.81387	0.75201	0.68140	0.60076	0.50771
8	0.95940	0.91967	0.87709	0.82739	0.76922	0.70189	0.62361	0.53184
9	0.96260	0.92589	0.88646	0.84023	0.78564	0.72156	0.64574	0.55538
10	0.96561	0.93178	0.89534	0.85247	0.80137	0.74049	0.66717	0.57835
11	0.96827	0.93702	0.90333	0.86362	0.81536	0.75808	0.68714	0.59989
12	0.97078	0.94194	0.91078	0.87390	0.82907	0.77389	0.70490	0.61852
13	0.97324	0.94675	0.91809	0.88404	0.84215	0.78930	0.72282	0.63812
14	0.97563	0.95146	0.92527	0.89401	0.85510	0.80548	0.74089	0.65775
15	0.97794	0.95604	0.93227	0.90378	0.86788	0.82117	0.75900	0.67765
16	0.98019	0.96050	0.93911	0.91335	0.88045	0.83672	0.77713	0.69775
17	0.98237	0.96483	0.94574	0.92269	0.89278	0.85207	0.79519	0.71800
18	0.98447	0.96899	0.95217	0.93174	0.90481	0.86716	0.81309	0.73824
19	0.98649	0.97302	0.95837	0.94053	0.91652	0.88196	0.83076	0.75841
20	0.98842	0.97687	0.96433	0.94898	0.92786	0.89638	0.84812	0.77838
21	0.99055	0.98098	0.97054	0.95759	0.93932	0.91116	0.86633	0.79995
22	0.99254	0.98498	0.97672	0.96641	0.95144	0.92732	0.88713	0.82554
23	0.99443	0.98879	0.98260	0.97483	0.96314	0.94331	0.90829	0.85247
24	0.99616	0.99227	0.98801	0.98258	0.97409	0.95869	0.92949	0.88057

TABLE 226. MALE ADULT SURVIVORSHIP PROBABILITIES FROM AGE 20, $l(N)/l(20)$, FOR ANALYSIS BASED ON SURVIVAL OF FIRST HUSBAND. WEST MODEL

	Values of age N							
Level	25	30	35	40	45	50	55	60
1	0.90775	0.81376	0.71644	0.61671	0.51446	0.41781	0.32133	0.23441
2	0.91555	0.82888	0.73827	0.64426	0.54635	0.45186	0.35538	0.26582
3	0.92260	0.84263	0.75833	0.66986	0.57640	0.48444	0.38859	0.29739
4	0.92899	0.85523	0.77687	0.69379	0.60483	0.51570	0.42100	0.32837
5	0.93484	0.86681	0.79408	0.71620	0.63180	0.54574	0.45262	0.35933
6	0.94022	0.87757	0.81015	0.73732	0.65745	0.57463	0.48347	0.39008
7	0.94521	0.88756	0.82520	0.75724	0.68190	0.60247	0.51357	0.42038
8	0.94982	0.89689	0.83934	0.77611	0.70523	0.62930	0.54293	0.45037
9	0.95413	0.90565	0.85268	0.79404	0.72759	0.65524	0.57159	0.48002
10	0.95818	0.91387	0.86528	0.81108	0.74900	0.68028	0.59956	0.50927
11	0.96196	0.92163	0.87723	0.82732	0.76956	0.70451	0.62685	0.53810
12	0.96553	0.92896	0.88857	0.84283	0.78931	0.72795	0.65349	0.56651
13	0.96886	0.93581	0.89921	0.85765	0.80813	0.75058	0.67972	0.59495
14	0.97208	0.94246	0.90964	0.87177	0.82642	0.77225	0.70424	0.62106
15	0.97496	0.94849	0.91907	0.88483	0.84318	0.79225	0.72712	0.64570
16	0.97778	0.95436	0.92828	0.89763	0.85974	0.81224	0.75025	0.67094
17	0.98050	0.96002	0.93722	0.91013	0.87603	0.83212	0.77352	0.69665
18	0.98313	0.96553	0.94589	0.92231	0.89203	0.85180	0.79683	0.72271
19	0.98563	0.97082	0.95425	0.93411	0.90763	0.87117	0.82000	0.74897
20	0.98808	0.97589	0.96229	0.94550	0.92279	0.89038	0.84299	0.77526
21	0.99043	0.98074	0.96988	0.95622	0.93698	0.90809	0.86494	0.80080
22	0.99271	0.98543	0.97727	0.96643	0.95156	0.92733	0.88956	0.83108
23	0.99486	0.98981	0.98417	0.97678	0.96546	0.94623	0.91468	0.86306
24	0.99673	0.99365	0.99018	0.98552	0.97791	0.96384	0.93915	0.89580

Annex VIII

VALUES OF PROBABILITY OF SURVIVING FROM BIRTH, FROM COALE-DEMENY MODEL LIFE TABLES FOR FEMALES, MALES AND BOTH SEXES COMBINED

TABLE 227. FEMALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, NORTH MODEL

	Probability of surviving from birth, $l(x)$									
Level	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.68027	0.59707	0.54585	0.50719	0.47783	0.41664	0.38879	0.36233	0.33468	0.30483
2	0.70798	0.62931	0.58089	0.54433	0.51658	0.45647	0.42870	0.40206	0.37199	0.34257
3	0.73285	0.65879	0.61320	0.57878	0.55265	0.49416	0.46675	0.44023	0.41205	0.38134
4	0.75539	0.68592	0.64316	0.61087	0.58637	0.52990	0.50309	0.47693	0.44891	0.41822
5	0.77595	0.71104	0.67108	0.64091	0.61801	0.56387	0.53785	0.51225	0.48463	0.45421
6	0.79483	0.73440	0.69720	0.66912	0.64780	0.59622	0.57115	0.54626	0.51920	0.48931
7	0.81226	0.75622	0.72173	0.69569	0.67592	0.62708	0.60308	0.57905	0.55272	0.52359
8	0.82842	0.77668	0.74483	0.72078	0.70253	0.65657	0.63373	0.61067	0.58521	0.55689
9	0.84347	0.79591	0.76664	0.74454	0.72777	0.68479	0.66320	0.64120	0.61672	0.58941
10	0.85753	0.81405	0.78729	0.76708	0.75175	0.71182	0.69154	0.67068	0.64728	0.62111
11	0.87070	0.83119	0.80687	0.78851	0.77457	0.73775	0.71883	0.69917	0.67693	0.65200
12	0.88305	0.84739	0.82544	0.80886	0.79628	0.76259	0.74507	0.72666	0.70565	0.68204
13	0.89451	0.86319	0.84391	0.82936	0.81831	0.78822	0.77212	0.75483	0.73483	0.71235
14	0.90498	0.87810	0.86140	0.84874	0.83906	0.81261	0.79809	0.78216	0.76351	0.74247
15	0.91512	0.89247	0.87813	0.86714	0.85864	0.83564	0.82270	0.80820	0.79099	0.77169
16	0.92492	0.90605	0.89388	0.88447	0.87710	0.85736	0.84599	0.83295	0.81724	0.79939
17	0.93435	0.91887	0.90871	0.90078	0.89450	0.87787	0.86803	0.85646	0.84230	0.82617
18	0.94337	0.93096	0.92267	0.91615	0.91093	0.89724	0.88890	0.87879	0.86622	0.85184
19	0.95199	0.94236	0.93583	0.93064	0.92645	0.91536	0.90868	0.90004	0.88904	0.87645
20	0.96018	0.95311	0.94822	0.94432	0.94114	0.93294	0.92747	0.92027	0.91084	0.90004
21	0.96795	0.96323	0.95993	0.95727	0.95508	0.94946	0.94553	0.93957	0.93171	0.92269
22	0.97525	0.97287	0.97111	0.96968	0.96850	0.96545	0.96272	0.95834	0.95204	0.94482
23	0.98177	0.98036	0.97935	0.97852	0.97783	0.97598	0.97427	0.97140	0.96725	0.96225
24	0.98782	0.98713	0.98662	0.98620	0.98585	0.98491	0.98393	0.98211	0.97936	0.97601

	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.27314	0.24056	0.21028	0.18219	0.15376	0.12154	0.08638	0.05134	0.02245	0.00624
2	0.31083	0.27692	0.24498	0.21492	0.18396	0.14841	0.10867	0.06771	0.03214	0.01031
3	0.34807	0.31331	0.28013	0.24847	0.21537	0.17688	0.13295	0.08627	0.04385	0.01574
4	0.38478	0.34957	0.31550	0.28264	0.24776	0.20675	0.15903	0.10692	0.05759	0.02268
5	0.42091	0.38560	0.35101	0.31728	0.28096	0.23783	0.18675	0.12954	0.07336	0.03122
6	0.45643	0.42132	0.38653	0.35225	0.31482	0.26998	0.21596	0.15404	0.09116	0.04144
7	0.49130	0.45669	0.42198	0.38744	0.34920	0.30303	0.24652	0.18029	0.11089	0.05341
8	0.52552	0.49165	0.45728	0.42274	0.38400	0.33685	0.27828	0.20818	0.13263	0.06717
9	0.55908	0.52616	0.49237	0.45808	0.41909	0.37133	0.31112	0.23761	0.15620	0.08273
10	0.59197	0.56020	0.52719	0.49338	0.45441	0.40636	0.34491	0.26846	0.18157	0.10010
11	0.62418	0.59373	0.56170	0.52858	0.48985	0.44184	0.37956	0.30062	0.20865	0.11928
12	0.65567	0.62669	0.59579	0.56355	0.52529	0.47760	0.41494	0.33401	0.23755	0.14026
13	0.68727	0.65965	0.62971	0.59821	0.56027	0.51292	0.44980	0.36683	0.26580	0.16121
14	0.71897	0.69292	0.66416	0.63346	0.59587	0.54869	0.48500	0.39998	0.29447	0.18242
15	0.74971	0.72543	0.69809	0.66846	0.63155	0.58495	0.52120	0.43471	0.32522	0.20584
16	0.77945	0.75710	0.73138	0.70309	0.66716	0.62155	0.55826	0.47092	0.35807	0.23157
17	0.80816	0.78785	0.76396	0.73723	0.70258	0.65836	0.59603	0.50852	0.39296	0.25968
18	0.83582	0.81786	0.79574	0.77079	0.73768	0.69522	0.63441	0.54735	0.42980	0.29018
19	0.86240	0.84651	0.82668	0.80366	0.77237	0.73199	0.67317	0.58722	0.46845	0.32303
20	0.88809	0.87441	0.85678	0.83586	0.80654	0.76855	0.71214	0.62795	0.50872	0.35812
21	0.91278	0.90138	0.88601	0.86728	0.84013	0.80477	0.75118	0.66931	0.55040	0.39537
22	0.93700	0.92794	0.91492	0.89845	0.87367	0.84077	0.78990	0.71052	0.59225	0.43330
23	0.95675	0.95007	0.93995	0.92628	0.90485	0.87604	0.83030	0.75679	0.64336	0.48391
24	0.97234	0.96780	0.96043	0.94992	0.93251	0.90879	0.86985	0.80481	0.69996	0.54801

TABLE 228. MALE PROBABILITY OF SURVIVING FROM BIRTH, $(_{x}p_{0})$: NORTH MODEL

Level	$_{1}p_{0}$	$_{2}p_{0}$	$_{3}p_{0}$	$_{4}p_{0}$	$_{5}p_{0}$	$_{10}p_{0}$	$_{15}p_{0}$	$_{20}p_{0}$	$_{25}p_{0}$	$_{30}p_{0}$
1	0.62883	0.54784	0.49858	0.46197	0.43413	0.37865	0.35414	0.33041	0.29946	0.26970
2	0.66077	0.58341	0.53637	0.50141	0.47482	0.41960	0.39484	0.37056	0.33867	0.30777
3	0.68944	0.61599	0.57133	0.53813	0.51289	0.45855	0.43384	0.40930	0.37655	0.34525
4	0.71541	0.64603	0.60083	0.57247	0.54862	0.49563	0.47123	0.44669	0.41407	0.38206
5	0.73911	0.67387	0.63419	0.60470	0.58227	0.53101	0.50711	0.48278	0.45025	0.41816
6	0.76087	0.69979	0.66265	0.63504	0.61405	0.56480	0.54158	0.51762	0.48544	0.45353
7	0.78096	0.72404	0.68942	0.66369	0.64412	0.59713	0.57473	0.55129	0.51965	0.48816
8	0.79959	0.74678	0.71467	0.69080	0.67265	0.62809	0.60662	0.58383	0.55292	0.52203
9	0.81694	0.76819	0.73854	0.71652	0.69976	0.65778	0.63734	0.61530	0.58527	0.55516
10	0.83314	0.78839	0.76117	0.74095	0.72557	0.68627	0.66694	0.64574	0.61672	0.58754
11	0.84833	0.80749	0.78286	0.76421	0.75017	0.71365	0.69548	0.67519	0.64730	0.61918
12	0.86256	0.82556	0.80306	0.78633	0.77368	0.73992	0.72297	0.70365	0.67697	0.65002
13	0.87589	0.84328	0.82304	0.80870	0.79749	0.76689	0.75116	0.73264	0.70693	0.68090
14	0.88772	0.85920	0.84186	0.82897	0.81916	0.79161	0.77715	0.75958	0.73511	0.71031
15	0.89926	0.87436	0.85954	0.84838	0.83990	0.81537	0.80222	0.78571	0.76263	0.73921
16	0.91045	0.89005	0.87707	0.86725	0.85969	0.83814	0.82634	0.81097	0.78941	0.76751
17	0.92126	0.90473	0.89368	0.88518	0.87855	0.85994	0.84949	0.83534	0.81539	0.79513
18	0.93167	0.91855	0.90937	0.90218	0.89652	0.88076	0.87169	0.85880	0.84055	0.82201
19	0.94164	0.93156	0.92416	0.91829	0.91362	0.90065	0.89294	0.88135	0.86485	0.84811
20	0.95115	0.94378	0.93811	0.93355	0.92989	0.91963	0.91327	0.90300	0.88829	0.87339
21	0.96019	0.95524	0.95126	0.94801	0.94538	0.93776	0.93273	0.92379	0.91088	0.89784
22	0.96870	0.96595	0.96365	0.96175	0.96021	0.95510	0.95137	0.94378	0.93272	0.92160
23	0.97647	0.97484	0.97341	0.97222	0.97124	0.96781	0.96513	0.95938	0.95113	0.94280
24	0.98394	0.98310	0.98234	0.98170	0.98117	0.97916	0.97745	0.97326	0.96721	0.96113

Level	$_{35}p_{0}$	$_{40}p_{0}$	$_{45}p_{0}$	$_{50}p_{0}$	$_{55}p_{0}$	$_{60}p_{0}$	$_{65}p_{0}$	$_{70}p_{0}$	$_{75}p_{0}$	$_{80}p_{0}$
1	0.24112	0.21886	0.18115	0.15008	0.11976	0.08902	0.05958	0.03338	0.01292	0.00273
2	0.27782	0.24689	0.21406	0.18033	0.14670	0.11191	0.07755	0.04575	0.01952	0.00503
3	0.31434	0.28218	0.24767	0.21174	0.17522	0.13673	0.09766	0.06022	0.02784	0.00832
4	0.35056	0.31754	0.28179	0.24411	0.20509	0.16329	0.11978	0.07676	0.03795	0.01277
5	0.38638	0.35286	0.31626	0.27725	0.23612	0.19162	0.14377	0.09531	0.04993	0.01851
6	0.42175	0.38804	0.35095	0.31099	0.26816	0.22095	0.16952	0.11581	0.06380	0.02565
7	0.45661	0.42300	0.38574	0.34521	0.30104	0.25171	0.19690	0.13819	0.07957	0.03430
8	0.49095	0.45768	0.42055	0.37978	0.33463	0.28363	0.22577	0.16236	0.09724	0.04453
9	0.52472	0.49202	0.45529	0.41461	0.36881	0.31651	0.25601	0.18824	0.11680	0.05641
10	0.55791	0.52598	0.48990	0.44959	0.40348	0.35025	0.28752	0.21574	0.13820	0.06998
11	0.59051	0.55952	0.52432	0.48466	0.43853	0.38474	0.32017	0.24477	0.16140	0.08526
12	0.62244	0.59256	0.55843	0.51967	0.47386	0.42012	0.35425	0.27569	0.18687	0.10276
13	0.65419	0.62524	0.59209	0.55428	0.50871	0.45451	0.38735	0.30553	0.21139	0.11961
14	0.68474	0.65694	0.62489	0.58794	0.54258	0.48838	0.41967	0.33494	0.23580	0.13670
15	0.71496	0.68850	0.65780	0.62201	0.57727	0.52321	0.45349	0.36631	0.26248	0.15595
16	0.74472	0.71981	0.69069	0.65636	0.61249	0.55909	0.48902	0.39966	0.29152	0.17753
17	0.77395	0.75074	0.72342	0.69083	0.64822	0.59587	0.52585	0.43492	0.32293	0.20156
18	0.80255	0.78118	0.75586	0.72527	0.68425	0.63336	0.56391	0.47197	0.35668	0.22813
19	0.83043	0.81104	0.78788	0.75951	0.72040	0.67137	0.60299	0.51064	0.39267	0.25725
20	0.85760	0.84024	0.81936	0.79341	0.75649	0.70970	0.64283	0.55072	0.43074	0.28887
21	0.88396	0.86871	0.85022	0.82685	0.79231	0.74814	0.68334	0.59196	0.47070	0.32290
22	0.90970	0.89667	0.88065	0.85956	0.82723	0.78521	0.72320	0.63189	0.50981	0.35899
23	0.93361	0.92323	0.91008	0.89229	0.86376	0.82069	0.76283	0.68162	0.56213	0.40614
24	0.95434	0.94663	0.93667	0.92272	0.89894	0.86700	0.81344	0.73627	0.62294	0.46701

TABLE 229. PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR BOTH SEXES COMBINED, NORTH MODEL

Level	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.65392	0.57180	0.52163	0.48402	0.45544	0.39732	0.37104	0.34398	0.31664	0.28858
2	0.68379	0.60579	0.55808	0.52234	0.49519	0.43758	0.41135	0.38592	0.35589	0.32523
3	0.71061	0.63686	0.59175	0.55795	0.53228	0.47592	0.44989	0.42438	0.39403	0.36285
4	0.73491	0.66548	0.62301	0.59120	0.56703	0.51234	0.48677	0.46144	0.43106	0.39969
5	0.75708	0.69200	0.65218	0.62236	0.59970	0.54703	0.52210	0.49715	0.46701	0.43574
6	0.77743	0.71667	0.67950	0.65166	0.63051	0.58012	0.55600	0.53159	0.50190	0.47098
7	0.79622	0.73973	0.70518	0.67929	0.65963	0.61173	0.58853	0.56483	0.53578	0.50541
8	0.81365	0.76136	0.72938	0.70542	0.68722	0.64198	0.61984	0.59692	0.56867	0.53903
9	0.82988	0.78171	0.75224	0.73018	0.71342	0.67095	0.64995	0.62793	0.60061	0.57186
10	0.84503	0.80090	0.77391	0.75369	0.73834	0.69873	0.67891	0.65790	0.63162	0.60391
11	0.85924	0.81905	0.79446	0.77606	0.76207	0.72540	0.70683	0.68688	0.66135	0.63518
12	0.87255	0.83620	0.81397	0.79732	0.78466	0.75097	0.73375	0.71487	0.69095	0.66563
13	0.88497	0.85299	0.83242	0.81877	0.80764	0.77729	0.76138	0.74346	0.72053	0.69624
14	0.89613	0.86841	0.85139	0.83861	0.82886	0.80185	0.78936	0.77059	0.74896	0.72599
15	0.90699	0.88329	0.86860	0.85753	0.84904	0.82525	0.81220	0.79668	0.77648	0.75495
16	0.91750	0.89783	0.88526	0.87564	0.86818	0.84751	0.83592	0.82169	0.80098	0.78006
17	0.92764	0.91162	0.90101	0.89278	0.88633	0.86868	0.85853	0.84564	0.82851	0.81027
18	0.93737	0.92460	0.91585	0.90899	0.90354	0.88879	0.88008	0.86835	0.85307	0.83656
19	0.94668	0.93682	0.92985	0.92431	0.91987	0.90792	0.90061	0.89046	0.87664	0.86193
20	0.95555	0.94833	0.94304	0.93880	0.93537	0.92612	0.92019	0.91142	0.89928	0.88638
21	0.96397	0.95913	0.95548	0.95252	0.95011	0.94346	0.93889	0.93148	0.92104	0.90996
22	0.97194	0.96932	0.96728	0.96561	0.96425	0.96014	0.95690	0.95088	0.94214	0.93292
23	0.97905	0.97753	0.97630	0.97529	0.97445	0.97179	0.96958	0.96524	0.95899	0.95228
24	0.98583	0.98506	0.98442	0.98389	0.98345	0.98196	0.98061	0.97757	0.97313	0.96838

Level	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.25673	0.22585	0.19535	0.16574	0.13634	0.10488	0.07265	0.04214	0.01756	0.00444
2	0.29392	0.26153	0.22914	0.19720	0.16487	0.12971	0.09273	0.05646	0.02567	0.00760
3	0.33079	0.29736	0.26349	0.22965	0.19480	0.15651	0.11487	0.07292	0.03584	0.01193
4	0.36725	0.33316	0.29823	0.26290	0.22590	0.18448	0.13892	0.09197	0.04733	0.01760
5	0.40322	0.36883	0.33321	0.29677	0.25799	0.21405	0.16473	0.11200	0.06135	0.02470
6	0.43866	0.40427	0.36830	0.33111	0.29092	0.24486	0.19217	0.13445	0.07714	0.03335
7	0.47353	0.43943	0.40341	0.36580	0.32453	0.27675	0.22110	0.15872	0.09486	0.04362
8	0.50781	0.47425	0.43846	0.40073	0.35871	0.30959	0.25138	0.18471	0.11450	0.05557
9	0.54148	0.50867	0.47337	0.43581	0.39333	0.34325	0.28289	0.21232	0.13601	0.06924
10	0.57452	0.54267	0.50809	0.47095	0.42832	0.37762	0.31551	0.24145	0.15935	0.08467
11	0.60693	0.57620	0.54255	0.50608	0.46356	0.41259	0.34914	0.27201	0.18444	0.10185
12	0.63864	0.60929	0.57665	0.54107	0.49894	0.44815	0.38385	0.30413	0.21159	0.12119
13	0.67032	0.64202	0.61044	0.57570	0.53386	0.48330	0.41781	0.33545	0.23793	0.13990
14	0.70143	0.67449	0.64404	0.61014	0.56857	0.51779	0.45153	0.36666	0.26441	0.15900
15	0.73191	0.70651	0.67745	0.64466	0.60372	0.55332	0.48861	0.39967	0.29308	0.18028
16	0.76166	0.73800	0.71053	0.67913	0.63915	0.58955	0.52219	0.43442	0.32398	0.20389
17	0.79063	0.76884	0.74319	0.71346	0.67473	0.62635	0.56009	0.47082	0.35709	0.22991
18	0.81877	0.79897	0.77538	0.74747	0.71031	0.66353	0.59829	0.50874	0.39234	0.25839
19	0.84605	0.82834	0.80683	0.78105	0.74575	0.70094	0.63722	0.54799	0.42963	0.28933
20	0.87247	0.85690	0.83761	0.81411	0.78090	0.73840	0.67665	0.58839	0.46877	0.32265
21	0.89801	0.88464	0.86767	0.84657	0.81585	0.77576	0.71643	0.62969	0.50957	0.35822
22	0.92301	0.91192	0.89736	0.87853	0.84988	0.81231	0.75527	0.67024	0.55002	0.39421
23	0.94489	0.93632	0.92465	0.90887	0.88260	0.85045	0.79031	0.71828	0.60175	0.44807
24	0.96312	0.95693	0.94825	0.93598	0.91533	0.88738	0.84198	0.76970	0.66051	0.50457

Note: Values of $l(x)$ based on a sex ratio at birth of 105 males per 100 females

TABLE 230. FEMALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, SOUTH MODEL

Level	Probability of surviving from birth, $l(x)$									
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.69300	0.55531	0.49192	0.45845	0.43943	0.39982	0.38056	0.35552	0.32645	0.29785
2	0.71551	0.58633	0.53685	0.49544	0.47760	0.43860	0.41940	0.39429	0.36492	0.33575
3	0.73585	0.61504	0.55941	0.53005	0.51336	0.47538	0.45646	0.43156	0.40225	0.37290
4	0.75437	0.64178	0.58992	0.56254	0.54699	0.51034	0.49188	0.46744	0.43850	0.40928
5	0.77135	0.66676	0.61860	0.59317	0.57872	0.54365	0.52579	0.50201	0.47368	0.44487
6	0.78702	0.69022	0.64565	0.62212	0.60875	0.57544	0.55831	0.53536	0.50785	0.47968
7	0.80153	0.71231	0.67124	0.64955	0.63722	0.60585	0.58953	0.56754	0.54105	0.51372
8	0.81504	0.73318	0.69550	0.67560	0.66429	0.63496	0.61954	0.59863	0.57330	0.54699
9	0.82766	0.75295	0.71855	0.70039	0.69007	0.66283	0.64842	0.62868	0.60464	0.57951
10	0.83955	0.77177	0.74065	0.72422	0.71489	0.69031	0.67685	0.65815	0.63535	0.61131
11	0.84956	0.78923	0.76145	0.74679	0.73845	0.71633	0.70401	0.68682	0.66568	0.64319
12	0.85953	0.80603	0.78140	0.76839	0.76100	0.74127	0.73010	0.71444	0.69500	0.67416
13	0.86925	0.82219	0.80052	0.78909	0.78259	0.76519	0.75517	0.74104	0.72334	0.70420
14	0.87870	0.83774	0.81883	0.80892	0.80328	0.78815	0.77927	0.76666	0.75072	0.73333
15	0.88786	0.85268	0.83649	0.82794	0.82308	0.81018	0.80244	0.79134	0.77717	0.76157
16	0.89673	0.86705	0.85339	0.84617	0.84207	0.83135	0.82471	0.81511	0.80272	0.78892
17	0.90529	0.88101	0.86975	0.86375	0.86029	0.85168	0.84615	0.83802	0.82741	0.81543
18	0.91361	0.89454	0.88559	0.88077	0.87791	0.87131	0.86688	0.86022	0.85138	0.84125
19	0.92285	0.90801	0.90094	0.89708	0.89474	0.88949	0.88592	0.88076	0.87374	0.86562
20	0.93199	0.92070	0.91525	0.91224	0.91036	0.90641	0.90363	0.89962	0.89406	0.88754
21	0.94110	0.93283	0.92878	0.92651	0.92507	0.92222	0.92015	0.91715	0.91291	0.90786
22	0.95013	0.94436	0.94151	0.93989	0.93883	0.93689	0.93542	0.93328	0.93019	0.92646
23	0.95904	0.95528	0.95339	0.95230	0.95158	0.95035	0.94937	0.94795	0.94584	0.94325
24	0.96774	0.96550	0.96436	0.96369	0.96324	0.96254	0.96195	0.96109	0.95976	0.95810

Level	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.27066	0.24473	0.22102	0.19851	0.17246	0.14202	0.10228	0.06086	0.02557	0.00598
2	0.30779	0.28085	0.25588	0.23187	0.20377	0.17049	0.12633	0.07876	0.03598	0.00992
3	0.34454	0.31694	0.29106	0.26586	0.23605	0.20031	0.15218	0.09878	0.04641	0.01516
4	0.38084	0.35292	0.32642	0.30032	0.26911	0.23129	0.17966	0.12081	0.06287	0.02183
5	0.41665	0.38869	0.36185	0.33510	0.30282	0.26327	0.20862	0.14476	0.07936	0.03004
6	0.45192	0.42419	0.39725	0.37018	0.33704	0.29611	0.23892	0.17050	0.09784	0.03987
7	0.48664	0.45937	0.43256	0.40524	0.37165	0.32964	0.27042	0.19793	0.11827	0.05136
8	0.52078	0.49418	0.46771	0.44042	0.40656	0.36387	0.30300	0.22693	0.14062	0.06457
9	0.55434	0.52858	0.50265	0.47558	0.44169	0.39857	0.33654	0.25740	0.16482	0.07952
10	0.58725	0.56239	0.53707	0.51033	0.47670	0.43345	0.37082	0.28913	0.19055	0.09599
11	0.62047	0.59675	0.57217	0.54587	0.51229	0.46882	0.40536	0.32108	0.21671	0.11294
12	0.65289	0.63047	0.60680	0.58104	0.54788	0.50455	0.44076	0.35449	0.24482	0.13184
13	0.68450	0.66350	0.64092	0.61594	0.58340	0.54053	0.47690	0.38927	0.27487	0.15278
14	0.71528	0.69582	0.67447	0.65044	0.61876	0.57666	0.51368	0.42530	0.30681	0.17579
15	0.74523	0.72740	0.70740	0.68449	0.65388	0.61284	0.55098	0.46247	0.34056	0.20090
16	0.77434	0.75822	0.73970	0.71803	0.68869	0.64899	0.58867	0.50066	0.37603	0.22811
17	0.80265	0.78830	0.77134	0.75104	0.72314	0.68503	0.62667	0.53974	0.41312	0.25759
18	0.83032	0.81780	0.80249	0.78368	0.75725	0.72089	0.66456	0.57963	0.45137	0.28944
19	0.85641	0.84569	0.83210	0.81494	0.79031	0.75612	0.70268	0.61998	0.49266	0.32387
20	0.88200	0.87308	0.85936	0.84413	0.82198	0.79050	0.74107	0.66226	0.53501	0.36106
21	0.90690	0.89970	0.88487	0.87168	0.85189	0.82381	0.77904	0.70618	0.58384	0.41070
22	0.92195	0.91637	0.90841	0.89733	0.88025	0.85372	0.81613	0.74994	0.63604	0.46123
23	0.94001	0.93592	0.92975	0.92081	0.90658	0.88586	0.85200	0.79359	0.68449	0.51710
24	0.95396	0.95116	0.94868	0.94284	0.93053	0.91382	0.88612	0.83657	0.74254	0.57880

TABLE 231. MALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, SOUTH MODEL

Level	Probability of surviving from birth, $l(x)$									
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.66445	0.53955	0.48194	0.45107	0.43401	0.39926	0.38400	0.36199	0.33087	0.30238
2	0.68878	0.57082	0.51641	0.48725	0.47115	0.43683	0.42157	0.39948	0.36805	0.33906
3	0.71075	0.59976	0.54857	0.52113	0.50598	0.47244	0.45733	0.43539	0.40401	0.37487
4	0.73076	0.62670	0.57869	0.55296	0.53876	0.50626	0.49143	0.46984	0.43879	0.40980
5	0.74911	0.65187	0.60702	0.58298	0.56971	0.53846	0.52401	0.50293	0.47247	0.44387
6	0.76604	0.67551	0.63375	0.61137	0.59901	0.56917	0.55520	0.53475	0.50508	0.47709
7	0.78172	0.69776	0.65903	0.63828	0.62681	0.59852	0.58509	0.56540	0.53669	0.50945
8	0.79632	0.71878	0.68301	0.66384	0.65326	0.62661	0.61378	0.59493	0.56732	0.54107
9	0.80996	0.73868	0.70580	0.68818	0.67845	0.65353	0.64135	0.62341	0.59704	0.57186
10	0.82268	0.75752	0.72754	0.71030	0.70134	0.67848	0.66701	0.64993	0.62468	0.60052
11	0.83458	0.77468	0.74751	0.73294	0.72490	0.70400	0.69325	0.67724	0.65340	0.63058
12	0.84441	0.79168	0.76765	0.75466	0.74749	0.72855	0.71853	0.70363	0.68148	0.66009
13	0.85496	0.80845	0.78700	0.77549	0.76914	0.75214	0.74289	0.72914	0.70869	0.68883
14	0.86525	0.82442	0.80558	0.79548	0.78991	0.77482	0.76636	0.75379	0.73507	0.71680
15	0.87522	0.83978	0.82343	0.81467	0.80983	0.79662	0.78896	0.77758	0.76063	0.74400
16	0.88487	0.85455	0.84056	0.83307	0.82893	0.81758	0.81072	0.80054	0.78537	0.77040
17	0.89419	0.86875	0.85701	0.85073	0.84725	0.83772	0.83167	0.82270	0.80931	0.79603
18	0.90324	0.88267	0.87307	0.86788	0.86498	0.85725	0.85202	0.84426	0.83285	0.82110
19	0.91182	0.89858	0.89165	0.88762	0.88525	0.87921	0.87480	0.86836	0.85911	0.84948
20	0.92390	0.91265	0.90700	0.90377	0.90183	0.89709	0.89345	0.88814	0.88064	0.87268
21	0.93395	0.92580	0.92155	0.91906	0.91752	0.91396	0.91104	0.90680	0.90090	0.89453
22	0.94395	0.93833	0.93528	0.93345	0.93230	0.92976	0.92749	0.92425	0.91980	0.91490
23	0.95383	0.95020	0.94816	0.94690	0.94609	0.94439	0.94273	0.94038	0.93720	0.93363
24	0.96352	0.96137	0.96012	0.95933	0.95882	0.95777	0.95664	0.95506	0.95296	0.95054

Level	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.27675	0.25110	0.22380	0.19568	0.16511	0.13177	0.09393	0.05659	0.02538	0.00684
2	0.31266	0.28600	0.25729	0.22732	0.19421	0.15750	0.11508	0.07202	0.03416	0.01038
3	0.34800	0.32065	0.29089	0.25940	0.22411	0.18436	0.13767	0.08906	0.04461	0.01484
4	0.38274	0.35500	0.32448	0.29180	0.25465	0.21219	0.16156	0.10761	0.05630	0.02029
5	0.41686	0.38898	0.35799	0.32440	0.28569	0.24085	0.18662	0.12758	0.06982	0.02675
6	0.45034	0.42254	0.39133	0.35710	0.31713	0.27020	0.21271	0.14887	0.08451	0.03427
7	0.48318	0.45566	0.42446	0.38983	0.34886	0.30014	0.23972	0.17139	0.10054	0.04284
8	0.51537	0.48830	0.45732	0.42251	0.38081	0.33057	0.26756	0.19504	0.11785	0.05244
9	0.54690	0.52046	0.48987	0.45509	0.41288	0.36141	0.29613	0.21973	0.13640	0.06319
10	0.57826	0.55042	0.52025	0.48557	0.44305	0.39060	0.32340	0.24356	0.15458	0.07397
11	0.60713	0.58195	0.55223	0.51769	0.47483	0.42131	0.35212	0.26878	0.17394	0.08556
12	0.63776	0.61361	0.58470	0.55057	0.50757	0.45322	0.38229	0.29567	0.19509	0.09863
13	0.66774	0.64475	0.61683	0.58333	0.54046	0.48558	0.41329	0.32378	0.21772	0.11301
14	0.69704	0.67531	0.64854	0.61589	0.57340	0.51829	0.44502	0.35304	0.24181	0.12878
15	0.72563	0.70527	0.67979	0.64816	0.60629	0.55125	0.47740	0.38337	0.26732	0.14595
16	0.75349	0.73458	0.71052	0.68008	0.63905	0.58436	0.51029	0.41466	0.29417	0.16452
17	0.78062	0.76323	0.74068	0.71158	0.67160	0.61755	0.54362	0.44681	0.32232	0.18448
18	0.80724	0.79144	0.77051	0.74289	0.70413	0.65098	0.57755	0.48000	0.35191	0.20599
19	0.83273	0.82010	0.80547	0.77995	0.74302	0.69119	0.61887	0.52130	0.38997	0.23473
20	0.86275	0.85103	0.83457	0.81136	0.77681	0.72736	0.65753	0.56162	0.42906	0.26614
21	0.88632	0.87651	0.86232	0.84163	0.80988	0.76341	0.69699	0.60093	0.47153	0.30179
22	0.90836	0.90042	0.88836	0.87061	0.84206	0.79926	0.73717	0.64835	0.51785	0.34262
23	0.92867	0.92253	0.91300	0.89795	0.87300	0.83454	0.77786	0.69484	0.56843	0.38965
24	0.94782	0.94256	0.93531	0.92326	0.90228	0.86883	0.81872	0.74330	0.62368	0.44418

TABLE 232. PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR BOTH SEXES COMBINED: SOUTH MODEL

	Probability of surviving from birth, $l(x)$									
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)	l(20)	l(25)	l(30)
1	0.67857	0.54721	0.48680	0.45466	0.43665	0.39953	0.38232	0.35883	0.32871	0.30017
2	0.70181	0.57838	0.52150	0.49124	0.47429	0.43769	0.42051	0.39694	0.36652	0.33744
3	0.72299	0.60721	0.55385	0.52548	0.50957	0.47387	0.45690	0.43352	0.40115	0.37090
4	0.74227	0.63404	0.58416	0.55763	0.54277	0.50836	0.49164	0.46866	0.43864	0.40954
5	0.75995	0.65913	0.61266	0.58795	0.57410	0.54099	0.52487	0.50248	0.47306	0.44433
6	0.77627	0.68268	0.63955	0.61661	0.60376	0.57222	0.55671	0.53504	0.50643	0.47835
7	0.79138	0.70485	0.66498	0.64377	0.63188	0.60209	0.58725	0.56644	0.53881	0.51134
8	0.80545	0.72580	0.68910	0.66957	0.65864	0.63068	0.61658	0.59673	0.57023	0.54385
9	0.81859	0.74564	0.71203	0.69413	0.68411	0.65809	0.64479	0.62598	0.60074	0.57559
10	0.83090	0.76441	0.73382	0.71709	0.70794	0.68425	0.67180	0.65393	0.62988	0.60578
11	0.84237	0.78177	0.75430	0.73969	0.73150	0.71001	0.69849	0.68191	0.65939	0.63673
12	0.85278	0.79878	0.77435	0.76135	0.75408	0.73475	0.72417	0.70890	0.68806	0.66695
13	0.86194	0.81515	0.79359	0.78212	0.77570	0.75850	0.74888	0.73494	0.71583	0.69632
14	0.87181	0.83091	0.81206	0.80203	0.79642	0.78132	0.77265	0.76006	0.74270	0.72486
15	0.88138	0.84607	0.82980	0.82114	0.81629	0.80323	0.79553	0.78429	0.76869	0.75237
16	0.89065	0.86064	0.84681	0.83945	0.83533	0.82429	0.81754	0.80764	0.79383	0.77943
17	0.89960	0.87473	0.86322	0.85708	0.85361	0.84452	0.83873	0.83017	0.81813	0.80549
18	0.90829	0.88845	0.87917	0.87416	0.87128	0.86410	0.85926	0.85204	0.84178	0.83092
19	0.91822	0.90333	0.89618	0.89223	0.88989	0.88422	0.88022	0.87440	0.86624	0.85735
20	0.92784	0.91657	0.91102	0.90790	0.90599	0.90163	0.89841	0.89373	0.88718	0.87992
21	0.93743	0.92922	0.92507	0.92269	0.92120	0.91798	0.91548	0.91184	0.90675	0.90103
22	0.94696	0.94127	0.93831	0.93659	0.93548	0.93323	0.93135	0.92865	0.92486	0.92053
23	0.95637	0.95267	0.95071	0.94953	0.94876	0.94729	0.94596	0.94407	0.94141	0.93832
24	0.96557	0.96338	0.96218	0.96145	0.96097	0.96009	0.95923	0.95800	0.95627	0.95422

	l(35)	l(40)	l(45)	l(50)	l(55)	l(60)	l(65)	l(70)	l(75)	l(80)
1	0.27377	0.24799	0.22244	0.19706	0.16869	0.13676	0.09800	0.05467	0.02537	0.00842
2	0.31028	0.28348	0.25660	0.22951	0.19887	0.16383	0.12056	0.07530	0.03504	0.01015
3	0.34631	0.31884	0.29097	0.26255	0.22993	0.19214	0.14474	0.09380	0.04646	0.01499
4	0.38181	0.35398	0.32542	0.29595	0.26170	0.22150	0.17038	0.11404	0.05960	0.02104
5	0.41675	0.38883	0.35987	0.32961	0.29404	0.25178	0.19735	0.13596	0.07447	0.02835
6	0.45111	0.42334	0.39421	0.36344	0.32684	0.28283	0.22549	0.15942	0.09101	0.03700
7	0.48486	0.45746	0.42841	0.39734	0.35997	0.31454	0.25469	0.18433	0.10918	0.04699
8	0.51800	0.49116	0.46238	0.43124	0.39337	0.34681	0.28484	0.21059	0.12895	0.05837
9	0.55052	0.52442	0.49610	0.46508	0.42693	0.37953	0.31584	0.23810	0.15026	0.07115
10	0.58162	0.55625	0.52845	0.49764	0.45946	0.41150	0.34653	0.26578	0.17212	0.08471
11	0.61363	0.58916	0.56195	0.53140	0.49310	0.44448	0.37809	0.29429	0.19480	0.09892
12	0.64514	0.62183	0.59568	0.56543	0.52723	0.47825	0.41081	0.32436	0.21934	0.11482
13	0.67591	0.65389	0.62858	0.59923	0.56140	0.51238	0.44431	0.35572	0.24559	0.13240
14	0.70593	0.68531	0.66118	0.63274	0.59552	0.54676	0.47851	0.38828	0.27351	0.15171
15	0.73519	0.71606	0.69325	0.66588	0.62930	0.58129	0.51329	0.42195	0.30304	0.17275
16	0.76366	0.74611	0.72475	0.69859	0.66326	0.61588	0.54852	0.45661	0.33410	0.19553
17	0.79136	0.77545	0.75563	0.73082	0.69674	0.65046	0.58413	0.49214	0.36661	0.22004
18	0.81849	0.80429	0.78610	0.76278	0.73003	0.68508	0.61999	0.52835	0.40042	0.24620
19	0.84684	0.83463	0.81845	0.79701	0.76606	0.72286	0.65975	0.56943	0.44006	0.27821
20	0.87116	0.86081	0.84666	0.82734	0.79876	0.75815	0.69828	0.61095	0.48220	0.31439
21	0.89391	0.88538	0.87331	0.85628	0.83037	0.79248	0.73699	0.65380	0.52729	0.35491
22	0.91498	0.90820	0.89824	0.88364	0.86068	0.82680	0.77568	0.69790	0.57550	0.40047
23	0.93420	0.92906	0.92117	0.90910	0.88938	0.85957	0.81402	0.74301	0.62695	0.45182
24	0.95138	0.94773	0.94183	0.93232	0.91606	0.89078	0.85159	0.78879	0.68167	0.50984

Note: Values of $l(x)$ based on a sex ratio at birth of 105 males per 100 females.

TABLE 233. FEMALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, EAST MODEL

Level	Probability of surviving from birth, $l(x)$									
	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)	l(20)	l(25)	l(30)
1	0.57215	0.49832	0.46694	0.44635	0.43206	0.40070	0.38442	0.36355	0.33791	0.30981
2	0.60670	0.53331	0.50096	0.48504	0.47123	0.44000	0.42364	0.40256	0.37632	0.34781
3	0.63820	0.56971	0.54059	0.52189	0.50823	0.47748	0.46123	0.44019	0.41405	0.38508
4	0.66712	0.60186	0.57412	0.55592	0.54229	0.51329	0.49737	0.47650	0.45083	0.42162
5	0.69382	0.63204	0.60378	0.58854	0.57659	0.54757	0.53197	0.51157	0.48600	0.45741
6	0.71859	0.66045	0.63574	0.61953	0.60828	0.58042	0.56532	0.54548	0.52050	0.49245
7	0.74167	0.68730	0.66418	0.64902	0.63849	0.61194	0.59744	0.57827	0.55405	0.52675
8	0.76326	0.71272	0.69124	0.67714	0.66736	0.64224	0.62841	0.61002	0.58669	0.56031
9	0.78352	0.73685	0.71702	0.70400	0.69497	0.67139	0.65828	0.64077	0.61846	0.59316
10	0.80258	0.75981	0.74163	0.72970	0.72142	0.69946	0.68713	0.67057	0.64938	0.62529
11	0.82060	0.78213	0.76586	0.75518	0.74777	0.72768	0.71612	0.70036	0.68011	0.65710
12	0.83700	0.80317	0.78879	0.77935	0.77280	0.75464	0.74401	0.72942	0.71056	0.68904
13	0.85297	0.82332	0.81071	0.80244	0.79670	0.78044	0.77077	0.75738	0.73998	0.72006
14	0.86831	0.84259	0.83166	0.82449	0.81952	0.80513	0.79643	0.78427	0.76834	0.75013
15	0.88302	0.86105	0.85168	0.84555	0.84129	0.82879	0.82101	0.81010	0.79576	0.77925
16	0.89710	0.87865	0.87080	0.86565	0.86208	0.85133	0.84457	0.83491	0.82214	0.80741
17	0.91058	0.89569	0.88926	0.88498	0.88194	0.87295	0.86716	0.85876	0.84758	0.83465
18	0.92343	0.91194	0.90684	0.90341	0.90092	0.89365	0.88881	0.88166	0.87207	0.86097
19	0.93567	0.92730	0.92334	0.92096	0.91905	0.91347	0.90957	0.90387	0.89566	0.88639
20	0.94732	0.94189	0.93942	0.93769	0.93638	0.93246	0.92949	0.92481	0.91838	0.91095
21	0.95907	0.95550	0.95384	0.95266	0.95176	0.94899	0.94684	0.94349	0.93881	0.93325
22	0.96937	0.96716	0.96611	0.96536	0.96478	0.96296	0.96148	0.95910	0.95574	0.95173
23	0.97856	0.97734	0.97675	0.97633	0.97599	0.97492	0.97398	0.97243	0.97021	0.96754
24	0.98640	0.98581	0.98554	0.98534	0.98517	0.98462	0.98410	0.98322	0.98192	0.98034

Level	l(55)	l(60)	l(65)	l(70)	l(75)	l(80)	l(85)	l(90)	l(95)	l(100)
1	0.28138	0.25342	0.22729	0.20212	0.17847	0.13850	0.09777	0.05676	0.02473	0.00673
2	0.31852	0.28943	0.26192	0.23504	0.20908	0.16579	0.12030	0.07300	0.03408	0.01043
3	0.35532	0.32547	0.29692	0.26865	0.23970	0.19448	0.14481	0.09121	0.04517	0.01523
4	0.39170	0.36143	0.33216	0.30279	0.26819	0.22442	0.17056	0.11133	0.05805	0.02123
5	0.42762	0.39724	0.36753	0.33734	0.30140	0.25546	0.19804	0.13328	0.07271	0.02850
6	0.46306	0.43282	0.40295	0.37210	0.33520	0.28748	0.22692	0.15699	0.08915	0.03712
7	0.49797	0.46813	0.43834	0.40727	0.36950	0.32034	0.25707	0.18236	0.10736	0.04715
8	0.53236	0.50313	0.47364	0.44247	0.40420	0.35395	0.28839	0.20931	0.12731	0.05862
9	0.56619	0.53778	0.50880	0.47773	0.43921	0.38821	0.32078	0.23775	0.14896	0.07156
10	0.59947	0.57205	0.54376	0.51299	0.47446	0.42302	0.35414	0.26761	0.17228	0.08600
11	0.63241	0.60600	0.57845	0.54800	0.50948	0.45773	0.38758	0.29896	0.19608	0.10105
12	0.66575	0.64051	0.61373	0.58360	0.54506	0.49290	0.42138	0.32823	0.22020	0.11637
13	0.69830	0.67441	0.64863	0.61906	0.58081	0.52864	0.45626	0.36032	0.24625	0.13343
14	0.73002	0.70764	0.68306	0.65429	0.61661	0.56484	0.49213	0.39398	0.27426	0.15237
15	0.76089	0.74016	0.71695	0.68920	0.65239	0.60140	0.52890	0.42915	0.30427	0.17326
16	0.79087	0.77192	0.75025	0.72371	0.68804	0.63823	0.56646	0.46576	0.33624	0.19620
17	0.82000	0.80292	0.78293	0.75780	0.72352	0.67523	0.60471	0.50373	0.37016	0.22124
18	0.84825	0.83311	0.81493	0.79136	0.75869	0.71226	0.64348	0.54287	0.40590	0.24835
19	0.87562	0.86250	0.84621	0.82434	0.79348	0.74921	0.68263	0.58304	0.44335	0.27752
20	0.90215	0.89108	0.87676	0.85670	0.82783	0.78599	0.72203	0.62407	0.48234	0.30866
21	0.92641	0.91739	0.90516	0.88729	0.86098	0.82239	0.76232	0.66589	0.52630	0.34391
22	0.94667	0.93974	0.92990	0.91475	0.89187	0.85781	0.80364	0.71563	0.57750	0.39280
23	0.96408	0.95913	0.95166	0.93943	0.92036	0.89153	0.84447	0.76500	0.63347	0.44698
24	0.97822	0.97501	0.96981	0.96061	0.94567	0.92264	0.88389	0.81553	0.69434	0.51067

TABLE 234. MALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, EAST MODEL

Level	Probability of surviving from birth, $l(x)$									
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.49494	0.42963	0.40248	0.38524	0.37364	0.34679	0.33560	0.31937	0.29680	0.27530
2	0.53551	0.47104	0.44424	0.42721	0.41478	0.38847	0.37694	0.36011	0.33662	0.31412
3	0.57249	0.50961	0.48346	0.46686	0.45473	0.42835	0.41664	0.39946	0.37537	0.35221
4	0.60644	0.54570	0.52044	0.50441	0.49269	0.46655	0.45481	0.43747	0.41309	0.38953
5	0.63778	0.57960	0.55541	0.54005	0.52883	0.50321	0.49154	0.47421	0.44978	0.42608
6	0.66686	0.61156	0.58857	0.57397	0.56330	0.53841	0.52692	0.50975	0.48547	0.46185
7	0.69396	0.64178	0.62008	0.60630	0.59624	0.57225	0.56103	0.54414	0.52020	0.49684
8	0.71931	0.67042	0.65009	0.63718	0.62775	0.60483	0.59395	0.57744	0.55400	0.53106
9	0.74309	0.69762	0.67872	0.66671	0.65794	0.63621	0.62573	0.60970	0.58660	0.56452
10	0.76547	0.72352	0.70608	0.69500	0.68691	0.66647	0.65643	0.64096	0.61892	0.59724
11	0.78641	0.74870	0.73302	0.72307	0.71579	0.69695	0.68733	0.67233	0.65100	0.62999
12	0.80562	0.77196	0.75797	0.74909	0.74259	0.72523	0.71610	0.70170	0.68120	0.66096
13	0.82416	0.79439	0.78201	0.77415	0.76841	0.75256	0.74398	0.73023	0.71073	0.69139
14	0.84204	0.81599	0.80536	0.79828	0.79326	0.77896	0.77097	0.75802	0.73955	0.72123
15	0.85923	0.83676	0.82742	0.82148	0.81715	0.80443	0.79705	0.78494	0.76763	0.75044
16	0.87574	0.85671	0.84879	0.84377	0.84009	0.82896	0.82224	0.81101	0.79495	0.77898
17	0.89158	0.87586	0.86933	0.86518	0.86214	0.85259	0.84655	0.83625	0.82151	0.80684
18	0.90672	0.89442	0.88918	0.88580	0.88329	0.87533	0.86997	0.86064	0.84728	0.83396
19	0.92117	0.91227	0.90827	0.90559	0.90357	0.89718	0.89252	0.88417	0.87223	0.86032
20	0.93494	0.92913	0.92637	0.92447	0.92300	0.91818	0.91422	0.90688	0.89638	0.88591
21	0.94857	0.94466	0.94271	0.94132	0.94023	0.93649	0.93325	0.92710	0.91834	0.90947
22	0.96108	0.95865	0.95738	0.95645	0.95571	0.95305	0.95056	0.94562	0.93857	0.93142
23	0.97240	0.97104	0.97029	0.96973	0.96928	0.96759	0.96576	0.96200	0.95665	0.95122
24	0.98218	0.98153	0.98115	0.98087	0.98064	0.97967	0.97843	0.97584	0.97210	0.96830

Level	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.25292	0.22780	0.20031	0.17150	0.14198	0.11139	0.07983	0.04863	0.02322	0.00739
2	0.29058	0.26395	0.23450	0.20317	0.17043	0.13381	0.09935	0.06244	0.03123	0.01073
3	0.32784	0.30011	0.26912	0.23565	0.20003	0.16165	0.12046	0.07782	0.04054	0.01487
4	0.36464	0.33615	0.30400	0.26872	0.23062	0.18876	0.14303	0.09470	0.05113	0.01984
5	0.40094	0.37200	0.33904	0.30243	0.26204	0.21698	0.16689	0.11300	0.06299	0.02567
6	0.43669	0.40758	0.37413	0.33646	0.29418	0.24619	0.19200	0.13263	0.07610	0.03240
7	0.47187	0.44286	0.40920	0.37078	0.32690	0.27627	0.21821	0.15357	0.09043	0.04006
8	0.50647	0.47777	0.44418	0.40530	0.36010	0.30711	0.24544	0.17569	0.10595	0.04859
9	0.54048	0.51229	0.47901	0.43994	0.39371	0.33862	0.27358	0.19892	0.12261	0.05805
10	0.57388	0.54640	0.51365	0.47463	0.42763	0.37070	0.30256	0.22321	0.14038	0.06843
11	0.60734	0.58064	0.54853	0.50968	0.46199	0.40333	0.33212	0.24808	0.15865	0.07923
12	0.63910	0.61322	0.58176	0.54304	0.49463	0.43425	0.36038	0.27165	0.17604	0.08962
13	0.67046	0.64559	0.61500	0.57670	0.52786	0.46600	0.38918	0.29656	0.19487	0.10110
14	0.70138	0.67768	0.64819	0.61057	0.56161	0.49859	0.41939	0.32283	0.21507	0.11375
15	0.73178	0.70942	0.68124	0.64457	0.59577	0.53192	0.45066	0.35044	0.23671	0.12764
16	0.76163	0.74074	0.71406	0.67859	0.63026	0.56589	0.48292	0.37935	0.25980	0.14282
17	0.79088	0.77160	0.74659	0.71253	0.66498	0.60043	0.51608	0.40952	0.28434	0.15933
18	0.81948	0.80190	0.77872	0.74633	0.69978	0.63536	0.55001	0.44083	0.31027	0.17718
19	0.84737	0.83159	0.81036	0.77981	0.73454	0.67057	0.58456	0.47315	0.33750	0.19633
20	0.87454	0.86062	0.84146	0.81291	0.76915	0.70590	0.61960	0.50635	0.36594	0.21676
21	0.89979	0.88779	0.87085	0.84474	0.80320	0.74164	0.65623	0.54253	0.39951	0.24149
22	0.92361	0.91386	0.89971	0.87690	0.83889	0.78085	0.69797	0.58571	0.43932	0.27418
23	0.94527	0.93779	0.92655	0.90747	0.87378	0.82088	0.74173	0.63290	0.48604	0.31365
24	0.96414	0.95885	0.95056	0.93553	0.90704	0.85930	0.78736	0.68455	0.54004	0.36216

TABLE 235 PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR BOTH SEXES COMBINED, EAST MODEL

	Probability of surviving from birth, $l(x)$									
Level	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.53260	0.46313	0.43592	0.41504	0.40162	0.37308	0.35941	0.34092	0.31685	0.29213
2	0.57023	0.50239	0.47385	0.45541	0.44231	0.41360	0.39972	0.38081	0.35608	0.33055
3	0.60434	0.53892	0.51132	0.49350	0.48082	0.45251	0.43839	0.41932	0.39423	0.36824
4	0.63603	0.57309	0.54662	0.52953	0.51737	0.48934	0.47554	0.45650	0.43135	0.40518
5	0.66511	0.60514	0.57998	0.56370	0.55212	0.52484	0.51126	0.49243	0.46744	0.44136
6	0.69209	0.63560	0.61157	0.59619	0.58524	0.55890	0.54565	0.52717	0.50255	0.47677
7	0.71723	0.66398	0.64159	0.62713	0.61684	0.59161	0.57879	0.56078	0.53671	0.51143
8	0.74074	0.69105	0.67016	0.65667	0.64707	0.62307	0.61075	0.59333	0.56994	0.54532
9	0.76281	0.71675	0.69740	0.68490	0.67600	0.65537	0.64360	0.62683	0.60229	0.57849
10	0.78357	0.74122	0.72342	0.71192	0.70374	0.68256	0.67140	0.65540	0.63177	0.61092
11	0.80299	0.76500	0.74903	0.73873	0.73134	0.71194	0.70137	0.68600	0.66519	0.64321
12	0.82092	0.78718	0.77300	0.76385	0.75732	0.73957	0.72971	0.71522	0.69552	0.67465
13	0.83821	0.80850	0.79600	0.78794	0.78220	0.76615	0.75704	0.74349	0.72499	0.70537
14	0.85485	0.82896	0.81808	0.81106	0.80606	0.79172	0.78338	0.77082	0.75361	0.73532
15	0.87083	0.84859	0.83925	0.83322	0.82892	0.81628	0.80873	0.79721	0.78135	0.76449
16	0.88615	0.86741	0.85952	0.85448	0.85081	0.83987	0.83313	0.82266	0.80821	0.79284
17	0.90084	0.88553	0.87905	0.87483	0.87179	0.86252	0.85660	0.84723	0.83422	0.82040
18	0.91487	0.90295	0.89779	0.89439	0.89188	0.88426	0.87915	0.87089	0.85937	0.84713
19	0.92824	0.91960	0.91571	0.91308	0.91112	0.90512	0.90083	0.89368	0.88365	0.87303
20	0.94097	0.93533	0.93273	0.93091	0.92952	0.92514	0.92166	0.91562	0.90711	0.89812
21	0.95309	0.94994	0.94813	0.94685	0.94585	0.94258	0.93987	0.93509	0.92832	0.92106
22	0.96512	0.96280	0.96163	0.96079	0.96013	0.95789	0.95588	0.95219	0.94694	0.94132
23	0.97580	0.97411	0.97344	0.97294	0.97255	0.97116	0.96976	0.96708	0.96326	0.95918
24	0.98423	0.98362	0.98329	0.98305	0.98284	0.98208	0.98121	0.97943	0.97689	0.97417

	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.26680	0.24029	0.21347	0.18643	0.15934	0.12461	0.08858	0.05259	0.02395	0.00706
2	0.30430	0.27637	0.24787	0.21871	0.18684	0.15043	0.10956	0.06759	0.03262	0.01058
3	0.34134	0.31248	0.28268	0.25174	0.21742	0.17766	0.13224	0.08435	0.04279	0.01504
4	0.37783	0.34845	0.31773	0.28537	0.24894	0.20615	0.15644	0.10281	0.05450	0.02051
5	0.41395	0.38431	0.35293	0.31945	0.28123	0.23573	0.18208	0.12289	0.06773	0.02705
6	0.44935	0.41969	0.38818	0.35389	0.31418	0.26633	0.20903	0.14452	0.08246	0.03470
7	0.48460	0.45518	0.42341	0.38857	0.34768	0.29776	0.23716	0.16761	0.09868	0.04350
8	0.51909	0.49014	0.45855	0.42343	0.38161	0.32995	0.26639	0.19208	0.11636	0.05348
9	0.55302	0.52472	0.49354	0.45837	0.41590	0.36281	0.29660	0.21786	0.13546	0.06464
10	0.58636	0.55891	0.52833	0.49334	0.45047	0.39622	0.32772	0.24486	0.15594	0.07700
11	0.61956	0.59301	0.56312	0.52837	0.48515	0.42986	0.35917	0.27233	0.17690	0.08987
12	0.65209	0.62653	0.59735	0.56282	0.51922	0.46284	0.38998	0.29924	0.19760	0.10266
13	0.68404	0.63964	0.63140	0.59736	0.55368	0.49655	0.42190	0.32766	0.21993	0.11687
14	0.71535	0.69229	0.66519	0.63189	0.58843	0.53090	0.45487	0.35753	0.24394	0.13258
15	0.74597	0.72441	0.69865	0.66634	0.62338	0.56581	0.48882	0.38883	0.26966	0.14989
16	0.77589	0.75594	0.73171	0.70039	0.65844	0.60117	0.52367	0.42150	0.29708	0.16885
17	0.80508	0.78687	0.76431	0.73462	0.69353	0.63691	0.55931	0.45547	0.32620	0.18952
18	0.83351	0.81712	0.79636	0.76829	0.72851	0.67287	0.59560	0.49060	0.35691	0.21189
19	0.86115	0.84666	0.82784	0.80153	0.76329	0.70893	0.63239	0.52675	0.38913	0.23593
20	0.88800	0.87547	0.85867	0.83427	0.79777	0.74496	0.66956	0.56377	0.42272	0.26158
21	0.91277	0.90222	0.88758	0.86549	0.83138	0.78103	0.70796	0.60368	0.46064	0.29242
22	0.93485	0.92648	0.91443	0.89536	0.86473	0.81828	0.74951	0.64908	0.50672	0.33194
23	0.95444	0.94819	0.93879	0.92503	0.89650	0.85493	0.79185	0.69733	0.55795	0.37868
24	0.97100	0.96673	0.95995	0.94776	0.92588	0.89019	0.83444	0.74834	0.61530	0.43460

Note: Values of $l(x)$ based on a sex ratio at birth of 105 males per 100 females.

TABLE 236. FEMALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, WEST MODEL

Level	Probability of surviving from birth, $l(x)$									
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.63483	0.59000	0.51199	0.48742	0.46883	0.43456	0.40972	0.37943	0.34460	0.30927
2	0.66638	0.58557	0.54936	0.52595	0.50824	0.47445	0.44978	0.41949	0.38435	0.34838
3	0.69481	0.61829	0.58399	0.56183	0.54506	0.51206	0.48783	0.45786	0.42283	0.38666
4	0.72064	0.64856	0.61625	0.59534	0.57958	0.54763	0.52404	0.49465	0.46007	0.42407
5	0.74427	0.67671	0.64643	0.62686	0.61205	0.58135	0.55856	0.52998	0.49612	0.46062
6	0.76602	0.70300	0.67476	0.65651	0.64270	0.61339	0.59153	0.56394	0.53103	0.49631
7	0.78614	0.72763	0.70145	0.68451	0.67169	0.64389	0.62307	0.59661	0.56486	0.53114
8	0.80482	0.75084	0.72665	0.71101	0.69918	0.67298	0.65329	0.62808	0.59765	0.56515
9	0.82226	0.77271	0.75051	0.73616	0.72530	0.70074	0.68227	0.65842	0.62944	0.59829
10	0.83857	0.79340	0.77315	0.76007	0.75017	0.72737	0.71011	0.68769	0.66028	0.63066
11	0.85388	0.81300	0.79468	0.78285	0.77389	0.75285	0.73687	0.71598	0.69022	0.66224
12	0.86829	0.83163	0.81519	0.80457	0.79654	0.77729	0.76263	0.74327	0.71928	0.69306
13	0.88169	0.84939	0.83492	0.82536	0.81848	0.80100	0.78771	0.76990	0.74769	0.72328
14	0.89452	0.86720	0.85496	0.84705	0.84106	0.82559	0.81387	0.79749	0.77646	0.75363
15	0.90661	0.88364	0.87324	0.86646	0.86127	0.84773	0.83740	0.82284	0.80416	0.78333
16	0.91823	0.89936	0.89066	0.88490	0.88041	0.86874	0.85980	0.84703	0.83052	0.81194
17	0.92934	0.91419	0.90709	0.90232	0.89854	0.88868	0.88110	0.87010	0.85575	0.83944
18	0.93996	0.92820	0.92260	0.91878	0.91571	0.90762	0.90136	0.89211	0.87998	0.86585
19	0.95006	0.94143	0.93724	0.93436	0.93201	0.92561	0.92065	0.91311	0.90298	0.89121
20	0.95966	0.95392	0.95109	0.94912	0.94749	0.94275	0.93906	0.93317	0.92510	0.91557
21	0.96907	0.96559	0.96385	0.96263	0.96160	0.95835	0.95574	0.95158	0.94564	0.93847
22	0.97738	0.97530	0.97425	0.97350	0.97286	0.97072	0.96898	0.96614	0.96216	0.95724
23	0.98484	0.98377	0.98321	0.98282	0.98248	0.98123	0.98019	0.97846	0.97598	0.97285
24	0.99106	0.99061	0.99037	0.99020	0.99006	0.98945	0.98893	0.98803	0.98671	0.98499

Level	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.27556	0.23916	0.20729	0.17814	0.14931	0.11387	0.07819	0.04721	0.02226	0.00737
2	0.31164	0.27581	0.24212	0.21076	0.17598	0.13981	0.09910	0.06235	0.03128	0.01136
3	0.34937	0.31259	0.27754	0.24436	0.20705	0.16751	0.12208	0.07959	0.04211	0.01632
4	0.38666	0.34936	0.31336	0.27875	0.23931	0.19677	0.14697	0.09887	0.05475	0.02293
5	0.42345	0.38601	0.34948	0.31375	0.27257	0.22742	0.17363	0.12010	0.06923	0.03067
6	0.45963	0.42245	0.38565	0.34922	0.30668	0.25929	0.20192	0.14319	0.08553	0.03979
7	0.49513	0.45862	0.42191	0.38504	0.34149	0.29224	0.23171	0.16806	0.10363	0.05035
8	0.53019	0.49446	0.45813	0.42111	0.37689	0.32614	0.26287	0.19460	0.12348	0.06235
9	0.56483	0.52993	0.49424	0.45733	0.41277	0.36087	0.29527	0.22272	0.14505	0.07584
10	0.59865	0.56500	0.53019	0.49364	0.44903	0.39632	0.32882	0.25233	0.16828	0.09079
11	0.63186	0.59963	0.56592	0.52996	0.48558	0.43239	0.36339	0.28333	0.19311	0.10722
12	0.66444	0.63382	0.60140	0.56623	0.52233	0.46899	0.39890	0.31563	0.21949	0.12511
13	0.69647	0.66756	0.63656	0.60234	0.55916	0.50587	0.43503	0.34890	0.24711	0.14424
14	0.72840	0.70104	0.67138	0.63799	0.59545	0.54215	0.47053	0.38145	0.27404	0.16298
15	0.76029	0.73493	0.70686	0.67452	0.63216	0.57964	0.50742	0.41367	0.30277	0.18323
16	0.79119	0.76798	0.74173	0.71074	0.67015	0.61765	0.54539	0.45151	0.33352	0.20545
17	0.82106	0.80014	0.77593	0.74655	0.70743	0.65603	0.58432	0.48888	0.36626	0.22970
18	0.84988	0.83137	0.80939	0.78183	0.74460	0.69465	0.62405	0.52768	0.40095	0.25602
19	0.87767	0.86164	0.84203	0.81653	0.78144	0.73337	0.66444	0.56775	0.43751	0.28442
20	0.90448	0.89098	0.87384	0.85057	0.81788	0.77205	0.70531	0.60893	0.47580	0.31485
21	0.92997	0.91919	0.90468	0.88392	0.85396	0.81090	0.74697	0.65101	0.51568	0.34725
22	0.95413	0.94598	0.93443	0.91591	0.88787	0.84926	0.79063	0.69378	0.55697	0.38152
23	0.96682	0.96119	0.95466	0.94074	0.91928	0.88619	0.83458	0.73698	0.59947	0.41758
24	0.98270	0.97927	0.97362	0.96347	0.94708	0.92052	0.87767	0.80130	0.68201	0.50461

TABLE 237. MALE PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, WEST MODEL

	Probability of surviving from birth, $l(x)$									
Level	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$	$l(25)$	$l(30)$
1	0.58093	0.50308	0.46898	0.44665	0.43005	0.40102	0.38160	0.35676	0.33385	0.29032
2	0.61657	0.54152	0.50865	0.48712	0.47117	0.44211	0.42258	0.39740	0.34384	0.32940
3	0.64868	0.57690	0.54546	0.52488	0.50957	0.48093	0.46151	0.43631	0.40254	0.36765
4	0.67785	0.60967	0.57980	0.56024	0.54371	0.51769	0.49858	0.47360	0.43997	0.40504
5	0.70454	0.64015	0.61193	0.59348	0.57976	0.55258	0.53393	0.50938	0.47619	0.44154
6	0.72911	0.66865	0.64217	0.62482	0.61194	0.58575	0.56770	0.54375	0.51125	0.47718
7	0.75183	0.69537	0.67064	0.65445	0.64242	0.61737	0.60000	0.57680	0.54520	0.51195
8	0.77294	0.72052	0.69756	0.68253	0.67135	0.64754	0.63095	0.60862	0.57808	0.54587
9	0.79263	0.74425	0.72307	0.70919	0.69888	0.67639	0.66064	0.63926	0.60995	0.57895
10	0.81105	0.76671	0.74728	0.73457	0.72511	0.70401	0.68916	0.66882	0.64085	0.61122
11	0.82835	0.78800	0.77032	0.75875	0.75015	0.73048	0.71657	0.69734	0.67063	0.64269
12	0.84463	0.80822	0.79228	0.78184	0.77408	0.75588	0.74296	0.72489	0.69991	0.67340
13	0.86058	0.82912	0.81534	0.80632	0.79961	0.78315	0.77147	0.75456	0.73107	0.70613
14	0.87547	0.84833	0.83644	0.82866	0.82287	0.80820	0.79734	0.78167	0.75985	0.73670
15	0.88864	0.86523	0.85498	0.84826	0.84327	0.82988	0.82018	0.80571	0.78554	0.76421
16	0.90143	0.88164	0.87292	0.86720	0.86293	0.85103	0.84234	0.82912	0.81070	0.79128
17	0.91379	0.89790	0.89056	0.88561	0.88184	0.87145	0.86377	0.85187	0.83526	0.81782
18	0.92570	0.91330	0.90736	0.90321	0.90004	0.89112	0.88467	0.87391	0.85917	0.84379
19	0.93713	0.92796	0.92332	0.92002	0.91744	0.91008	0.90444	0.89524	0.88240	0.86912
20	0.94807	0.94179	0.93847	0.93606	0.93413	0.92823	0.92366	0.91584	0.90493	0.89376
21	0.95909	0.95508	0.95285	0.95121	0.94989	0.94536	0.94174	0.93531	0.92636	0.91730
22	0.96925	0.96675	0.96531	0.96422	0.96334	0.96004	0.95734	0.95234	0.94540	0.93847
23	0.97836	0.97719	0.97636	0.97573	0.97521	0.97303	0.97119	0.96758	0.96261	0.95773
24	0.98668	0.98605	0.98566	0.98535	0.98510	0.98385	0.98273	0.98041	0.97723	0.97419

	$l(35)$	$l(40)$	$l(45)$	$l(50)$	$l(55)$	$l(60)$	$l(65)$	$l(70)$	$l(75)$	$l(80)$
1	0.25560	0.22002	0.18354	0.14906	0.11464	0.08363	0.05422	0.03036	0.01343	0.00392
2	0.29339	0.25603	0.21712	0.17957	0.14123	0.10564	0.07048	0.04149	0.01953	0.00631
3	0.33087	0.29227	0.25149	0.21137	0.16955	0.12967	0.08963	0.05452	0.02706	0.00949
4	0.36793	0.32858	0.28645	0.24424	0.19939	0.15552	0.11037	0.06943	0.03607	0.01358
5	0.40449	0.36482	0.32183	0.27799	0.23056	0.18304	0.13299	0.08619	0.04660	0.01863
6	0.44052	0.40092	0.35749	0.31246	0.26289	0.21207	0.15738	0.10475	0.05868	0.02471
7	0.47598	0.43678	0.39332	0.34751	0.29623	0.24248	0.18343	0.12506	0.07232	0.03189
8	0.51084	0.47236	0.42922	0.38301	0.33044	0.27411	0.21102	0.14705	0.08750	0.04020
9	0.54509	0.50760	0.46512	0.41887	0.36540	0.30686	0.24006	0.17066	0.10421	0.04967
10	0.57872	0.54247	0.50095	0.45499	0.40100	0.34061	0.27043	0.19580	0.12243	0.06032
11	0.61173	0.57693	0.53665	0.49129	0.43713	0.37524	0.30203	0.22241	0.14213	0.07217
12	0.64412	0.61096	0.57217	0.52769	0.47371	0.41064	0.33477	0.25040	0.16326	0.08522
13	0.67581	0.64700	0.60979	0.56636	0.51289	0.44893	0.37047	0.28128	0.18685	0.10007
14	0.71104	0.68144	0.64599	0.60365	0.55049	0.48547	0.40646	0.31063	0.20934	0.11431
15	0.74051	0.71292	0.67936	0.63833	0.58585	0.52025	0.43729	0.33944	0.23186	0.12890
16	0.76966	0.74425	0.71283	0.67345	0.62205	0.55629	0.47179	0.37023	0.25634	0.14518
17	0.79819	0.77532	0.74627	0.70886	0.65894	0.59346	0.50788	0.40295	0.28295	0.16323
18	0.82663	0.80602	0.77956	0.74460	0.69636	0.63159	0.54542	0.43755	0.31158	0.18314
19	0.85429	0.83626	0.81255	0.77991	0.73412	0.67051	0.58427	0.47392	0.34223	0.20495
20	0.88131	0.86593	0.84513	0.81525	0.77205	0.71002	0.62422	0.51188	0.37481	0.22866
21	0.90734	0.89437	0.87637	0.84935	0.80890	0.74900	0.66439	0.55098	0.40931	0.25436
22	0.93070	0.92076	0.90621	0.88314	0.84717	0.79148	0.71057	0.59046	0.45378	0.29018
23	0.95227	0.94512	0.93416	0.91556	0.88503	0.83908	0.75995	0.63154	0.50591	0.33422
24	0.97079	0.96622	0.95876	0.94496	0.92076	0.87826	0.81150	0.71014	0.56701	0.38934

TABLE 23B. PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR BOTH SEXES COMBINED, WEST MODEL

Level	Probability of surviving from birth, $l(x)$									
	l(1)	l(2)	l(3)	l(5)	l(10)	l(15)	l(20)	l(25)	l(30)	
1	0.60722	0.52598	0.48996	0.48653	0.44896	0.41738	0.39531	0.36781	0.33397	0.29956
2	0.64086	0.56300	0.52850	0.50606	0.48922	0.45788	0.43584	0.40817	0.37384	0.33865
3	0.67118	0.59709	0.56425	0.54290	0.52888	0.49611	0.47434	0.44682	0.41243	0.37692
4	0.69872	0.62864	0.59758	0.57738	0.56273	0.53229	0.51099	0.48386	0.44977	0.41432
5	0.72392	0.65798	0.62876	0.60976	0.59551	0.56661	0.54594	0.51942	0.48591	0.45084
6	0.74711	0.68540	0.65806	0.64027	0.62694	0.59923	0.57932	0.55359	0.52089	0.48651
7	0.76856	0.71111	0.68566	0.66911	0.65669	0.63030	0.61125	0.58646	0.55479	0.52131
8	0.78849	0.73530	0.71175	0.69642	0.68492	0.65994	0.64184	0.61811	0.58762	0.55526
9	0.80708	0.75813	0.73645	0.72234	0.71176	0.68828	0.67119	0.64860	0.61945	0.58838
10	0.82447	0.77972	0.75989	0.74700	0.73753	0.71540	0.69937	0.67802	0.65032	0.62070
11	0.84080	0.80019	0.78220	0.77050	0.76173	0.74139	0.72647	0.70642	0.68028	0.65222
12	0.85617	0.81963	0.80345	0.79292	0.78503	0.76632	0.75255	0.73385	0.70935	0.68299
13	0.87087	0.83900	0.82489	0.81570	0.80841	0.79185	0.77939	0.76204	0.73917	0.71448
14	0.88476	0.85753	0.84547	0.83763	0.83174	0.81658	0.80540	0.78938	0.76804	0.74493
15	0.89740	0.87421	0.86388	0.85713	0.85205	0.83858	0.82857	0.81406	0.79462	0.77351
16	0.90962	0.89028	0.88157	0.87583	0.87145	0.85966	0.85085	0.83785	0.82036	0.80135
17	0.92137	0.90584	0.89862	0.89376	0.88998	0.87985	0.87222	0.86076	0.84525	0.82836
18	0.93265	0.92058	0.91479	0.91080	0.90766	0.89916	0.89270	0.88278	0.86927	0.85455
19	0.94343	0.93453	0.93010	0.92701	0.92454	0.91763	0.91234	0.90395	0.89243	0.87989
20	0.95372	0.94770	0.94462	0.94243	0.94063	0.93531	0.93117	0.92429	0.91476	0.90439
21	0.96395	0.96020	0.95821	0.95678	0.95560	0.95169	0.94856	0.94324	0.93576	0.92762
22	0.97321	0.97092	0.96967	0.96874	0.96796	0.96524	0.96301	0.95907	0.95357	0.94762
23	0.98162	0.98039	0.97970	0.97918	0.97875	0.97702	0.97558	0.97288	0.96913	0.96510
24	0.98881	0.98827	0.98795	0.98771	0.98751	0.98658	0.98575	0.98412	0.98185	0.97945

Level	l(30)	l(40)	l(45)	l(50)	l(55)	l(60)	l(65)	l(70)	l(75)	l(80)
1	0.26436	0.22935	0.19512	0.16324	0.13008	0.09838	0.06591	0.03857	0.01773	0.00560
2	0.30229	0.26567	0.22931	0.19478	0.15818	0.12230	0.08464	0.05166	0.02526	0.00877
3	0.33989	0.30218	0.26419	0.22746	0.18784	0.14812	0.10545	0.06674	0.03440	0.01291
4	0.37706	0.33871	0.29957	0.26107	0.21886	0.17564	0.12822	0.08379	0.04518	0.01814
5	0.41373	0.37515	0.33529	0.29543	0.25105	0.20468	0.15281	0.10273	0.05763	0.02450
6	0.44986	0.41142	0.37122	0.33039	0.28425	0.23510	0.17910	0.12350	0.07177	0.03206
7	0.48541	0.44743	0.40726	0.36581	0.31830	0.26675	0.20698	0.14603	0.08759	0.04089
8	0.52037	0.48314	0.44332	0.40159	0.35309	0.29949	0.23631	0.17024	0.10505	0.05100
9	0.55471	0.51849	0.47932	0.43763	0.38850	0.33320	0.26699	0.19605	0.12413	0.06243
10	0.58844	0.55346	0.51521	0.47384	0.42442	0.36778	0.29891	0.22337	0.14479	0.07518
11	0.62158	0.58800	0.55092	0.51015	0.46076	0.40311	0.33196	0.25212	0.16699	0.08926
12	0.65403	0.62211	0.58642	0.54648	0.49743	0.43915	0.36605	0.28221	0.19068	0.10467
13	0.68727	0.65702	0.62284	0.58391	0.53546	0.47670	0.40196	0.31426	0.21624	0.12161
14	0.71950	0.69100	0.65837	0.62040	0.57242	0.51411	0.43868	0.34517	0.24090	0.13800
15	0.75015	0.72365	0.69277	0.65398	0.60873	0.54922	0.47149	0.37662	0.26645	0.15540
16	0.78016	0.75582	0.72693	0.69164	0.64551	0.58622	0.50769	0.40987	0.29400	0.17457
17	0.80944	0.78742	0.76074	0.72724	0.68261	0.62398	0.54516	0.44486	0.32338	0.19565
18	0.83797	0.81838	0.79411	0.76265	0.71989	0.66235	0.58377	0.48151	0.35517	0.21869
19	0.86569	0.84864	0.82693	0.79777	0.75720	0.70117	0.62337	0.51969	0.38870	0.24371
20	0.89261	0.87814	0.85913	0.83247	0.79440	0.74027	0.66377	0.55922	0.42407	0.27070
21	0.91827	0.90647	0.89017	0.86621	0.83088	0.77919	0.70467	0.60016	0.46168	0.30026
22	0.94066	0.93159	0.91851	0.89814	0.86702	0.81966	0.74962	0.64288	0.50150	0.33976
23	0.96034	0.95303	0.94415	0.92784	0.90173	0.86001	0.79635	0.69980	0.56198	0.38737
24	0.97659	0.97258	0.96600	0.95398	0.93359	0.89847	0.84377	0.75558	0.62510	0.44556

Note: Values of $l(x)$ based on a sex ratio at birth of 105 males per 100 females.

Annex IX

VALUES OF PROBABILITY OF SURVIVING FOR BOTH SEXES COMBINED, FOR AGES UP TO 15, FROM COALE-DEMENY MODEL LIFE TABLES USING DIFFERENT SEX RATIOS AT BIRTH

TABLE 239. NORTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $_1(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.02

	Probability of surviving, $_1(x)$						
Level	$_1(1)$	$_1(2)$	$_1(3)$	$_1(4)$	$_1(5)$	$_1(10)$	$_1(15)$
1	0.65429	0.57221	0.52198	0.48435	0.45576	0.39745	0.37129
2	0.68814	0.60613	0.55840	0.52265	0.49549	0.43785	0.41160
3	0.71092	0.63717	0.59205	0.55825	0.53257	0.47617	0.45013
4	0.73520	0.66577	0.62330	0.59147	0.56730	0.51259	0.48700
5	0.75734	0.69227	0.65245	0.62262	0.59996	0.54727	0.52232
6	0.77768	0.71692	0.67973	0.65191	0.63075	0.58035	0.55621
7	0.79645	0.73997	0.70541	0.67953	0.65986	0.61195	0.58876
8	0.81386	0.76158	0.72960	0.70564	0.68744	0.64218	0.62004
9	0.83007	0.78191	0.75245	0.73039	0.71362	0.67115	0.65014
10	0.84521	0.80109	0.77410	0.75388	0.73853	0.69891	0.67911
11	0.85940	0.81922	0.79464	0.77623	0.76224	0.72558	0.70703
12	0.87270	0.83636	0.81413	0.79746	0.78483	0.75134	0.73391
13	0.88510	0.85313	0.83357	0.81892	0.80779	0.77744	0.76153
14	0.89626	0.86855	0.85153	0.83875	0.82901	0.80200	0.78751
15	0.90711	0.88342	0.86874	0.85766	0.84917	0.82540	0.81235
16	0.91761	0.89797	0.88539	0.87577	0.86830	0.84765	0.83606
17	0.92774	0.91172	0.90112	0.89290	0.88644	0.86881	0.85866
18	0.93746	0.92469	0.91595	0.90909	0.90365	0.88891	0.88020
19	0.94676	0.93690	0.92993	0.92440	0.91997	0.90803	0.90073
20	0.95562	0.94839	0.94311	0.93838	0.93545	0.92621	0.92029
21	0.96403	0.95919	0.95555	0.95219	0.95018	0.94355	0.93898
22	0.97199	0.96937	0.96734	0.96567	0.96431	0.96022	0.95698
23	0.97909	0.97757	0.97635	0.97533	0.97450	0.97185	0.96965
24	0.98586	0.98509	0.98445	0.98392	0.98348	0.98200	0.98065

TABLE 240. NORTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $_1(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.03

	Probability of surviving, $_1(x)$						
Level	$_1(1)$	$_1(2)$	$_1(3)$	$_1(4)$	$_1(5)$	$_1(10)$	$_1(15)$
1	0.65416	0.57209	0.52186	0.48424	0.45565	0.39736	0.37120
2	0.68802	0.60602	0.55830	0.52255	0.49539	0.43776	0.41151
3	0.71082	0.63707	0.59195	0.55815	0.53247	0.47609	0.45005
4	0.73510	0.66568	0.62320	0.59138	0.56721	0.51251	0.48692
5	0.75725	0.69218	0.65236	0.62253	0.59987	0.54719	0.52225
6	0.77759	0.71683	0.67966	0.65182	0.63067	0.58027	0.55614
7	0.79637	0.73989	0.70533	0.67945	0.65978	0.61188	0.58869
8	0.81379	0.76150	0.72952	0.70556	0.68736	0.64211	0.61997
9	0.83000	0.78184	0.75234	0.73032	0.71355	0.67108	0.65007
10	0.84515	0.80103	0.77401	0.75382	0.73846	0.69885	0.67905
11	0.85934	0.81916	0.79458	0.77618	0.76218	0.72552	0.70698
12	0.87265	0.83631	0.81408	0.79742	0.78477	0.75108	0.73385
13	0.88506	0.85308	0.83352	0.81887	0.80774	0.77739	0.76148
14	0.89622	0.86851	0.85148	0.83870	0.82896	0.80195	0.78746
15	0.90707	0.88338	0.86869	0.85762	0.84913	0.82535	0.81230
16	0.91757	0.89793	0.88533	0.87573	0.86826	0.84760	0.83601
17	0.92770	0.91169	0.90108	0.89286	0.88640	0.86877	0.85862
18	0.93743	0.92466	0.91592	0.90906	0.90361	0.88887	0.88016
19	0.94673	0.93688	0.92990	0.92437	0.91995	0.90799	0.90069
20	0.95559	0.94837	0.94309	0.93885	0.93543	0.92618	0.92026
21	0.96401	0.95917	0.95553	0.95257	0.95015	0.94352	0.93895
22	0.97197	0.96935	0.96732	0.96565	0.96429	0.96019	0.95696
23	0.97908	0.97755	0.97633	0.97532	0.97446	0.97183	0.96963
24	0.98585	0.98508	0.98444	0.98391	0.98347	0.98199	0.98064

TABLE 241. NORTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED. SEX RATIO AT BIRTH OF 1.04

	Probability of surviving, $l(x)$						
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.65404	0.57197	0.52175	0.48413	0.45555	0.39727	0.37112
2	0.68391	0.60590	0.55819	0.52244	0.49529	0.43767	0.41143
3	0.71071	0.63697	0.59185	0.55805	0.53238	0.47600	0.44997
4	0.73500	0.66558	0.62310	0.59129	0.56712	0.51242	0.48684
5	0.75716	0.69209	0.65223	0.62244	0.59978	0.54711	0.52217
6	0.77751	0.71675	0.67958	0.65174	0.63059	0.58020	0.55607
7	0.79630	0.73981	0.70525	0.67937	0.65970	0.61181	0.58862
8	0.81372	0.76143	0.72945	0.70549	0.68729	0.64205	0.61990
9	0.82994	0.78177	0.75231	0.73025	0.71349	0.67101	0.65001
10	0.84509	0.80096	0.77395	0.75375	0.73840	0.69879	0.67899
11	0.85929	0.81910	0.79452	0.77612	0.76213	0.72546	0.70692
12	0.87260	0.83626	0.81403	0.79737	0.78472	0.75103	0.73380
13	0.88501	0.85303	0.83347	0.81882	0.80769	0.77734	0.76143
14	0.89618	0.86846	0.85143	0.83866	0.82891	0.80190	0.78747
15	0.90703	0.88333	0.86863	0.85757	0.84908	0.82530	0.81225
16	0.91734	0.89789	0.88530	0.87569	0.86822	0.84756	0.83597
17	0.92767	0.91166	0.90104	0.89262	0.88636	0.86872	0.85857
18	0.93740	0.92463	0.91588	0.90902	0.90358	0.88883	0.88012
19	0.94671	0.93685	0.92988	0.92434	0.91990	0.90795	0.90065
20	0.95557	0.94835	0.94306	0.93882	0.93540	0.92615	0.92023
21	0.96399	0.95915	0.95550	0.95254	0.95013	0.94349	0.93892
22	0.97195	0.96934	0.96730	0.96563	0.96427	0.96017	0.95693
23	0.97906	0.97754	0.97632	0.97530	0.97447	0.97181	0.96961
24	0.98584	0.98507	0.98443	0.98390	0.98346	0.98197	0.98062

TABLE 242. NORTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.05

	Probability of surviving, $l(x)$						
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.65392	0.57185	0.52163	0.48402	0.45544	0.39718	0.37104
2	0.68379	0.60580	0.55808	0.52234	0.49519	0.43758	0.41135
3	0.71061	0.63686	0.59175	0.55795	0.53228	0.47592	0.44989
4	0.73491	0.66548	0.62301	0.59120	0.56703	0.51234	0.48677
5	0.75708	0.69200	0.65218	0.62236	0.59970	0.54703	0.52210
6	0.77743	0.71667	0.67950	0.65166	0.63051	0.58012	0.55600
7	0.79622	0.73973	0.70518	0.67929	0.65963	0.61173	0.58855
8	0.81365	0.76136	0.72938	0.70542	0.68722	0.64198	0.61984
9	0.82988	0.78171	0.75224	0.73018	0.71342	0.67095	0.64995
10	0.84503	0.80090	0.77391	0.75369	0.73834	0.69873	0.67893
11	0.85924	0.81905	0.79446	0.77606	0.76207	0.72540	0.70687
12	0.87255	0.83620	0.81397	0.79732	0.78466	0.75097	0.73375
13	0.88497	0.85299	0.83342	0.81877	0.80764	0.77729	0.76138
14	0.89615	0.86841	0.85139	0.83861	0.82886	0.80185	0.78736
15	0.90699	0.88329	0.86860	0.85753	0.84904	0.82525	0.81227
16	0.91750	0.89783	0.88526	0.87564	0.86818	0.84751	0.83592
17	0.92764	0.91162	0.90101	0.89278	0.88633	0.86863	0.85853
18	0.93737	0.92460	0.91585	0.90899	0.90354	0.88879	0.88008
19	0.94668	0.93682	0.92985	0.92431	0.91987	0.90792	0.90061
20	0.95555	0.94833	0.94304	0.93880	0.93537	0.92612	0.92019
21	0.96397	0.95913	0.95548	0.95252	0.95011	0.94346	0.93889
22	0.97194	0.96932	0.96728	0.96561	0.96425	0.96014	0.95690
23	0.97905	0.97753	0.97630	0.97529	0.97445	0.97179	0.96958
24	0.98583	0.98506	0.98442	0.98389	0.98345	0.98196	0.98061

TABLE 243. NORTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.06

Level	Probability of surviving, $l(x)$						
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.65380	0.57173	0.52152	0.48392	0.45534	0.39709	0.37096
2	0.68368	0.60569	0.55798	0.52224	0.49509	0.43749	0.41127
3	0.71051	0.63676	0.59165	0.55786	0.53219	0.47583	0.44981
4	0.73481	0.66539	0.62292	0.59111	0.56694	0.51226	0.48669
5	0.75699	0.69191	0.65209	0.62227	0.59961	0.54696	0.52203
6	0.77715	0.71659	0.67942	0.65158	0.63043	0.58005	0.55593
7	0.79615	0.73966	0.70510	0.67922	0.65955	0.61166	0.58849
8	0.81358	0.76129	0.72931	0.70535	0.68715	0.64191	0.61978
9	0.82981	0.78164	0.75218	0.73012	0.71335	0.67089	0.64989
10	0.84497	0.80084	0.77384	0.75363	0.73827	0.69867	0.67888
11	0.85918	0.81899	0.79441	0.77600	0.76201	0.72534	0.70681
12	0.87250	0.83615	0.81392	0.79726	0.78461	0.75092	0.73369
13	0.88492	0.85294	0.83337	0.81872	0.80759	0.77724	0.76133
14	0.89609	0.86837	0.85134	0.83856	0.82881	0.80180	0.78731
15	0.90695	0.88325	0.86856	0.85748	0.84899	0.82520	0.81216
16	0.91747	0.89781	0.88525	0.87560	0.86814	0.84746	0.83587
17	0.92761	0.91139	0.90097	0.89275	0.88629	0.86864	0.85848
18	0.93734	0.92457	0.91582	0.90896	0.90351	0.88875	0.88004
19	0.94666	0.93680	0.92982	0.92428	0.91984	0.90788	0.90054
20	0.95553	0.94830	0.94301	0.93877	0.93535	0.92609	0.92016
21	0.96395	0.95911	0.95546	0.95250	0.95008	0.94343	0.93886
22	0.97192	0.96930	0.96727	0.96559	0.96423	0.96012	0.95687
23	0.97904	0.97751	0.97629	0.97527	0.97443	0.97177	0.96965
24	0.98582	0.98505	0.98441	0.98388	0.98344	0.98195	0.98059

TABLE 244. NORTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.07

Level	Probability of surviving, $l(x)$						
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.65368	0.57162	0.52141	0.48381	0.45524	0.39700	0.37087
2	0.68357	0.60558	0.55787	0.52214	0.49499	0.43741	0.41119
3	0.71041	0.63666	0.59155	0.55776	0.53209	0.47575	0.44973
4	0.73472	0.66530	0.62282	0.59102	0.56685	0.51218	0.48662
5	0.75690	0.69182	0.65201	0.62219	0.59953	0.54688	0.52195
6	0.77727	0.71650	0.67934	0.65150	0.63035	0.57997	0.55586
7	0.79608	0.73958	0.70502	0.67914	0.65948	0.61159	0.58842
8	0.81351	0.76122	0.72923	0.70528	0.68708	0.64184	0.61971
9	0.82975	0.78158	0.75211	0.73005	0.71329	0.67082	0.64983
10	0.84492	0.80078	0.77378	0.75357	0.73821	0.69861	0.67882
11	0.85913	0.81893	0.79435	0.77594	0.76195	0.72529	0.70676
12	0.87245	0.83610	0.81387	0.79721	0.78456	0.75087	0.73364
13	0.88488	0.85289	0.83332	0.81868	0.80754	0.77719	0.76128
14	0.89605	0.86833	0.85129	0.83852	0.82877	0.80175	0.78726
15	0.90692	0.88321	0.86852	0.85744	0.84895	0.82516	0.81211
16	0.91744	0.89777	0.88519	0.87556	0.86810	0.84742	0.83583
17	0.92758	0.91156	0.90094	0.89271	0.88625	0.86860	0.85844
18	0.93732	0.92454	0.91579	0.90892	0.90348	0.88872	0.88000
19	0.94663	0.93677	0.92979	0.92425	0.91981	0.90785	0.90054
20	0.95551	0.94826	0.94299	0.93875	0.93532	0.92605	0.92012
21	0.96393	0.95909	0.95544	0.95248	0.95006	0.94341	0.93883
22	0.97191	0.96929	0.96725	0.96558	0.96421	0.96009	0.95685
23	0.97903	0.97750	0.97627	0.97526	0.97442	0.97175	0.96954
24	0.98581	0.98504	0.98440	0.98387	0.98343	0.98193	0.98058

TABLE 245 SOUTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, l(x), FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.02

	Probability of surviving, l(x)						
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.67858	0.54735	0.48688	0.45472	0.43669	0.39953	0.38229
2	0.70201	0.57849	0.52157	0.49130	0.47434	0.43770	0.42049
3	0.72347	0.60732	0.55393	0.52534	0.50963	0.47389	0.45689
4	0.74244	0.63415	0.58424	0.55770	0.54283	0.50827	0.49165
5	0.76011	0.65924	0.61275	0.58892	0.57417	0.54102	0.52489
6	0.77642	0.68279	0.63964	0.61869	0.60383	0.57227	0.55673
7	0.79152	0.70496	0.66507	0.64385	0.63196	0.60214	0.58728
8	0.80558	0.72590	0.68919	0.66966	0.65872	0.63074	0.61663
9	0.81872	0.74574	0.71211	0.69422	0.68420	0.65815	0.64484
10	0.83083	0.76422	0.73352	0.71719	0.70804	0.68433	0.67184
11	0.84149	0.78188	0.75441	0.73979	0.73160	0.71010	0.69857
12	0.85189	0.79888	0.77445	0.76145	0.75417	0.73484	0.72425
13	0.86204	0.81525	0.79369	0.78222	0.77579	0.75861	0.74896
14	0.87190	0.83101	0.81216	0.80213	0.79651	0.78141	0.77275
15	0.88147	0.84616	0.82989	0.82123	0.81638	0.80333	0.79563
16	0.89074	0.86073	0.84691	0.83955	0.83540	0.82439	0.81764
17	0.89968	0.87481	0.86334	0.85717	0.85370	0.84463	0.83883
18	0.90837	0.88854	0.87926	0.87426	0.87138	0.86421	0.85937
19	0.91829	0.90339	0.89624	0.89230	0.88996	0.88429	0.88030
20	0.92790	0.91663	0.91108	0.90796	0.90605	0.90170	0.89848
21	0.93748	0.92927	0.92512	0.92274	0.92125	0.91804	0.91556
22	0.94700	0.94137	0.93836	0.93663	0.93553	0.93328	0.93141
23	0.95640	0.95271	0.95074	0.94957	0.94880	0.94734	0.94601
24	0.96560	0.96341	0.96227	0.96148	0.96100	0.96013	0.95926

TABLE 246. SOUTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, l(x), FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.03

	Probability of surviving, l(x)						
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.67851	0.54731	0.48685	0.45470	0.43667	0.39953	0.38230
2	0.70194	0.57846	0.52155	0.49128	0.47432	0.43770	0.42050
3	0.72341	0.60728	0.55390	0.52552	0.50961	0.47388	0.45690
4	0.74239	0.63411	0.58422	0.55767	0.54281	0.50826	0.49165
5	0.76006	0.65920	0.61272	0.58799	0.57414	0.54101	0.52488
6	0.77637	0.68275	0.63961	0.61666	0.60380	0.57225	0.55673
7	0.79147	0.70492	0.66504	0.64383	0.63193	0.60213	0.58727
8	0.80554	0.72587	0.68916	0.66963	0.65869	0.63072	0.61661
9	0.81867	0.74570	0.71208	0.69419	0.68417	0.65813	0.64483
10	0.83079	0.76418	0.73349	0.71715	0.70801	0.68430	0.67185
11	0.84145	0.78184	0.75437	0.73976	0.73157	0.71007	0.69855
12	0.85185	0.79885	0.77442	0.76142	0.75414	0.73481	0.72422
13	0.86200	0.81521	0.79365	0.78218	0.77576	0.75856	0.74893
14	0.87187	0.83098	0.81213	0.80210	0.79648	0.78138	0.77271
15	0.88144	0.84613	0.82986	0.82120	0.81635	0.80329	0.79561
16	0.89071	0.86070	0.84688	0.83952	0.83540	0.82436	0.81761
17	0.89965	0.87478	0.86338	0.85714	0.85367	0.84459	0.83881
18	0.90834	0.88851	0.87923	0.87422	0.87134	0.86417	0.85934
19	0.91826	0.90337	0.89622	0.89228	0.88993	0.88427	0.88027
20	0.92788	0.91661	0.91106	0.90794	0.90603	0.90168	0.89846
21	0.93747	0.92926	0.92511	0.92272	0.92123	0.91802	0.91552
22	0.94699	0.94130	0.93834	0.93662	0.93551	0.93327	0.93139
23	0.95639	0.95270	0.95073	0.94955	0.94879	0.94732	0.94600
24	0.96559	0.96340	0.96220	0.96147	0.96099	0.96011	0.95925

TABLE 247. SOUTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.04

	Probability of surviving, $l(x)$						
Level	$l(1)$	$l(2)$	$l(3)$	$l(5)$	$l(10)$	$l(15)$	
1	0.67844	0.54727	0.48683	0.45468	0.43666	0.39953	0.38231
2	0.70188	0.57842	0.52152	0.49126	0.47431	0.43769	0.42050
3	0.72305	0.60724	0.55388	0.52550	0.50959	0.47388	0.45690
4	0.74233	0.63408	0.58419	0.55765	0.54279	0.50825	0.49165
5	0.76001	0.65916	0.61269	0.58797	0.57412	0.54100	0.52488
6	0.77632	0.68272	0.63958	0.61663	0.60378	0.57224	0.55672
7	0.79143	0.70489	0.66501	0.64380	0.63191	0.60211	0.58726
8	0.80549	0.72583	0.68913	0.66960	0.65866	0.63070	0.61660
9	0.81863	0.74567	0.71205	0.69416	0.68414	0.65811	0.64481
10	0.83094	0.76454	0.73385	0.71762	0.70798	0.68427	0.67183
11	0.84241	0.78185	0.75464	0.73972	0.73154	0.71004	0.69852
12	0.85302	0.79881	0.77439	0.76139	0.75411	0.73478	0.72420
13	0.86297	0.81518	0.79362	0.78215	0.77573	0.75853	0.74890
14	0.87184	0.83094	0.81209	0.80206	0.79645	0.78135	0.77268
15	0.88141	0.84610	0.82983	0.82117	0.81632	0.80326	0.79556
16	0.89068	0.86067	0.84684	0.83949	0.83537	0.82432	0.81757
17	0.89963	0.87475	0.86325	0.85717	0.85364	0.84456	0.83876
18	0.90832	0.88845	0.87920	0.87419	0.87131	0.86414	0.85930
19	0.91824	0.90335	0.89620	0.89225	0.88991	0.88424	0.88025
20	0.92786	0.91659	0.91104	0.90792	0.90601	0.90165	0.89844
21	0.93745	0.92924	0.92509	0.92271	0.92122	0.91803	0.91590
22	0.94697	0.94128	0.93833	0.93660	0.93550	0.93325	0.93137
23	0.95638	0.95268	0.95072	0.94954	0.94878	0.94731	0.94598
24	0.96558	0.96339	0.96219	0.96146	0.96098	0.96010	0.95924

TABLE 248. SOUTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.05

	Probability of surviving, $l(x)$						
Level	$l(1)$	$l(2)$	$l(3)$	$l(5)$	$l(10)$	$l(15)$	
1	0.67837	0.54723	0.48680	0.45466	0.43665	0.39953	0.38232
2	0.70181	0.57838	0.52150	0.49124	0.47429	0.43769	0.42051
3	0.72299	0.60721	0.55385	0.52548	0.50957	0.47387	0.45690
4	0.74227	0.63404	0.58416	0.55763	0.54277	0.50825	0.49164
5	0.75995	0.65913	0.61266	0.58795	0.57410	0.54099	0.52487
6	0.77627	0.68268	0.63955	0.61661	0.60376	0.57222	0.55671
7	0.79138	0.70485	0.66498	0.64377	0.63188	0.60209	0.58725
8	0.80545	0.72580	0.68910	0.66957	0.65864	0.63068	0.61658
9	0.81859	0.74564	0.71201	0.69413	0.68411	0.65809	0.64479
10	0.83090	0.76441	0.73382	0.71708	0.70794	0.68425	0.67180
11	0.84237	0.78177	0.75460	0.73969	0.73150	0.71001	0.69849
12	0.85298	0.79870	0.77435	0.76133	0.75408	0.73475	0.72417
13	0.86294	0.81515	0.79359	0.78212	0.77570	0.75850	0.74888
14	0.87181	0.83091	0.81206	0.80203	0.79642	0.78132	0.77263
15	0.88138	0.84607	0.82980	0.82114	0.81629	0.80323	0.79553
16	0.89065	0.86064	0.84681	0.83946	0.83533	0.82429	0.81754
17	0.89960	0.87473	0.86322	0.85703	0.85361	0.84452	0.83873
18	0.90829	0.88845	0.87917	0.87416	0.87128	0.86410	0.85926
19	0.91822	0.90333	0.89618	0.89223	0.88989	0.88422	0.88022
20	0.92784	0.91657	0.91102	0.90790	0.90599	0.90163	0.89841
21	0.93743	0.92922	0.92507	0.92269	0.92120	0.91798	0.91548
22	0.94696	0.94127	0.93831	0.93659	0.93548	0.93323	0.93135
23	0.95637	0.95267	0.95071	0.94953	0.94876	0.94729	0.94596
24	0.96557	0.96338	0.96218	0.96145	0.96097	0.96009	0.95923

TABLE 249. SOUTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.06

	Probability of surviving, $l(x)$						
Level	$l(1)$	$l(2)$	$l(3)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$
1	0.67830	0.54720	0.48678	0.45465	0.43664	0.39953	0.38232
2	0.70175	0.57834	0.52147	0.49122	0.47428	0.43768	0.42051
3	0.72293	0.60717	0.55383	0.52545	0.50956	0.47386	0.45690
4	0.74222	0.63401	0.58414	0.55761	0.54275	0.50824	0.49164
5	0.75990	0.65909	0.61264	0.58792	0.57408	0.54097	0.52487
6	0.77622	0.68265	0.63952	0.61658	0.60373	0.57221	0.55670
7	0.79133	0.70482	0.66495	0.64375	0.63186	0.60207	0.58724
8	0.80540	0.72577	0.68907	0.66954	0.65861	0.63066	0.61657
9	0.81853	0.74560	0.71198	0.69410	0.68409	0.65806	0.64478
10	0.83086	0.76437	0.73378	0.71755	0.70791	0.68432	0.67178
11	0.84133	0.78174	0.75427	0.73966	0.73147	0.70994	0.69847
12	0.85174	0.79874	0.77432	0.76132	0.75404	0.73472	0.72414
13	0.86150	0.81531	0.79356	0.78209	0.77566	0.75847	0.74885
14	0.87077	0.83088	0.81203	0.80200	0.79639	0.78129	0.77262
15	0.88135	0.84604	0.82976	0.82111	0.81626	0.80329	0.79550
16	0.89062	0.86066	0.84678	0.83942	0.83530	0.82426	0.81751
17	0.89957	0.87470	0.86319	0.85705	0.85357	0.84449	0.83869
18	0.90827	0.88843	0.87914	0.87413	0.87125	0.86407	0.85923
19	0.91820	0.90331	0.89615	0.89221	0.88987	0.88420	0.88019
20	0.92782	0.91655	0.91100	0.90788	0.90597	0.90161	0.89839
21	0.93742	0.92921	0.92505	0.92267	0.92118	0.91796	0.91546
22	0.94694	0.94125	0.93830	0.93657	0.93546	0.93322	0.93133
23	0.95635	0.95266	0.95069	0.94952	0.94875	0.94728	0.94595
24	0.96536	0.96337	0.96217	0.96144	0.96096	0.96008	0.95921

TABLE 250. SOUTH MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.07

	Probability of surviving, $l(x)$						
Level	$l(1)$	$l(2)$	$l(3)$	$l(5)$	$l(10)$	$l(15)$	$l(20)$
1	0.67824	0.54716	0.48676	0.45463	0.43662	0.39953	0.38235
2	0.70169	0.57831	0.52145	0.49120	0.47426	0.43768	0.42052
3	0.72287	0.60714	0.55380	0.52543	0.50954	0.47386	0.45690
4	0.74216	0.63397	0.58411	0.55758	0.54273	0.50823	0.49164
5	0.75985	0.65906	0.61261	0.58790	0.57406	0.54096	0.52486
6	0.77617	0.68261	0.63949	0.61656	0.60371	0.57219	0.55670
7	0.79128	0.70478	0.66492	0.64372	0.63183	0.60206	0.58723
8	0.80536	0.72573	0.68904	0.66952	0.65858	0.63064	0.61636
9	0.81851	0.74557	0.71195	0.69407	0.68406	0.65804	0.64476
10	0.83062	0.76404	0.73335	0.71702	0.70788	0.68419	0.67176
11	0.84129	0.78170	0.75424	0.73963	0.73144	0.70995	0.69844
12	0.85171	0.79871	0.77429	0.76129	0.75401	0.73469	0.72411
13	0.86187	0.81508	0.79353	0.78205	0.77563	0.75844	0.74882
14	0.87174	0.83085	0.81200	0.80197	0.79635	0.78125	0.77259
15	0.88132	0.84601	0.82973	0.82108	0.81623	0.80317	0.79547
16	0.89059	0.86058	0.84675	0.83939	0.83527	0.82423	0.81747
17	0.89955	0.87467	0.86316	0.85701	0.85354	0.84446	0.83866
18	0.90824	0.88840	0.87911	0.87410	0.87122	0.86404	0.85919
19	0.91818	0.90329	0.89613	0.89218	0.88984	0.88417	0.88017
20	0.92780	0.91653	0.91098	0.90786	0.90595	0.90159	0.89836
21	0.93740	0.92919	0.92504	0.92265	0.92116	0.91795	0.91544
22	0.94693	0.94124	0.93828	0.93656	0.93545	0.93320	0.93132
23	0.95634	0.95265	0.95068	0.94950	0.94874	0.94726	0.94593
24	0.96535	0.96336	0.96216	0.96143	0.96095	0.96007	0.95920

TABLE 251. EAST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED. SEX RATIO AT BIRTH OF 1.02

	Probability of surviving $l(x)$						
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.53316	0.48363	0.43439	0.41549	0.40205	0.37347	0.35976
2	0.57075	0.50285	0.47429	0.45583	0.44272	0.41397	0.40005
3	0.60501	0.53936	0.51174	0.49390	0.48121	0.45267	0.43871
4	0.63647	0.57350	0.54701	0.52990	0.51773	0.48968	0.47584
5	0.66552	0.60556	0.58134	0.56405	0.55247	0.52517	0.51155
6	0.69246	0.63576	0.61192	0.59652	0.58556	0.55920	0.54592
7	0.71751	0.66431	0.64191	0.62744	0.61715	0.59189	0.57905
8	0.74106	0.69136	0.67046	0.65696	0.64735	0.62334	0.61100
9	0.76310	0.71704	0.69768	0.68517	0.67627	0.65362	0.64184
10	0.78384	0.74148	0.72367	0.71217	0.70399	0.68280	0.67162
11	0.80323	0.76524	0.74927	0.73896	0.73162	0.71216	0.70158
12	0.82115	0.78741	0.77372	0.76407	0.75754	0.73978	0.72991
13	0.83842	0.80871	0.79621	0.78815	0.78241	0.76636	0.75724
14	0.85504	0.82915	0.81827	0.81125	0.80625	0.79191	0.78357
15	0.87100	0.84877	0.83942	0.83339	0.82910	0.81646	0.80891
16	0.88631	0.86757	0.85968	0.85460	0.85097	0.84003	0.83329
17	0.90098	0.88567	0.87919	0.87498	0.87194	0.86266	0.85675
18	0.91499	0.90307	0.89792	0.89453	0.89209	0.88439	0.87929
19	0.92834	0.91971	0.91582	0.91319	0.91123	0.90524	0.90096
20	0.94106	0.93546	0.93283	0.93101	0.92962	0.92524	0.92177
21	0.95376	0.95002	0.94821	0.94693	0.94593	0.94267	0.93997
22	0.96518	0.96286	0.96170	0.96086	0.96019	0.95797	0.95596
23	0.97544	0.97415	0.97348	0.97299	0.97260	0.97121	0.96982
24	0.98426	0.98365	0.98332	0.98308	0.98288	0.98212	0.98125

TABLE 252. EAST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED. SEX RATIO AT BIRTH OF 1.03

	Probability of surviving $l(x)$						
Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.53297	0.48346	0.43423	0.41534	0.40191	0.37334	0.35960
2	0.57053	0.50270	0.47415	0.45569	0.44258	0.41385	0.39994
3	0.60485	0.53927	0.51160	0.49377	0.48108	0.45255	0.43860
4	0.63633	0.57336	0.54688	0.52978	0.51761	0.48957	0.47574
5	0.66538	0.60543	0.58022	0.56393	0.55235	0.52506	0.51145
6	0.69234	0.63564	0.61180	0.59641	0.58545	0.55910	0.54583
7	0.71746	0.66420	0.64180	0.62734	0.61705	0.59180	0.57896
8	0.74096	0.69125	0.67036	0.65686	0.64726	0.62325	0.61092
9	0.76300	0.71694	0.69758	0.68507	0.67618	0.65353	0.64176
10	0.78375	0.74139	0.72359	0.71209	0.70390	0.68272	0.67155
11	0.80315	0.76516	0.74919	0.73888	0.73154	0.71208	0.70151
12	0.82107	0.78733	0.77315	0.76399	0.75747	0.73971	0.72984
13	0.83835	0.80864	0.79614	0.78808	0.78234	0.76629	0.75717
14	0.85498	0.82909	0.81821	0.81119	0.80619	0.79185	0.78351
15	0.87094	0.84871	0.83937	0.83333	0.82904	0.81640	0.80885
16	0.88626	0.86751	0.85963	0.85454	0.85092	0.83997	0.83323
17	0.90093	0.88562	0.87914	0.87493	0.87189	0.86261	0.85670
18	0.91495	0.90303	0.89787	0.89447	0.89197	0.88435	0.87925
19	0.92831	0.91967	0.91579	0.91316	0.91119	0.90520	0.90091
20	0.94103	0.93541	0.93279	0.93098	0.92959	0.92521	0.92174
21	0.95374	0.94999	0.94819	0.94690	0.94590	0.94268	0.93994
22	0.96516	0.96284	0.96168	0.96083	0.96017	0.95794	0.95593
23	0.97543	0.97414	0.97347	0.97298	0.97258	0.97120	0.96980
24	0.98425	0.98364	0.98331	0.98307	0.98287	0.98210	0.98124

TABLE 253. EAST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $\ell(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.04

	Probability of surviving, $\ell(x)$						
Level	$\ell(1)$	$\ell(2)$	$\ell(3)$	$\ell(4)$	$\ell(5)$	$\ell(10)$	$\ell(15)$
1	0.53278	0.46350	0.43407	0.41519	0.40176	0.37321	0.35953
2	0.57040	0.50254	0.47400	0.45555	0.44245	0.41372	0.39983
3	0.60470	0.53907	0.51146	0.49363	0.48095	0.45243	0.43849
4	0.63618	0.57322	0.54675	0.52965	0.51749	0.48946	0.47564
5	0.66525	0.60530	0.58010	0.56381	0.55224	0.52495	0.51135
6	0.69221	0.63552	0.61169	0.59630	0.58534	0.55900	0.54574
7	0.71734	0.66409	0.64169	0.62724	0.61695	0.59170	0.57887
8	0.74085	0.69115	0.67026	0.65676	0.64716	0.62310	0.61084
9	0.76290	0.71685	0.69749	0.68498	0.67609	0.65345	0.64168
10	0.78366	0.74130	0.72350	0.71200	0.70382	0.68264	0.67147
11	0.80307	0.76508	0.74911	0.73881	0.73146	0.71201	0.70144
12	0.82100	0.78725	0.77307	0.76392	0.75739	0.73964	0.72978
13	0.83728	0.80757	0.79607	0.78801	0.78227	0.76622	0.75711
14	0.85191	0.82602	0.81814	0.81112	0.80613	0.79178	0.78345
15	0.86509	0.84265	0.83931	0.83327	0.82898	0.81634	0.80879
16	0.87681	0.85746	0.85957	0.85449	0.85086	0.83992	0.83318
17	0.88689	0.87058	0.87909	0.87488	0.87184	0.86257	0.85665
18	0.89491	0.88209	0.89083	0.89443	0.89193	0.88431	0.87920
19	0.92827	0.91963	0.91575	0.91312	0.91113	0.90516	0.90087
20	0.94100	0.93538	0.93276	0.93095	0.92953	0.92517	0.92170
21	0.95171	0.94997	0.94816	0.94687	0.94588	0.94261	0.93991
22	0.96114	0.96282	0.96165	0.96081	0.96013	0.95792	0.95591
23	0.97541	0.97412	0.97345	0.97296	0.97256	0.97118	0.96978
24	0.98424	0.98363	0.98330	0.98306	0.98286	0.98209	0.98122

TABLE 254. EAST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $\ell(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.05

	Probability of surviving, $\ell(x)$						
Level	$\ell(1)$	$\ell(2)$	$\ell(3)$	$\ell(4)$	$\ell(5)$	$\ell(10)$	$\ell(15)$
1	0.53260	0.46343	0.43392	0.41504	0.40162	0.37308	0.35941
2	0.57023	0.50239	0.47385	0.45541	0.44231	0.41360	0.39972
3	0.60454	0.53892	0.51132	0.49350	0.48082	0.45231	0.43839
4	0.63603	0.57309	0.54662	0.52953	0.51737	0.48935	0.47554
5	0.66511	0.60518	0.57998	0.56370	0.55212	0.52484	0.51126
6	0.69209	0.63540	0.61157	0.59619	0.58524	0.55890	0.54565
7	0.71723	0.66398	0.64159	0.62713	0.61684	0.59161	0.57879
8	0.74074	0.69105	0.67016	0.65667	0.64707	0.62307	0.61075
9	0.76281	0.71675	0.69740	0.68489	0.67600	0.65337	0.64160
10	0.78357	0.74122	0.72347	0.71192	0.70374	0.68256	0.67140
11	0.80299	0.76500	0.74903	0.73873	0.73138	0.71194	0.70137
12	0.82092	0.78718	0.77300	0.76385	0.75732	0.73957	0.72971
13	0.83721	0.80750	0.79600	0.78794	0.78220	0.76615	0.75704
14	0.85183	0.82596	0.81808	0.81106	0.80606	0.79172	0.78338
15	0.86503	0.84259	0.83925	0.83322	0.82892	0.81628	0.80873
16	0.87675	0.85741	0.85952	0.85444	0.85081	0.83987	0.83313
17	0.88683	0.87053	0.87905	0.87483	0.87179	0.86252	0.85660
18	0.89487	0.89205	0.89779	0.89439	0.89188	0.88426	0.87915
19	0.92824	0.91960	0.91571	0.91308	0.91112	0.90512	0.90083
20	0.94097	0.93535	0.93273	0.93091	0.92952	0.92514	0.92166
21	0.95169	0.94994	0.94813	0.94685	0.94585	0.94258	0.93987
22	0.96112	0.96280	0.96163	0.96079	0.96013	0.95789	0.95588
23	0.97540	0.97411	0.97344	0.97294	0.97255	0.97116	0.96976
24	0.98423	0.98362	0.98329	0.98305	0.98284	0.98208	0.98121

TABLE 255. EAST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.06

Level	Probability of surviving, $l(x)$						
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.53242	0.46297	0.43377	0.41490	0.40148	0.37295	0.35929
2	0.57006	0.50223	0.47331	0.45528	0.44218	0.41348	0.39960
3	0.60438	0.53878	0.51119	0.49337	0.48070	0.45219	0.43828
4	0.63589	0.57296	0.54649	0.52941	0.51725	0.48923	0.47544
5	0.66498	0.60505	0.57986	0.56358	0.55204	0.52474	0.51116
6	0.69197	0.63529	0.61146	0.59608	0.58513	0.55880	0.54556
7	0.71712	0.66387	0.64148	0.62703	0.61674	0.59151	0.57870
8	0.74064	0.69095	0.67006	0.65657	0.64697	0.62299	0.61067
9	0.76271	0.71666	0.69731	0.68481	0.67591	0.65328	0.64153
10	0.78348	0.74113	0.72333	0.71184	0.70366	0.68249	0.67133
11	0.80291	0.76492	0.74896	0.73865	0.73131	0.71186	0.70130
12	0.82085	0.78711	0.77293	0.76377	0.75725	0.73959	0.72964
13	0.83814	0.80843	0.79594	0.78788	0.78214	0.76609	0.75698
14	0.85479	0.82890	0.81802	0.81100	0.80600	0.79166	0.78332
15	0.87077	0.84854	0.83919	0.83316	0.82886	0.81623	0.80868
16	0.88610	0.86736	0.85947	0.85439	0.85076	0.83981	0.83307
17	0.90080	0.88548	0.87900	0.87479	0.87175	0.86247	0.85655
18	0.91483	0.90291	0.89775	0.89434	0.89184	0.88422	0.87911
19	0.92820	0.91956	0.91568	0.91305	0.91108	0.90508	0.90079
20	0.94094	0.93552	0.93270	0.93088	0.92949	0.92511	0.92163
21	0.95366	0.94992	0.94813	0.94682	0.94582	0.94255	0.93984
22	0.96310	0.96078	0.96161	0.96077	0.96011	0.95787	0.95586
23	0.97539	0.97409	0.97342	0.97293	0.97253	0.97114	0.96975
24	0.98422	0.98361	0.98328	0.98303	0.98283	0.98207	0.98120

TABLE 256. EAST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.07

Level	Probability of surviving, $l(x)$						
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.53223	0.46281	0.43362	0.41476	0.40134	0.37283	0.35918
2	0.56990	0.50208	0.47357	0.45514	0.44203	0.41336	0.39950
3	0.60423	0.53864	0.51105	0.49325	0.48057	0.45208	0.43818
4	0.63575	0.57283	0.54637	0.52929	0.51713	0.48912	0.47534
5	0.66485	0.60493	0.57974	0.56347	0.55190	0.52463	0.51107
6	0.69185	0.63517	0.61135	0.59597	0.58502	0.55870	0.54547
7	0.71700	0.66377	0.64138	0.62693	0.61665	0.59142	0.57861
8	0.74054	0.69085	0.66996	0.65648	0.64688	0.62290	0.61059
9	0.76262	0.71657	0.69722	0.68472	0.67582	0.65320	0.64145
10	0.78339	0.74105	0.72325	0.71176	0.70358	0.68240	0.67126
11	0.80283	0.76484	0.74888	0.73858	0.73123	0.71179	0.70125
12	0.82077	0.78703	0.77285	0.76370	0.75718	0.73943	0.72958
13	0.83807	0.80836	0.79587	0.78781	0.78207	0.76602	0.75692
14	0.85473	0.82884	0.81796	0.81094	0.80594	0.79160	0.78326
15	0.87072	0.84848	0.83913	0.83310	0.82881	0.81617	0.80862
16	0.88605	0.86730	0.85942	0.85433	0.85071	0.83976	0.83302
17	0.90075	0.88543	0.87895	0.87474	0.87170	0.86242	0.85650
18	0.91479	0.90286	0.89771	0.89430	0.89180	0.88418	0.87907
19	0.92817	0.91953	0.91564	0.91301	0.91104	0.90504	0.90075
20	0.94092	0.93529	0.93267	0.93083	0.92946	0.92507	0.92159
21	0.95364	0.94989	0.94808	0.94679	0.94579	0.94252	0.93981
22	0.96308	0.96276	0.96159	0.96075	0.96009	0.95785	0.95583
23	0.97537	0.97408	0.97341	0.97291	0.97252	0.97113	0.96973
24	0.98421	0.98360	0.98327	0.98302	0.98282	0.98206	0.98118

TABLE 257. WEST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.02.

	Probability of surviving, $l(x)$						
Level	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.60761	0.52630	0.49027	0.46683	0.44924	0.41762	0.39552
2	0.64122	0.56332	0.52880	0.50634	0.48949	0.45811	0.43604
3	0.67151	0.59738	0.56453	0.54317	0.52713	0.49634	0.47453
4	0.69903	0.62892	0.59784	0.57763	0.56247	0.53251	0.51118
5	0.72420	0.65824	0.62901	0.61000	0.59574	0.56682	0.54612
6	0.74738	0.68565	0.65830	0.64050	0.62716	0.59943	0.57949
7	0.76881	0.71135	0.68589	0.66933	0.65690	0.63049	0.61142
8	0.78872	0.73552	0.71196	0.69662	0.68512	0.66013	0.64200
9	0.80729	0.75833	0.73665	0.72254	0.71195	0.68846	0.67134
10	0.82467	0.77992	0.76008	0.74719	0.73751	0.71557	0.69953
11	0.84098	0.80037	0.78237	0.77068	0.76190	0.74155	0.72661
12	0.85634	0.81980	0.80362	0.79309	0.78519	0.76647	0.75269
13	0.87103	0.83915	0.82503	0.81584	0.80895	0.79198	0.77950
14	0.88490	0.85787	0.84560	0.83776	0.83187	0.81670	0.80552
15	0.89753	0.87434	0.86401	0.85726	0.85218	0.83871	0.82870
16	0.90974	0.89041	0.88170	0.87596	0.87158	0.85979	0.85098
17	0.92148	0.90596	0.89874	0.89388	0.89010	0.87997	0.87234
18	0.93275	0.92069	0.91490	0.91091	0.90778	0.89928	0.89283
19	0.94353	0.93462	0.93021	0.92711	0.92465	0.91774	0.91246
20	0.95380	0.94779	0.94471	0.94257	0.94093	0.93541	0.93128
21	0.96403	0.96028	0.95829	0.95686	0.95568	0.95179	0.94867
22	0.97327	0.97088	0.96973	0.96881	0.96805	0.96532	0.96310
23	0.98166	0.98044	0.97975	0.97923	0.97880	0.97708	0.97564
24	0.98884	0.98830	0.98799	0.98775	0.98755	0.98662	0.98579

TABLE 258. WEST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.03

	Probability of surviving, $l(x)$						
Level	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.60748	0.52619	0.49016	0.46673	0.44915	0.41754	0.39543
2	0.64110	0.56321	0.52870	0.50624	0.48940	0.45804	0.43597
3	0.67140	0.59728	0.56444	0.54308	0.52705	0.49626	0.47447
4	0.69892	0.62882	0.59775	0.57755	0.56239	0.53243	0.51112
5	0.72411	0.65815	0.62893	0.60992	0.59566	0.56675	0.54606
6	0.74729	0.68557	0.65822	0.64043	0.62709	0.59936	0.57943
7	0.76873	0.71127	0.68581	0.66925	0.65683	0.63043	0.61136
8	0.78864	0.73545	0.71188	0.69655	0.68505	0.66007	0.64195
9	0.80722	0.75826	0.73658	0.72247	0.71189	0.68840	0.67129
10	0.82460	0.77985	0.76007	0.74713	0.73745	0.71551	0.69948
11	0.84092	0.80031	0.78231	0.77062	0.76184	0.74149	0.72656
12	0.85628	0.81975	0.80356	0.79303	0.78514	0.76642	0.75264
13	0.87097	0.83910	0.82496	0.81579	0.80890	0.79194	0.77946
14	0.88485	0.85782	0.84556	0.83771	0.83183	0.81666	0.80548
15	0.89749	0.87429	0.86397	0.85722	0.85213	0.83867	0.82866
16	0.90970	0.89036	0.88165	0.87591	0.87154	0.85975	0.85094
17	0.92145	0.90592	0.89870	0.89384	0.89006	0.87993	0.87230
18	0.93272	0.92066	0.91486	0.91087	0.90774	0.89924	0.89278
19	0.94349	0.93459	0.93017	0.92708	0.92461	0.91770	0.91242
20	0.95377	0.94776	0.94468	0.94249	0.94072	0.93538	0.93124
21	0.96400	0.96025	0.95826	0.95683	0.95565	0.95175	0.94863
22	0.97325	0.97096	0.96971	0.96879	0.96802	0.96530	0.96307
23	0.98165	0.98043	0.97973	0.97922	0.97879	0.97706	0.97562
24	0.98883	0.98829	0.98797	0.98773	0.98754	0.98660	0.98578

TABLE 259. WEST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.04

Level	Probability of surviving, $l(x)$						
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.60735	0.52607	0.49006	0.46663	0.44905	0.41746	0.39538
2	0.64098	0.56311	0.52860	0.50615	0.48931	0.45796	0.43591
3	0.67129	0.59718	0.56434	0.54299	0.52696	0.49618	0.47441
4	0.69882	0.62873	0.59766	0.57746	0.56234	0.53236	0.51106
5	0.72401	0.65807	0.62885	0.60984	0.59558	0.56668	0.54600
6	0.74720	0.68548	0.65814	0.64035	0.62701	0.59929	0.57938
7	0.76864	0.71119	0.68574	0.66918	0.65676	0.63036	0.61130
8	0.78856	0.73538	0.71181	0.69649	0.68499	0.66001	0.64190
9	0.80715	0.75820	0.73652	0.72241	0.71183	0.68834	0.67226
10	0.82454	0.77979	0.75996	0.74706	0.73739	0.71546	0.69942
11	0.84086	0.80025	0.78226	0.77056	0.76178	0.74144	0.72652
12	0.85622	0.81969	0.80351	0.79298	0.78508	0.76637	0.75260
13	0.87092	0.83905	0.82493	0.81575	0.80845	0.79189	0.77943
14	0.88480	0.85757	0.84551	0.83797	0.83178	0.81662	0.80544
15	0.89744	0.87425	0.86393	0.85718	0.85209	0.83862	0.82862
16	0.90966	0.89032	0.88161	0.87587	0.87149	0.85971	0.85089
17	0.92141	0.90588	0.89866	0.89380	0.89002	0.87989	0.87226
18	0.93268	0.92062	0.91483	0.91084	0.90770	0.89920	0.89274
19	0.94346	0.93456	0.93014	0.92704	0.92458	0.91767	0.91232
20	0.95375	0.94773	0.94465	0.94246	0.94068	0.93534	0.93120
21	0.96358	0.96021	0.95824	0.95680	0.95563	0.95172	0.94860
22	0.97323	0.97084	0.96969	0.96876	0.96800	0.96527	0.96304
23	0.98163	0.98041	0.97971	0.97920	0.97877	0.97704	0.97560
24	0.98882	0.98828	0.98796	0.98772	0.98753	0.98659	0.98576

TABLE 260. WEST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, $l(x)$, FOR AGES UP TO 15, BOTH SEXES COMBINED: SEX RATIO AT BIRTH OF 1.05

Level	Probability of surviving, $l(x)$						
	$l(1)$	$l(2)$	$l(3)$	$l(4)$	$l(5)$	$l(10)$	$l(15)$
1	0.60722	0.52596	0.48996	0.46653	0.44896	0.41738	0.39531
2	0.64086	0.56300	0.52850	0.50606	0.48922	0.45788	0.43584
3	0.67118	0.59709	0.56425	0.54290	0.52688	0.49611	0.47434
4	0.69872	0.62864	0.59758	0.57738	0.56223	0.53229	0.51099
5	0.72392	0.65798	0.62876	0.60976	0.59551	0.56661	0.54594
6	0.74711	0.68540	0.65806	0.64027	0.62694	0.59923	0.57932
7	0.76856	0.71111	0.68566	0.66911	0.65669	0.63030	0.61125
8	0.78849	0.73531	0.71175	0.69642	0.68492	0.65994	0.64184
9	0.80706	0.75813	0.73645	0.72234	0.71176	0.68828	0.67119
10	0.82447	0.77972	0.75989	0.74700	0.73733	0.71540	0.69937
11	0.84080	0.80019	0.78220	0.77050	0.76173	0.74139	0.72647
12	0.85617	0.81963	0.80345	0.79292	0.78503	0.76632	0.75253
13	0.87087	0.83900	0.82489	0.81570	0.80841	0.79185	0.77939
14	0.88476	0.85753	0.84547	0.83765	0.83174	0.81658	0.80540
15	0.89740	0.87421	0.86388	0.85713	0.85205	0.83858	0.82857
16	0.90962	0.89028	0.88157	0.87583	0.87145	0.85966	0.85085
17	0.92137	0.90584	0.89862	0.89376	0.88996	0.87985	0.87222
18	0.93265	0.92058	0.91479	0.91080	0.90766	0.89916	0.89270
19	0.94343	0.93453	0.93011	0.92701	0.92454	0.91763	0.91234
20	0.95372	0.94770	0.94462	0.94243	0.94065	0.93531	0.93117
21	0.96355	0.96020	0.95821	0.95678	0.95560	0.95169	0.94856
22	0.97321	0.97092	0.96967	0.96874	0.96798	0.96524	0.96301
23	0.98162	0.98039	0.97970	0.97918	0.97873	0.97702	0.97558
24	0.98881	0.98827	0.98795	0.98771	0.98751	0.98658	0.98575

TABLE 261. WEST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, l(x), FOR AGES UP TO 15, BOTH SEXES COMBINED, SEX RATIO AT BIRTH OF 1.06

Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.60709	0.52385	0.48985	0.46644	0.44887	0.41730	0.39525
2	0.64074	0.56290	0.52841	0.50596	0.48913	0.45780	0.43578
3	0.67107	0.59699	0.56416	0.54281	0.52679	0.49604	0.47429
4	0.69852	0.62854	0.59749	0.57729	0.56215	0.53222	0.51093
5	0.72382	0.65789	0.62868	0.60968	0.59543	0.56654	0.54588
6	0.74702	0.68532	0.65799	0.64020	0.62687	0.59916	0.57926
7	0.76848	0.71103	0.68559	0.66904	0.65662	0.63024	0.61119
8	0.78841	0.73523	0.71168	0.69635	0.68485	0.65988	0.64179
9	0.80701	0.75806	0.73639	0.72228	0.71170	0.68822	0.67113
10	0.82440	0.77966	0.75983	0.74694	0.73727	0.71534	0.69932
11	0.84074	0.80013	0.78214	0.77044	0.76167	0.74133	0.72642
12	0.85611	0.81958	0.80340	0.79287	0.78498	0.76627	0.75250
13	0.87062	0.83805	0.82384	0.81565	0.80833	0.79181	0.77935
14	0.88431	0.85549	0.84343	0.83758	0.83170	0.81663	0.80536
15	0.89736	0.87216	0.86384	0.85709	0.85200	0.83854	0.82853
16	0.90958	0.89014	0.88153	0.87579	0.87141	0.85962	0.85081
17	0.92133	0.90580	0.89858	0.89372	0.88994	0.87984	0.87218
18	0.93262	0.92055	0.91475	0.91076	0.90763	0.89912	0.89266
19	0.94340	0.93449	0.93007	0.92698	0.92451	0.91759	0.91230
20	0.95369	0.94767	0.94459	0.94239	0.94062	0.93527	0.93113
21	0.96393	0.96018	0.95818	0.95675	0.95557	0.95166	0.94853
22	0.97319	0.97090	0.96964	0.96872	0.96796	0.96522	0.96299
23	0.98160	0.98038	0.97968	0.97917	0.97873	0.97701	0.97555
24	0.98880	0.98826	0.98794	0.98770	0.98750	0.98656	0.98573

TABLE 262. WEST MODEL VALUES FOR PROBABILITY OF SURVIVING FROM BIRTH, l(x), FOR AGES UP TO 15, BOTH SEXES COMBINED, SEX RATIO AT BIRTH OF 1.07

Level	l(1)	l(2)	l(3)	l(4)	l(5)	l(10)	l(15)
1	0.60696	0.52578	0.48975	0.46634	0.44878	0.41722	0.39518
2	0.64063	0.56283	0.52831	0.50587	0.48905	0.45773	0.43572
3	0.67096	0.59689	0.56407	0.54273	0.52671	0.49596	0.47422
4	0.69852	0.62845	0.59740	0.57721	0.56207	0.53215	0.51087
5	0.72373	0.65781	0.62860	0.60960	0.59535	0.56647	0.54582
6	0.74694	0.68524	0.65791	0.64012	0.62679	0.59910	0.57921
7	0.76840	0.71096	0.68552	0.66897	0.65653	0.63018	0.61114
8	0.78834	0.73516	0.71161	0.69628	0.68479	0.65982	0.64174
9	0.80694	0.75799	0.73632	0.72221	0.71164	0.68847	0.67108
10	0.82434	0.77960	0.75977	0.74688	0.73721	0.71529	0.69928
11	0.84068	0.80007	0.78208	0.77039	0.76161	0.74128	0.72637
12	0.85605	0.81952	0.80334	0.79282	0.78493	0.76622	0.75246
13	0.87057	0.83801	0.82379	0.81561	0.80872	0.79177	0.77931
14	0.88427	0.85544	0.84338	0.83754	0.83165	0.81649	0.80532
15	0.89732	0.87212	0.86380	0.85705	0.85196	0.83850	0.82849
16	0.90954	0.89020	0.88144	0.87575	0.87137	0.85958	0.85077
17	0.92130	0.90576	0.89854	0.89368	0.88990	0.87937	0.87214
18	0.93258	0.92051	0.91472	0.91073	0.90759	0.89909	0.89262
19	0.94337	0.93446	0.93004	0.92694	0.92447	0.91756	0.91227
20	0.95366	0.94764	0.94456	0.94236	0.94059	0.93524	0.93109
21	0.96391	0.96015	0.95816	0.95672	0.95554	0.95163	0.94850
22	0.97317	0.97088	0.96962	0.96870	0.96793	0.96519	0.96298
23	0.98159	0.98036	0.97966	0.97915	0.97872	0.97699	0.97553
24	0.98879	0.98825	0.98793	0.98769	0.98749	0.98655	0.98572

Annex X

FIVE-YEAR AND TEN-YEAR SURVIVORSHIP PROBABILITIES FROM COALE-DEMENY MODEL LIFE TABLES[a]

TABLE 263. FEMALE FIVE-YEAR SURVIVORSHIP PROBABILITIES $_5S_{x,\,x+5}$: NORTH MODEL

Age x	Survivorship probability, $_5S_{x,\,x+5}$, for mortality level							
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
0	0.74013	0.76570	0.78829	0.80846	0.82663	0.84313	0.85821	0.87206
5	0.90045	0.90969	0.91794	0.92539	0.93218	0.93839	0.94410	0.94938
10	0.93257	0.93853	0.94388	0.94872	0.95315	0.95720	0.96096	0.96443
15	0.92796	0.93414	0.93969	0.94472	0.94930	0.95351	0.95740	0.96101
20	0.91763	0.92463	0.93090	0.93659	0.94178	0.94655	0.95094	0.95503
25	0.90378	0.91198	0.91936	0.92604	0.93215	0.93776	0.94293	0.94774
30	0.88861	0.89815	0.90673	0.91451	0.92160	0.92811	0.93414	0.93973
35	0.87763	0.88796	0.89725	0.90566	0.91333	0.92036	0.92688	0.93291
40	0.87057	0.88120	0.89073	0.89936	0.90725	0.91450	0.92119	0.92738
45	0.85599	0.86732	0.87752	0.88675	0.89518	0.90293	0.91008	0.91673
50	0.81947	0.83326	0.84566	0.85692	0.86719	0.87667	0.88541	0.89353
55	0.75525	0.77348	0.78988	0.80478	0.81840	0.83095	0.84257	0.85334
60	0.66237	0.68609	0.70755	0.72708	0.74495	0.76141	0.77665	0.79082
65	0.53580	0.56611	0.59356	0.61857	0.64150	0.66270	0.68232	0.70059
70	0.38481	0.42314	0.45796	0.48793	0.51543	0.54078	0.56432	0.58625
75+[a]	0.24468	0.26908	0.28773	0.30417	0.31885	0.33246	0.34540	0.35789
	9	*10*	*11*	*12*	*13*	*14*	*15*	*16*
0	0.88485	0.89670	0.90772	0.91799	0.92878	0.93907	0.94852	0.95700
5	0.95429	0.95886	0.96334	0.96715	0.97125	0.97519	0.97879	0.98206
10	0.96766	0.97068	0.97351	0.97617	0.97860	0.98110	0.98345	0.98567
15	0.96437	0.96751	0.97045	0.97321	0.97558	0.97812	0.98056	0.98288
20	0.95883	0.96239	0.96572	0.96885	0.97148	0.97432	0.97704	0.97966
25	0.95221	0.95639	0.96031	0.96398	0.96714	0.97042	0.97358	0.97662
30	0.94493	0.94979	0.95434	0.95862	0.96235	0.96610	0.96972	0.97321
35	0.93853	0.94378	0.94870	0.95331	0.95727	0.96118	0.96501	0.96872
40	0.93116	0.93855	0.94561	0.94835	0.95235	0.95619	0.95998	0.96371
45	0.92290	0.92869	0.93410	0.93919	0.94345	0.94737	0.95131	0.95523
50	0.90110	0.90819	0.91483	0.92106	0.92658	0.93108	0.93576	0.94049
55	0.86340	0.87279	0.88162	0.88997	0.89706	0.90313	0.90929	0.91550
60	0.80406	0.81644	0.82807	0.83912	0.84825	0.85614	0.86418	0.87233
65	0.71768	0.73370	0.74873	0.76315	0.77468	0.78471	0.79498	0.80549
70	0.60671	0.62589	0.64392	0.66154	0.67498	0.68672	0.69883	0.71127
75+[a]	0.37009	0.38213	0.39410	0.40636	0.41680	0.42674	0.43725	0.44829
	17	*18*	*19*	*20*	*21*	*22*	*23*	*24*
0	0.96464	0.97156	0.97785	0.98362	0.98894	0.99401	0.99649	0.99823
5	0.98507	0.98782	0.99035	0.99271	0.99490	0.99701	0.99818	0.99903
10	0.98774	0.98967	0.99149	0.99319	0.99478	0.99631	0.99763	0.99858
15	0.98508	0.98717	0.98914	0.99100	0.99275	0.99444	0.99639	0.99768
20	0.98217	0.98456	0.98681	0.98895	0.99098	0.99292	0.99528	0.99689
25	0.97954	0.98231	0.98494	0.98744	0.98979	0.99207	0.99436	0.99641
30	0.97655	0.97975	0.98278	0.98567	0.98839	0.99103	0.99365	0.99579
35	0.97231	0.97576	0.97907	0.98224	0.98524	0.98816	0.99119	0.99386
40	0.96738	0.97095	0.97440	0.97773	0.98092	0.98400	0.98741	0.99073
45	0.95911	0.96294	0.96668	0.97032	0.97383	0.97725	0.98119	0.98536
50	0.94522	0.94990	0.95451	0.95902	0.96339	0.96745	0.97256	0.97815
55	0.92172	0.92793	0.93406	0.94007	0.94592	0.95114	0.95815	0.96597
60	0.88055	0.88879	0.89697	0.90504	0.91294	0.92012	0.93011	0.94154
65	0.81614	0.82686	0.83757	0.84820	0.85865	0.86827	0.88220	0.89855
70	0.72396	0.73682	0.74974	0.76261	0.77536	0.78721	0.80511	0.82668
75+[a]	0.45980	0.47171	0.48393	0.49633	0.50882	0.52089	0.53681	0.55484

[a] Value listed for age 75+ is $T(75)/T(71)$.

TABLE 264. FEMALE FIVE-YEAR SURVIVORSHIP PROBABILITIES, ${}_5S_{x,x+5}$, SOUTH MODEL

Age x	Survivorship probability ${}_5S_{x,x+5}$ for mortality level:							
	1	2	3	4	5	6	7	8
0	0.73046	0.73838	0.78287	0.80461	0.82407	0.84166	0.85765	0.87228
5	0.92985	0.93648	0.94245	0.94788	0.95284	0.95741	0.96164	0.96556
10	0.94323	0.94836	0.95297	0.95720	0.96106	0.96465	0.96795	0.97104
15	0.93649	0.93805	0.93895	0.94436	0.94930	0.95386	0.95810	0.96204
20	0.91544	0.92289	0.92965	0.93580	0.94144	0.94663	0.95145	0.95594
25	0.91064	0.91846	0.92555	0.93199	0.93791	0.94336	0.94842	0.95312
30	0.90656	0.91469	0.92200	0.92867	0.93479	0.94044	0.94567	0.95054
35	0.90368	0.91181	0.91915	0.92583	0.93195	0.93760	0.94203	0.94771
40	0.90076	0.90874	0.91599	0.92257	0.92860	0.93416	0.93931	0.94411
45	0.88425	0.89316	0.90122	0.90856	0.91530	0.92154	0.92730	0.93266
50	0.84772	0.85910	0.86940	0.87877	0.88740	0.89535	0.90274	0.90962
55	0.77684	0.79309	0.80780	0.82124	0.83360	0.84503	0.85566	0.86558
60	0.66779	0.69096	0.71196	0.73116	0.74886	0.76523	0.78045	0.79465
65	0.52979	0.55946	0.58651	0.61131	0.63422	0.65542	0.67514	0.69358
70	0.36504	0.40003	0.43189	0.46113	0.48813	0.51319	0.53646	0.55826
75 + [a]	0.22194	0.24664	0.26706	0.28474	0.30058	0.31518	0.32887	0.34197

Age x	9	10	11	12	13	14	15	16
0	0.88573	0.89862	0.91067	0.92169	0.93178	0.94107	0.94964	0.95758
5	0.96922	0.97293	0.97633	0.97943	0.98228	0.98493	0.98736	0.98963
10	0.97392	0.97648	0.97922	0.98176	0.98412	0.98629	0.98832	0.99019
15	0.96572	0.96891	0.97244	0.97570	0.97873	0.98153	0.98414	0.98659
20	0.96013	0.96379	0.96774	0.97142	0.97484	0.97803	0.98102	0.98381
25	0.95752	0.96142	0.96546	0.96924	0.97279	0.97612	0.97924	0.98217
30	0.95508	0.95918	0.96325	0.96708	0.97069	0.97411	0.97732	0.98036
35	0.95227	0.95635	0.96032	0.96409	0.96767	0.97108	0.97431	0.97740
40	0.94861	0.95265	0.95642	0.96005	0.96354	0.96688	0.97009	0.97317
45	0.93768	0.94236	0.94644	0.95040	0.95424	0.95795	0.96155	0.96501
50	0.91604	0.92211	0.92724	0.93224	0.93712	0.94187	0.94647	0.95092
55	0.87486	0.88367	0.89101	0.89822	0.90528	0.91210	0.91877	0.92523
60	0.80796	0.82056	0.83100	0.84126	0.85133	0.86118	0.87080	0.88015
65	0.71088	0.72684	0.74031	0.75361	0.76675	0.77969	0.79237	0.80480
70	0.57870	0.59736	0.61297	0.62849	0.64391	0.65919	0.67427	0.68911
75 + [a]	0.35468	0.36690	0.37781	0.38909	0.40075	0.41274	0.42502	0.43754

Age x	17	18	19	20	21	22	23	24
0	0.96520	0.97244	0.97833	0.98339	0.98772	0.99133	0.99423	0.99647
5	0.99174	0.99369	0.99506	0.99610	0.99734	0.99818	0.99884	0.99933
10	0.99195	0.99362	0.99508	0.99625	0.99725	0.99807	0.99874	0.99925
15	0.98887	0.99103	0.99311	0.99469	0.99606	0.99720	0.99814	0.99886
20	0.98644	0.98892	0.99137	0.99327	0.99492	0.99634	0.99752	0.99844
25	0.98493	0.98756	0.99004	0.99211	0.99395	0.99556	0.99691	0.99802
30	0.98323	0.98597	0.98843	0.99069	0.99273	0.99454	0.99611	0.99742
35	0.98032	0.98361	0.98672	0.98821	0.99052	0.99263	0.99453	0.99619
40	0.97611	0.97894	0.98167	0.98443	0.98706	0.98957	0.99190	0.99405
45	0.96834	0.97148	0.97463	0.97796	0.98122	0.98441	0.98748	0.99040
50	0.95522	0.95925	0.96336	0.96781	0.97223	0.97659	0.98087	0.98503
55	0.93189	0.93729	0.94333	0.94992	0.95651	0.96306	0.96955	0.97592
60	0.88926	0.89768	0.90668	0.91660	0.92661	0.93673	0.94691	0.95708
65	0.81692	0.82858	0.84121	0.85535	0.86994	0.88501	0.90059	0.91668
70	0.70368	0.71791	0.73357	0.75208	0.77130	0.79169	0.81343	0.83677
75 + [a]	0.45024	0.46296	0.47703	0.49271	0.50913	0.52629	0.54414	0.56265

[a] Value fitted for age 75 + is $T(80)/T(75)$.

TABLE 265. FEMALE FIVE-YEAR SURVIVORSHIP PROBABILITIES $_5S_{x,x+5}$: EAST MODEL

Age x	Survivorship probability, $_5S_{x,x+5}$, for mortality level:							
	1	2	3	4	5	6	7	8
0	0.79507	0.81567	0.83383	0.85005	0.86466	0.87792	0.89003	0.90117
5	0.94279	0.94777	0.95232	0.95648	0.96031	0.96386	0.96717	0.97026
10	0.95268	0.95665	0.96028	0.96360	0.96665	0.96950	0.97214	0.97464
15	0.93782	0.94287	0.94766	0.95196	0.95595	0.95965	0.96309	0.96631
20	0.92339	0.92972	0.93589	0.94080	0.94571	0.95025	0.95450	0.95846
25	0.91272	0.91993	0.92651	0.93255	0.93812	0.94329	0.94811	0.95263
30	0.90462	0.91239	0.91949	0.92599	0.93201	0.93759	0.94279	0.94767
35	0.89586	0.90690	0.91422	0.92094	0.92715	0.93290	0.93822	0.94329
40	0.89128	0.90135	0.90871	0.91545	0.92168	0.92747	0.93286	0.93790
45	0.87467	0.88361	0.89176	0.89925	0.90618	0.91260	0.91859	0.92420
50	0.83061	0.84230	0.85294	0.86274	0.87181	0.88024	0.88809	0.89545
55	0.75735	0.77349	0.78825	0.80181	0.81439	0.82611	0.83702	0.84725
60	0.65404	0.67566	0.69545	0.71368	0.73058	0.74633	0.76104	0.77482
65	0.52734	0.55396	0.57832	0.60087	0.62173	0.64114	0.65931	0.67635
70	0.38606	0.41567	0.44288	0.46806	0.49133	0.51300	0.53331	0.55234
75+[*]	0.24077	0.26213	0.27814	0.29239	0.30538	0.31755	0.32913	0.34036

	9	10	11	12	13	14	15	16
0	0.91146	0.92099	0.93069	0.93968	0.94783	0.95527	0.96207	0.96832
5	0.97315	0.97587	0.97855	0.98115	0.98356	0.98579	0.98786	0.98978
10	0.97697	0.97916	0.98108	0.98317	0.98513	0.98698	0.98870	0.99032
15	0.96935	0.97220	0.97458	0.97730	0.97985	0.98225	0.98452	0.98664
20	0.96219	0.96570	0.96866	0.97196	0.97508	0.97801	0.98079	0.98340
25	0.95686	0.96084	0.96433	0.96798	0.97145	0.97474	0.97786	0.98081
30	0.95223	0.95653	0.96037	0.96418	0.96781	0.97129	0.97462	0.97779
35	0.94801	0.95245	0.95643	0.96035	0.96382	0.96734	0.97073	0.97401
40	0.94262	0.94709	0.95103	0.95483	0.95816	0.96164	0.96503	0.96833
45	0.92946	0.93442	0.93877	0.94265	0.94650	0.95031	0.95409	0.95779
50	0.90237	0.90889	0.91464	0.91964	0.92464	0.92962	0.93456	0.93945
55	0.85687	0.86594	0.87397	0.88084	0.88774	0.89464	0.90151	0.90833
60	0.78778	0.80003	0.81076	0.81989	0.82910	0.83835	0.84761	0.85683
65	0.69237	0.70750	0.72058	0.73162	0.74282	0.75415	0.76553	0.77697
70	0.57025	0.58715	0.60167	0.61370	0.62595	0.63844	0.65110	0.66389
75+[*]	0.35131	0.36213	0.37211	0.38110	0.39056	0.40055	0.41099	0.42190

	17	18	19	20	21	22	23	24
0	0.97458	0.98040	0.98567	0.99048	0.99372	0.99610	0.99784	0.99897
5	0.99158	0.99325	0.99483	0.99631	0.99741	0.99829	0.99887	0.99946
10	0.99185	0.99327	0.99462	0.99589	0.99710	0.99799	0.99872	0.99929
15	0.98866	0.99054	0.99233	0.99401	0.99575	0.99701	0.99806	0.99889
20	0.98587	0.98820	0.99040	0.99248	0.99456	0.99615	0.99748	0.99853
25	0.98360	0.98626	0.98875	0.99113	0.99338	0.99524	0.99684	0.99811
30	0.98082	0.98370	0.98644	0.98904	0.99147	0.99368	0.99565	0.99728
35	0.97716	0.98018	0.98308	0.98584	0.98847	0.99111	0.99354	0.99569
40	0.97155	0.97467	0.97767	0.98055	0.98348	0.98663	0.98969	0.99260
45	0.96344	0.96699	0.96844	0.97177	0.97535	0.97938	0.98345	0.98749
50	0.94426	0.94897	0.95356	0.95807	0.96288	0.96848	0.97424	0.98008
55	0.91506	0.92168	0.92815	0.93444	0.94139	0.94957	0.95812	0.96693
60	0.86601	0.87506	0.88395	0.89263	0.90251	0.91442	0.92711	0.94060
65	0.78840	0.79974	0.81095	0.82194	0.83498	0.85115	0.86890	0.88845
70	0.67674	0.68958	0.70234	0.71493	0.73038	0.75020	0.77259	0.79819
75+[*]	0.43321	0.44484	0.45673	0.46880	0.48291	0.50005	0.51893	0.53970

[*] Value listed for age 75 + is $T(80)/T(75)$.

TABLE 266 FEMALE FIVE-YEAR SURVIVORSHIP PROBABILITIES, ${}_5S_{x\ x+5}$, WEST MODEL

Age x	Survivorship probability, ${}_5S_{x\ x+5}$ for mortality level:							
	1	2	3	4	5	6	7	8
0	0.78861	0.81044	0.82959	0.84658	0.86180	0.87556	0.88806	0.89951
5	0.91453	0.92051	0.92586	0.93073	0.93518	0.93926	0.94304	0.94656
10	0.93470	0.94053	0.94579	0.95056	0.95493	0.95896	0.96268	0.96615
15	0.91748	0.92473	0.93127	0.93720	0.94264	0.94764	0.95227	0.95658
20	0.90310	0.91154	0.91915	0.92607	0.93240	0.93824	0.94363	0.94864
25	0.89135	0.90077	0.90925	0.91697	0.92404	0.93055	0.93656	0.94216
30	0.87971	0.89005	0.89937	0.90785	0.91561	0.92274	0.92935	0.93549
35	0.87075	0.88166	0.89149	0.90041	0.90857	0.91608	0.92304	0.92949
40	0.86332	0.87440	0.88438	0.89345	0.90175	0.90938	0.91644	0.92300
45	0.84179	0.85396	0.86494	0.87494	0.88409	0.89254	0.90034	0.90760
50	0.80191	0.81654	0.82976	0.84176	0.85276	0.86289	0.87227	0.88099
55	0.73818	0.75655	0.77315	0.78825	0.80212	0.81490	0.82677	0.83782
60	0.65292	0.67578	0.69640	0.71519	0.73240	0.74827	0.76299	0.77668
65	0.55399	0.57993	0.60346	0.62488	0.64457	0.66275	0.67962	0.69530
70	0.42652	0.45541	0.48176	0.50566	0.52765	0.54792	0.56675	0.58422
75+[a]	0.27283	0.28802	0.30142	0.31354	0.32477	0.33540	0.34571	0.35576

Age x	9	10	11	12	13	14	15	16
0	0.91004	0.91977	0.92840	0.93721	0.94529	0.95308	0.96125	0.96817
5	0.96982	0.97289	0.97575	0.97845	0.98100	0.98369	0.98603	0.98822
10	0.96937	0.97240	0.97524	0.97791	0.98042	0.98280	0.98525	0.98744
15	0.96060	0.96435	0.96789	0.97128	0.97451	0.97691	0.97996	0.98285
20	0.95351	0.95769	0.96180	0.96567	0.96927	0.97214	0.97572	0.97908
25	0.94757	0.95226	0.95685	0.96117	0.96518	0.96846	0.97237	0.97605
30	0.94123	0.94659	0.95162	0.95636	0.96077	0.96451	0.96865	0.97258
35	0.93552	0.94117	0.94645	0.95144	0.95608	0.96011	0.96457	0.96875
40	0.92911	0.93484	0.94023	0.94528	0.94999	0.95406	0.95810	0.96209
45	0.91438	0.92075	0.92669	0.93230	0.93755	0.94201	0.94636	0.95071
50	0.88914	0.89676	0.90392	0.91067	0.91694	0.92250	0.92742	0.93239
55	0.84812	0.85780	0.86689	0.87547	0.88345	0.89019	0.89662	0.90317
60	0.78945	0.80143	0.81269	0.82330	0.83317	0.84131	0.84916	0.85715
65	0.70999	0.72375	0.73670	0.74891	0.76028	0.76937	0.77830	0.78747
70	0.60062	0.61594	0.63036	0.64397	0.65662	0.66673	0.67647	0.68656
75+[a]	0.36575	0.37567	0.38561	0.39559	0.40550	0.41439	0.42345	0.43300

Age x	17	18	19	20	21	22	23	24
0	0.97441	0.98003	0.98514	0.98981	0.99356	0.99605	0.99787	0.99905
5	0.99024	0.99213	0.99388	0.99554	0.99695	0.99800	0.99883	0.99943
10	0.98950	0.99143	0.99323	0.99491	0.99646	0.99764	0.99859	0.99928
15	0.98552	0.98802	0.99036	0.99254	0.99470	0.99648	0.99785	0.99888
20	0.98223	0.98518	0.98794	0.99053	0.99309	0.99538	0.99713	0.99846
25	0.97954	0.98282	0.98589	0.98880	0.99168	0.99425	0.99633	0.99797
30	0.97633	0.97990	0.98328	0.98649	0.98968	0.99253	0.99502	0.99709
35	0.97218	0.97592	0.97951	0.98293	0.98632	0.98960	0.99267	0.99537
40	0.96600	0.96981	0.97352	0.97710	0.98066	0.98449	0.98829	0.99191
45	0.95502	0.95928	0.96347	0.96755	0.97164	0.97639	0.98133	0.98630
50	0.93774	0.94289	0.94796	0.95294	0.95798	0.96412	0.97067	0.97752
55	0.90968	0.91624	0.92276	0.92920	0.93574	0.94402	0.95309	0.96283
60	0.86524	0.87338	0.88151	0.88959	0.89788	0.90885	0.92163	0.93481
65	0.79681	0.80629	0.81583	0.82537	0.83536	0.84926	0.86517	0.88360
70	0.69691	0.70746	0.71815	0.72889	0.74021	0.75670	0.77591	0.79890
75+[a]	0.44309	0.45357	0.46442	0.47556	0.48730	0.50255	0.51971	0.53919

[a] Value listed for age 75+ is $T(80)/T(75)$

TABLE 247. MALE FIVE-YEAR SURVIVORSHIP PROBABILITIES, ${}_5S_{x,x+5}$, NORTH MODEL

Age x	Five-year probability ${}_5S_{x,x+5}$ for mortality level:							
	1	2	3	4	5	6	7	8
0	0.72799	0.75499	0.77876	0.79990	0.81891	0.83613	0.85182	0.86621
5	0.90158	0.91034	0.91863	0.92589	0.93249	0.93853	0.94410	0.94924
10	0.93417	0.93979	0.94481	0.94938	0.95354	0.95736	0.96088	0.96415
15	0.92012	0.92661	0.93244	0.93773	0.94256	0.94700	0.95108	0.95489
20	0.90361	0.91147	0.91853	0.92492	0.93074	0.93611	0.94105	0.94563
25	0.89750	0.90587	0.91339	0.92023	0.92645	0.93217	0.93745	0.94235
30	0.88677	0.89604	0.90438	0.91193	0.91884	0.92518	0.93101	0.93647
35	0.86761	0.87849	0.88824	0.89707	0.90515	0.91257	0.91943	0.92579
40	0.84280	0.85560	0.86706	0.87745	0.88700	0.89574	0.90381	0.91130
45	0.81466	0.82920	0.84230	0.85415	0.86497	0.87493	0.88412	0.89264
50	0.77372	0.79078	0.80616	0.82008	0.83281	0.84453	0.85535	0.86541
55	0.71175	0.73261	0.75137	0.76842	0.78400	0.79833	0.81160	0.82393
60	0.62557	0.65080	0.67358	0.69432	0.71327	0.73073	0.74692	0.76194
65	0.49806	0.52936	0.55777	0.58365	0.60750	0.62948	0.64986	0.66885
70	0.33801	0.37613	0.41063	0.44216	0.47122	0.49802	0.52291	0.54611
75+[a]	0.20905	0.23539	0.25795	0.27703	0.29353	0.30827	0.32187	0.33465

Age x	9	10	11	12	13	14	15	16
0	0.87948	0.89175	0.90315	0.91376	0.92480	0.93470	0.94381	0.95239
5	0.95402	0.95847	0.96264	0.96654	0.97038	0.97392	0.97724	0.98036
10	0.96720	0.97005	0.97271	0.97521	0.97744	0.97958	0.98166	0.98360
15	0.95843	0.96174	0.96485	0.96776	0.97019	0.97264	0.97507	0.97744
20	0.94991	0.95390	0.95765	0.96115	0.96406	0.96704	0.96997	0.97284
25	0.94691	0.95116	0.95516	0.95891	0.96200	0.96515	0.96826	0.97130
30	0.94153	0.94626	0.95068	0.95484	0.95831	0.96174	0.96513	0.96846
35	0.93171	0.93725	0.94245	0.94732	0.95146	0.95539	0.95927	0.96311
40	0.91826	0.92480	0.93093	0.93667	0.94171	0.94617	0.95061	0.95502
45	0.90059	0.90801	0.91497	0.92156	0.92727	0.93213	0.93704	0.94195
50	0.87478	0.88355	0.89177	0.89980	0.90633	0.91193	0.91761	0.92334
55	0.83540	0.84615	0.85623	0.86620	0.87403	0.88078	0.88756	0.89461
60	0.77596	0.78909	0.80144	0.81349	0.82289	0.83102	0.83936	0.84789
65	0.68664	0.70329	0.71896	0.73429	0.74606	0.75634	0.76700	0.77776
70	0.56783	0.58818	0.60728	0.62615	0.64022	0.65266	0.66545	0.67862
75+[a]	0.34891	0.35884	0.37056	0.38264	0.39264	0.40199	0.41198	0.42261

Age x	17	18	19	20	21	22	23	24
0	0.96030	0.96756	0.97425	0.98044	0.98620	0.99164	0.99671	0.99710
5	0.98328	0.98603	0.98860	0.99101	0.99328	0.99538	0.99685	0.99810
10	0.98561	0.98747	0.98924	0.99093	0.99253	0.99406	0.99364	0.99698
15	0.97976	0.98200	0.98417	0.98625	0.98823	0.99016	0.99273	0.99475
20	0.97564	0.97835	0.98096	0.98347	0.98586	0.98818	0.99132	0.99375
25	0.97427	0.97714	0.97992	0.98258	0.98512	0.98759	0.99075	0.99333
30	0.97171	0.97487	0.97792	0.98085	0.98365	0.98639	0.98957	0.99243
35	0.96686	0.97052	0.97407	0.97748	0.98075	0.98392	0.98733	0.99070
40	0.95936	0.96382	0.96777	0.97178	0.97565	0.97912	0.98312	0.98730
45	0.94483	0.95165	0.95639	0.96102	0.96549	0.96930	0.97430	0.97973
50	0.92908	0.93479	0.94044	0.94599	0.95139	0.95592	0.96230	0.96941
55	0.90164	0.90867	0.91564	0.92251	0.92924	0.93492	0.94323	0.95269
60	0.85452	0.86520	0.87387	0.88246	0.89090	0.89830	0.90936	0.92230
65	0.78879	0.79995	0.81114	0.82228	0.83326	0.84309	0.85808	0.87594
70	0.69208	0.70574	0.71949	0.73320	0.74681	0.75922	0.77851	0.80190
75+[a]	0.45383	0.44559	0.45780	0.47035	0.48313	0.49526	0.51240	0.53242

[a] Value based for age 75+ is $T(80)/T(75)$.

TABLE 26B. MALE FIVE-YEAR SURVIVORSHIP PROBABILITIES $_5S_{x,x+5}$, SOUTH MODEL

Age x	Survivorship probability, $_5S_{x,x+5}$ for mortality level:							
	1	2	3	4	5	6	7	8
0	0.74480	0.77054	0.79314	0.81319	0.83137	0.84739	0.86216	0.87567
5	0.93998	0.94540	0.95028	0.95471	0.95876	0.96250	0.96595	0.96915
10	0.95242	0.95649	0.96015	0.96350	0.96656	0.96939	0.97202	0.97446
15	0.92878	0.93481	0.94027	0.94524	0.94981	0.95402	0.95793	0.96156
20	0.91397	0.92128	0.92790	0.93392	0.93945	0.94455	0.94926	0.95366
25	0.91454	0.92167	0.92809	0.93395	0.93933	0.94427	0.94885	0.95313
30	0.91145	0.91858	0.92499	0.93086	0.93623	0.94118	0.94578	0.95005
35	0.89969	0.90751	0.91459	0.92103	0.92695	0.93240	0.93745	0.94216
40	0.88330	0.89199	0.89984	0.90699	0.91354	0.91959	0.92520	0.93043
45	0.86009	0.86983	0.87865	0.88669	0.89405	0.90086	0.90716	0.91304
50	0.82286	0.83437	0.84480	0.85431	0.86305	0.87111	0.87858	0.88555
55	0.76024	0.77501	0.78838	0.80060	0.81185	0.82221	0.83183	0.84080
60	0.66690	0.68640	0.70406	0.72019	0.73502	0.74875	0.76151	0.77341
65	0.54325	0.56750	0.58956	0.60969	0.62826	0.64544	0.66145	0.67637
70	0.39159	0.41948	0.44475	0.46792	0.48921	0.50896	0.52727	0.54438
75+[a]	0.24354	0.26106	0.27599	0.28932	0.30131	0.31253	0.32304	0.33311

Age x	9	10	11	12	13	14	15	16
0	0.88810	0.89911	0.91062	0.92117	0.93083	0.93972	0.94791	0.95551
5	0.97215	0.97512	0.97785	0.98038	0.98274	0.98495	0.98701	0.98894
10	0.97674	0.97878	0.98085	0.98278	0.98462	0.98635	0.98799	0.98953
15	0.96497	0.96786	0.97092	0.97393	0.97673	0.97942	0.98192	0.98427
20	0.95776	0.96124	0.96493	0.96857	0.97196	0.97516	0.97817	0.98100
25	0.95710	0.96048	0.96396	0.96743	0.97070	0.97381	0.97674	0.97951
30	0.95406	0.95743	0.96071	0.96419	0.96751	0.97065	0.97365	0.97649
35	0.94657	0.95029	0.95383	0.95760	0.96121	0.96466	0.96796	0.97112
40	0.93510	0.93943	0.94334	0.94739	0.95131	0.95512	0.95877	0.96229
45	0.91851	0.92325	0.92766	0.93206	0.93637	0.94057	0.94465	0.94861
50	0.89207	0.89773	0.90289	0.90800	0.91302	0.91793	0.92275	0.92744
55	0.84922	0.85647	0.86307	0.86961	0.87606	0.88240	0.88865	0.89475
60	0.78453	0.79406	0.80279	0.81143	0.82000	0.82846	0.83680	0.84497
65	0.69036	0.70224	0.71303	0.72388	0.73467	0.74537	0.75594	0.76634
70	0.56044	0.57404	0.58619	0.59850	0.61077	0.62300	0.63513	0.64711
75+[a]	0.34286	0.35153	0.35975	0.36834	0.37721	0.38637	0.39579	0.40545

Age x	17	18	19	20	21	22	23	24
0	0.96255	0.96929	0.97568	0.98193	0.98648	0.99034	0.99350	0.99597
5	0.99075	0.99247	0.99406	0.99534	0.99646	0.99742	0.99822	0.99886
10	0.99100	0.99260	0.99381	0.99500	0.99600	0.99703	0.99787	0.99858
15	0.98648	0.98858	0.99100	0.99281	0.99442	0.99585	0.99706	0.99808
20	0.98366	0.98619	0.98907	0.99126	0.99321	0.99493	0.99640	0.99763
25	0.98213	0.98464	0.98749	0.98980	0.99188	0.99376	0.99544	0.99688
30	0.97920	0.98178	0.98496	0.98752	0.98988	0.99206	0.99404	0.99580
35	0.97413	0.97702	0.98059	0.98356	0.98639	0.98905	0.99154	0.99380
40	0.96566	0.96892	0.97291	0.97647	0.97994	0.98334	0.98661	0.98972
45	0.95243	0.95615	0.96061	0.96491	0.96922	0.97357	0.97791	0.98223
50	0.93202	0.93648	0.94172	0.94711	0.95265	0.95834	0.96419	0.97020
55	0.90073	0.90658	0.91344	0.92070	0.92825	0.93609	0.94428	0.95282
60	0.85296	0.86083	0.87032	0.88032	0.89078	0.90178	0.91336	0.92560
65	0.77656	0.78664	0.79924	0.81260	0.82669	0.84171	0.85779	0.87514
70	0.65893	0.67063	0.68533	0.70174	0.71906	0.73784	0.75841	0.78118
75+[a]	0.41529	0.42538	0.43932	0.45209	0.46693	0.48303	0.50054	0.51962

[a] Value listed for age 75+ is $T(80)/T(75)$.

TABLE 269. MALE FIVE-YEAR SURVIVORSHIP PROBABILITIES, ${}_5S_{x,2+}$, EAST MODEL

Age	Survivorship probability: ${}_5T_{x,2+}$ for mortality level:							
x	1	2	3	4	5	6	7	8
0	0.78453	0.80717	0.82685	0.84430	0.85969	0.87364	0.88630	0.89788
5	0.94851	0.95289	0.95687	0.96051	0.96387	0.96698	0.96987	0.97258
10	0.95982	0.96295	0.96581	0.96844	0.97085	0.97310	0.97520	0.97715
15	0.94076	0.94530	0.94943	0.95324	0.95676	0.96002	0.96306	0.96589
20	0.92848	0.93399	0.93902	0.94364	0.94791	0.95187	0.95556	0.95901
25	0.92330	0.92925	0.93467	0.93964	0.94424	0.94851	0.95248	0.95620
30	0.91007	0.91703	0.92339	0.92922	0.93461	0.93960	0.94428	0.94864
35	0.89056	0.89887	0.90649	0.91347	0.91992	0.92590	0.93149	0.93671
40	0.86849	0.87806	0.88676	0.89476	0.90216	0.90902	0.91540	0.92139
45	0.84312	0.85361	0.86313	0.87189	0.87996	0.88749	0.89448	0.90102
50	0.80825	0.81970	0.83015	0.83977	0.84862	0.85686	0.86454	0.87171
55	0.75471	0.76789	0.78000	0.79110	0.80137	0.81091	0.81980	0.82815
60	0.67179	0.68800	0.70285	0.71649	0.72913	0.74089	0.75186	0.76216
65	0.55932	0.57896	0.59693	0.61348	0.62878	0.64300	0.65630	0.66877
70	0.42603	0.44796	0.46815	0.48666	0.50378	0.51976	0.53471	0.54871
75+[a]	0.26696	0.27948	0.29070	0.30090	0.31034	0.31932	0.32795	0.33629

Age	9	10	11	12	13	14	15	16
0	0.90852	0.91835	0.92740	0.93728	0.94541	0.95288	0.95976	0.96612
5	0.97511	0.97748	0.97985	0.98195	0.98394	0.98582	0.98760	0.98931
10	0.97899	0.98072	0.98227	0.98367	0.98511	0.98649	0.98783	0.98913
15	0.96857	0.97109	0.97328	0.97538	0.97745	0.97945	0.98140	0.98329
20	0.96224	0.96530	0.96800	0.97064	0.97302	0.97543	0.97778	0.98006
25	0.95968	0.96297	0.96592	0.96863	0.97126	0.97387	0.97638	0.97883
30	0.95273	0.95659	0.96012	0.96328	0.96637	0.96939	0.97232	0.97518
35	0.94161	0.94624	0.95050	0.95421	0.95786	0.96143	0.96493	0.96834
40	0.92702	0.93230	0.93716	0.94127	0.94533	0.94934	0.95337	0.95728
45	0.90718	0.91296	0.91822	0.92254	0.92688	0.93122	0.93553	0.93983
50	0.87846	0.88481	0.89055	0.89514	0.89978	0.90447	0.90918	0.91389
55	0.83596	0.84334	0.84992	0.85515	0.86046	0.86586	0.87132	0.87682
60	0.77181	0.78093	0.78890	0.79522	0.80187	0.80854	0.81530	0.82214
65	0.68049	0.69154	0.70102	0.70874	0.71664	0.72472	0.73293	0.74128
70	0.56188	0.57430	0.58486	0.59344	0.60226	0.61130	0.62054	0.62993
75+[a]	0.34443	0.35248	0.35989	0.36634	0.37316	0.38035	0.38792	0.39584

Age	17	18	19	20	21	22	23	24
0	0.97201	0.97780	0.98343	0.98856	0.99205	0.99491	0.99704	0.99851
5	0.99091	0.99243	0.99386	0.99523	0.99628	0.99730	0.99818	0.99889
10	0.99038	0.99158	0.99273	0.99383	0.99498	0.99608	0.99711	0.99804
15	0.98512	0.98689	0.98858	0.99020	0.99139	0.99268	0.99387	0.99674
20	0.98226	0.98438	0.98642	0.98837	0.99045	0.99246	0.99438	0.99613
25	0.98119	0.98346	0.98565	0.98775	0.98985	0.99200	0.99404	0.99590
30	0.97794	0.98061	0.98318	0.98563	0.98802	0.99053	0.99297	0.99511
35	0.97161	0.97486	0.97796	0.98094	0.98381	0.98699	0.99006	0.99294
40	0.96111	0.96484	0.96846	0.97197	0.97552	0.97962	0.98374	0.98779
45	0.94407	0.94824	0.95232	0.95629	0.96057	0.96577	0.97123	0.97693
50	0.91861	0.92326	0.92786	0.93236	0.93744	0.94390	0.95094	0.95863
55	0.88233	0.88782	0.89326	0.89861	0.90486	0.91299	0.92205	0.93224
60	0.82901	0.83589	0.84271	0.84945	0.85756	0.86816	0.88015	0.89388
65	0.74963	0.75804	0.76642	0.77471	0.78501	0.79851	0.81398	0.83197
70	0.63942	0.64898	0.65852	0.66801	0.68010	0.69606	0.71469	0.73674
75+[a]	0.40410	0.41266	0.42146	0.43048	0.44135	0.45533	0.47144	0.49020

[a] Value listed for age 75+ is $T(80)/T(75)$.

TABLE 270. MALE FIVE-YEAR SURVIVORSHIP PROBABILITIES, $_5S_{x,x+5}$, WEST MODEL

Age x	Survivorship probability, $_5S_{x,x+5}$ (for mortality level)							
	1	2	3	4	5	6	7	8
0	0.78598	0.80839	0.82791	0.84514	0.86050	0.87431	0.88685	0.89827
5	0.94170	0.94685	0.95148	0.95568	0.95953	0.96306	0.96633	0.96937
10	0.94345	0.94829	0.95266	0.95662	0.96024	0.96359	0.96667	0.96956
15	0.92179	0.92836	0.93432	0.93971	0.94466	0.94921	0.95343	0.95735
20	0.90238	0.91067	0.91815	0.92495	0.93117	0.93690	0.94220	0.94712
25	0.88887	0.89838	0.90695	0.91475	0.92187	0.92844	0.93452	0.94017
30	0.87123	0.88219	0.89209	0.90108	0.90932	0.91690	0.92391	0.93044
35	0.84849	0.86118	0.87261	0.88302	0.89255	0.90132	0.90944	0.91699
40	0.82417	0.83840	0.85122	0.86287	0.87345	0.88336	0.89246	0.90090
45	0.79284	0.80869	0.82297	0.83595	0.84784	0.85880	0.86894	0.87838
50	0.75188	0.76954	0.78552	0.80001	0.81329	0.82552	0.83684	0.84736
55	0.69526	0.71503	0.73291	0.74918	0.76410	0.77785	0.79061	0.80246
60	0.61357	0.63658	0.65732	0.67622	0.69354	0.70951	0.72431	0.73809
65	0.51773	0.54303	0.56594	0.58676	0.60585	0.62347	0.63983	0.65504
70	0.39621	0.42347	0.44803	0.47062	0.49123	0.51025	0.52797	0.54445
75+[a]	0.25310	0.26885	0.28214	0.29420	0.30500	0.31498	0.32409	0.33264

Age x	9	10	11	12	13	14	15	16
0	0.90877	0.91845	0.92741	0.93573	0.94694	0.95280	0.95842	0.96361
5	0.97219	0.97484	0.97732	0.97966	0.98222	0.98435	0.98620	0.98799
10	0.97223	0.97474	0.97710	0.97932	0.98161	0.98360	0.98535	0.98706
15	0.96100	0.96442	0.96765	0.97067	0.97353	0.97626	0.97869	0.98107
20	0.95172	0.95602	0.96006	0.96386	0.96748	0.97083	0.97392	0.97692
25	0.94544	0.95038	0.95501	0.95938	0.96343	0.96738	0.97094	0.97438
30	0.93652	0.94222	0.94758	0.95261	0.95730	0.96183	0.96591	0.96987
35	0.92403	0.93064	0.93684	0.94267	0.94815	0.95328	0.95793	0.96246
40	0.90878	0.91616	0.92309	0.92962	0.93584	0.94140	0.94643	0.95141
45	0.88719	0.89544	0.90319	0.91048	0.91761	0.92358	0.92903	0.93452
50	0.85718	0.86638	0.87503	0.88313	0.89119	0.89760	0.90354	0.90956
55	0.81355	0.82394	0.83370	0.84289	0.85193	0.85904	0.86569	0.87248
60	0.75097	0.76301	0.77434	0.78503	0.79540	0.80353	0.81117	0.81902
65	0.66924	0.68256	0.69510	0.70691	0.71827	0.72714	0.73552	0.74417
70	0.55983	0.57427	0.58786	0.60069	0.61291	0.62244	0.63147	0.64085
75+[a]	0.34255	0.35131	0.36002	0.36872	0.37769	0.38544	0.39303	0.40117

Age x	17	18	19	20	21	22	23	24
0	0.97174	0.97737	0.98257	0.98738	0.99139	0.99433	0.99645	0.99831
5	0.98969	0.99132	0.99289	0.99437	0.99570	0.99688	0.99794	0.99880
10	0.98872	0.99031	0.99184	0.99331	0.99467	0.99598	0.99720	0.99825
15	0.98334	0.98561	0.98775	0.98982	0.99181	0.99375	0.99557	0.99720
20	0.97982	0.98262	0.98531	0.98787	0.99033	0.99269	0.99490	0.99682
25	0.97770	0.98089	0.98395	0.98687	0.98958	0.99220	0.99462	0.99670
30	0.97370	0.97739	0.98093	0.98432	0.98743	0.99053	0.99340	0.99590
35	0.96688	0.97117	0.97531	0.97929	0.98292	0.98617	0.99046	0.99379
40	0.95632	0.96114	0.96582	0.97038	0.97458	0.97941	0.98427	0.98896
45	0.93998	0.94541	0.95075	0.95599	0.96090	0.96700	0.97344	0.98004
50	0.91563	0.92170	0.92774	0.93371	0.93948	0.94703	0.95520	0.96425
55	0.87938	0.88634	0.89332	0.90025	0.90724	0.91664	0.92728	0.93927
60	0.82702	0.83514	0.84333	0.85150	0.85990	0.87150	0.88493	0.90051
65	0.75305	0.76211	0.77127	0.78047	0.79012	0.80383	0.82012	0.83932
70	0.65050	0.66039	0.67044	0.68059	0.69132	0.70703	0.72545	0.74682
75+[a]	0.40982	0.41897	0.42857	0.43857	0.44913	0.46356	0.48056	0.50071

[a] Value listed for age 75+ is $T(80)/T(75)$.

TABLE 271. FEMALE 10-YEAR SURVIVORSHIP PROBABILITIES, $_{10}S_{x,x+10}$, NORTH MODEL

Age x	Survivorship probability $_{10}S_{x,x+10}$ for mortality level							
	1	2	3	4	5	6	7	8
0	0.66645	0.69855	0.72160	0.74815	0.77057	0.79119	0.81023	0.82791
5	0.83974	0.85377	0.86642	0.87794	0.88850	0.89823	0.90724	0.91561
10	0.86039	0.87672	0.88695	0.89625	0.90482	0.91270	0.92002	0.92682
15	0.85134	0.86374	0.87476	0.88481	0.89403	0.90254	0.91043	0.91779
20	0.82936	0.84324	0.85583	0.86732	0.87788	0.88764	0.89668	0.90512
25	0.80311	0.81910	0.83361	0.84683	0.85907	0.87034	0.88083	0.89061
30	0.77987	0.79752	0.81356	0.82823	0.84172	0.85420	0.86583	0.87668
35	0.76404	0.78248	0.79901	0.81452	0.82862	0.84167	0.85383	0.86516
40	0.74520	0.76428	0.78164	0.79753	0.81215	0.82573	0.83836	0.85016
45	0.70145	0.72270	0.74208	0.75983	0.77629	0.79158	0.80580	0.81913
50	0.61890	0.64450	0.66797	0.68963	0.70971	0.72847	0.74602	0.76249
55	0.50025	0.53067	0.55888	0.58514	0.60967	0.63269	0.65439	0.67484
60	0.35490	0.38840	0.41997	0.44975	0.47788	0.50459	0.52992	0.55405
65	0.20832	0.24067	0.27183	0.30182	0.33065	0.35838	0.38504	0.41072
70+[a]	0.08398	0.09895	0.11260	0.12535	0.13735	0.14879	0.15991	0.17081

Age x	9	10	11	12	13	14	15	16
0	0.84440	0.85981	0.87426	0.88783	0.90068	0.91278	0.92840	0.93983
5	0.92343	0.93075	0.93763	0.94410	0.95046	0.95676	0.96259	0.96799
10	0.93318	0.93915	0.94475	0.95002	0.95470	0.95965	0.96433	0.96879
15	0.92466	0.93112	0.93719	0.94290	0.94776	0.95300	0.95805	0.96289
20	0.91301	0.92042	0.92739	0.93395	0.93956	0.94551	0.95123	0.95676
25	0.89977	0.90837	0.91646	0.92410	0.93072	0.93752	0.94410	0.95046
30	0.88684	0.89639	0.90538	0.91386	0.92122	0.92859	0.93579	0.94277
35	0.87580	0.88578	0.89521	0.90407	0.91165	0.91907	0.92639	0.93357
40	0.86121	0.87162	0.88143	0.89068	0.89849	0.90586	0.91324	0.92057
45	0.83163	0.84342	0.85454	0.86505	0.87399	0.88205	0.89020	0.89839
50	0.77801	0.79265	0.80654	0.81972	0.83102	0.84086	0.85088	0.86102
55	0.69423	0.71258	0.73005	0.74679	0.76094	0.77321	0.78579	0.79861
60	0.57705	0.59903	0.62000	0.64037	0.65713	0.67182	0.68700	0.70265
65	0.43542	0.45922	0.48212	0.50485	0.52289	0.53887	0.55555	0.57292
70+[a]	0.18157	0.19229	0.20304	0.21417	0.22360	0.23258	0.24221	0.25246

Age x	17	18	19	20	21	22	23	24
0	0.95023	0.95972	0.96841	0.97644	0.98390	0.99104	0.99468	0.99725
5	0.97299	0.97761	0.98193	0.98594	0.98971	0.99333	0.99583	0.99761
10	0.97300	0.97697	0.98073	0.98425	0.98757	0.99077	0.99405	0.99626
15	0.96751	0.97192	0.97610	0.98005	0.98380	0.98740	0.99169	0.99457
20	0.96207	0.96713	0.97195	0.97653	0.98086	0.98505	0.98996	0.99334
25	0.95657	0.96241	0.96798	0.97328	0.97830	0.98317	0.98825	0.99221
30	0.94951	0.95600	0.96222	0.96816	0.97381	0.97930	0.98490	0.98967
35	0.94059	0.94741	0.95401	0.96036	0.96645	0.97235	0.97871	0.98465
40	0.92783	0.93496	0.94194	0.94871	0.95525	0.96161	0.96884	0.97625
45	0.90657	0.91470	0.92271	0.93055	0.93818	0.94564	0.95427	0.96385
50	0.87123	0.88144	0.89157	0.90154	0.91129	0.92078	0.93186	0.94486
55	0.81162	0.82473	0.83782	0.85080	0.86357	0.87597	0.89119	0.90930
60	0.71863	0.73490	0.75128	0.76766	0.78390	0.79892	0.82055	0.84802
65	0.59085	0.60924	0.62796	0.64685	0.66576	0.68351	0.71027	0.74282
70+[a]	0.26332	0.27473	0.28663	0.29892	0.31150	0.32381	0.34077	0.36064

[a] Value listed for age 70+ is $T(80)/T(70)$.

TABLE 272 FEMALE 10-YEAR SURVIVORSHIP PROBABILITIES $_{10}S_{x,x+10}$, SOUTH MODEL

Age x	Survivorship probability, $_{10}S_{x,x+10}$, for mortality level							
	1	2	3	4	5	6	7	8
0	0.67922	0.71020	0.73782	0.76267	0.78521	0.80581	0.82475	0.84224
5	0.87707	0.88811	0.89813	0.90730	0.91574	0.92356	0.93082	0.93759
10	0.87389	0.88486	0.89480	0.90393	0.91234	0.92014	0.92740	0.93418
15	0.84814	0.86110	0.87290	0.88373	0.89371	0.90295	0.91159	0.91965
20	0.83063	0.84764	0.86044	0.87215	0.88299	0.89301	0.90237	0.91112
25	0.82555	0.84011	0.85336	0.86551	0.87675	0.88727	0.89689	0.90598
30	0.81925	0.83403	0.84746	0.85979	0.87118	0.88175	0.89161	0.90084
35	0.81400	0.82860	0.84193	0.85415	0.86541	0.87587	0.88561	0.89475
40	0.79650	0.81166	0.82551	0.83821	0.84995	0.86087	0.87102	0.88054
45	0.74960	0.76732	0.78352	0.79842	0.81224	0.82510	0.83711	0.84837
50	0.65854	0.68134	0.70230	0.72169	0.73973	0.75660	0.77244	0.78735
55	0.51876	0.54799	0.57512	0.60046	0.62425	0.64664	0.66780	0.68784
60	0.35379	0.38636	0.41757	0.44696	0.47494	0.50154	0.52691	0.55116
65	0.19539	0.22180	0.25331	0.28189	0.30958	0.33653	0.36219	0.38720
70+[a]	0.07087	0.08554	0.09902	0.11161	0.12355	0.13502	0.14610	0.15696

Age x	9	10	11	12	13	14	15	16
0	0.85846	0.87429	0.88912	0.90273	0.91528	0.92688	0.93764	0.94764
5	0.94394	0.95004	0.95604	0.96157	0.96668	0.97142	0.97583	0.97992
10	0.94053	0.94652	0.95234	0.95791	0.96318	0.96807	0.97265	0.97691
15	0.92722	0.93383	0.94107	0.94782	0.95410	0.95997	0.96547	0.97062
20	0.91935	0.92660	0.93431	0.94154	0.94832	0.95468	0.96066	0.96627
25	0.91451	0.92218	0.92998	0.93733	0.94428	0.95084	0.95704	0.96288
30	0.90949	0.91732	0.92503	0.93235	0.93931	0.94593	0.95222	0.95820
35	0.90333	0.91107	0.91847	0.92551	0.93239	0.93892	0.94517	0.95117
40	0.88949	0.89774	0.90519	0.91243	0.91944	0.92623	0.93279	0.93912
45	0.85896	0.86896	0.87757	0.88600	0.89424	0.90227	0.91007	0.91765
50	0.80141	0.81484	0.82618	0.83736	0.84832	0.85906	0.86958	0.87982
55	0.70685	0.72510	0.74043	0.75563	0.77066	0.78548	0.80006	0.81434
60	0.57436	0.59642	0.61519	0.63398	0.65276	0.67145	0.69000	0.70834
65	0.41139	0.43418	0.45379	0.47364	0.49373	0.51396	0.53427	0.55460
70+[a]	0.16769	0.17842	0.18749	0.19730	0.20758	0.21828	0.22940	0.24091

Age x	17	18	19	20	21	22	23	24
0	0.95723	0.96631	0.97349	0.97975	0.98509	0.98953	0.99308	0.99580
5	0.98376	0.98735	0.99016	0.99256	0.99459	0.99626	0.99758	0.99858
10	0.98092	0.98470	0.98822	0.99096	0.99332	0.99528	0.99684	0.99811
15	0.97546	0.98004	0.98454	0.98799	0.99100	0.99355	0.99566	0.99731
20	0.97157	0.97661	0.98149	0.98543	0.98891	0.99192	0.99444	0.99646
25	0.96841	0.97370	0.97858	0.98287	0.98673	0.99013	0.99303	0.99544
30	0.96368	0.96932	0.97431	0.97901	0.98332	0.98722	0.99066	0.99362
35	0.95690	0.96241	0.96765	0.97242	0.97771	0.98228	0.98648	0.99026
40	0.94520	0.95102	0.95676	0.96273	0.96853	0.97413	0.97948	0.98450
45	0.92498	0.93189	0.93891	0.94647	0.95397	0.96136	0.96859	0.97558
50	0.88978	0.89910	0.90877	0.91934	0.92994	0.94052	0.95101	0.96132
55	0.82832	0.84139	0.85530	0.87069	0.88631	0.90213	0.91807	0.93404
60	0.72643	0.74380	0.76271	0.78401	0.80609	0.82901	0.85277	0.87733
65	0.57483	0.59485	0.61734	0.64329	0.67098	0.70065	0.73257	0.76704
70+[a]	0.25277	0.26485	0.27854	0.29424	0.31113	0.32927	0.34871	0.36951

[a] Value listed for age 70 + is $T(80)/T(70)$.

TABLE 27J. FEMALE 10-YEAR SURVIVORSHIP PROBABILITIES, ${}_{10}p_{x,\,x+10}$, EAST MODEL

$\frac{q_0}{x}$	*Survivorship probability, ${}_{10}p_{x,\,x+10}$, for mortality level*							
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
0	0.74959	0.77307	0.79407	0.81305	0.83034	0.84619	0.86081	0.87437
5	0.89810	0.90669	0.91449	0.92166	0.92828	0.93447	0.94024	0.94565
10	0.89344	0.90209	0.91001	0.91731	0.92407	0.93039	0.93628	0.94181
15	0.86597	0.87610	0.88552	0.89561	0.90405	0.91191	0.91927	0.92617
20	0.84280	0.85528	0.86674	0.87734	0.88719	0.89637	0.90497	0.91306
25	0.82567	0.83933	0.85191	0.86353	0.87434	0.88443	0.89387	0.90278
30	0.81312	0.82744	0.84061	0.85279	0.86412	0.87468	0.88460	0.89393
35	0.80294	0.81784	0.83175	0.84508	0.85753	0.86924	0.87528	0.88471
40	0.78132	0.79645	0.81034	0.82322	0.83521	0.84641	0.85692	0.86681
45	0.72651	0.74427	0.76061	0.77582	0.79002	0.80330	0.81579	0.82758
50	0.62906	0.65151	0.67233	0.69176	0.70999	0.72717	0.74335	0.75867
55	0.49534	0.52262	0.54819	0.57224	0.59498	0.61654	0.63700	0.65647
60	0.34490	0.37429	0.40219	0.42883	0.45422	0.47850	0.50176	0.52405
65	0.20059	0.23026	0.25815	0.28424	0.30847	0.33091	0.35161	0.37058
70+[a]	0.08239	0.09446	0.10576	0.11641	0.12652	0.13627	0.14578	0.15511

	9	*10*	*11*	*12*	*13*	*14*	*15*	*16*
0	0.88698	0.89877	0.91072	0.92197	0.93225	0.94170	0.95038	0.95842
5	0.95074	0.95553	0.96003	0.96464	0.96894	0.97295	0.97669	0.98020
10	0.94702	0.95194	0.95614	0.96085	0.96529	0.96946	0.97340	0.97709
15	0.93270	0.93885	0.94404	0.94989	0.95543	0.96066	0.96561	0.97027
20	0.92068	0.92788	0.93411	0.94084	0.94724	0.95331	0.95907	0.96453
25	0.91115	0.91908	0.92611	0.93331	0.94019	0.94676	0.95304	0.95903
30	0.90271	0.91104	0.91853	0.92578	0.93280	0.93957	0.94609	0.95238
35	0.89362	0.90203	0.90959	0.91661	0.92349	0.93023	0.93678	0.94316
40	0.87613	0.88496	0.89280	0.89987	0.90690	0.91386	0.92072	0.92746
45	0.83572	0.84928	0.85864	0.86690	0.87517	0.88343	0.89165	0.89980
50	0.77321	0.78704	0.79936	0.81006	0.82084	0.83167	0.84251	0.85333
55	0.67503	0.69277	0.70857	0.72220	0.73602	0.75002	0.76417	0.77429
60	0.54544	0.56602	0.58421	0.59985	0.61587	0.63223	0.64887	0.66571
65	0.39482	0.41341	0.43355	0.44899	0.46496	0.48146	0.49844	0.51582
70+[b]	0.16435	0.17357	0.18209	0.18974	0.19789	0.20648	0.21578	0.22552

	17	*18*	*19*	*20*	*21*	*22*	*23*	*24*
0	0.96617	0.97378	0.98057	0.98683	0.99114	0.99440	0.99681	0.99842
5	0.98349	0.98657	0.98948	0.99222	0.99452	0.99629	0.99769	0.99875
10	0.98059	0.98388	0.98699	0.98992	0.99286	0.99501	0.99679	0.99818
15	0.97469	0.97886	0.98280	0.98653	0.99034	0.99317	0.99555	0.99741
20	0.96971	0.97462	0.97926	0.98368	0.98797	0.99141	0.99433	0.99665
25	0.96474	0.97018	0.97535	0.98027	0.98490	0.98896	0.99250	0.99540
30	0.95862	0.96421	0.96975	0.97504	0.98005	0.98485	0.98922	0.99291
35	0.94936	0.95535	0.96112	0.96663	0.97215	0.97786	0.98330	0.98832
40	0.93409	0.94054	0.94681	0.95287	0.95924	0.96629	0.97351	0.98018
45	0.90785	0.91574	0.92346	0.93098	0.93918	0.94852	0.95812	0.96783
50	0.86405	0.87464	0.88504	0.89522	0.90644	0.91965	0.93344	0.94767
55	0.79245	0.80652	0.82043	0.83411	0.84961	0.86831	0.88818	0.90950
60	0.68276	0.69982	0.71683	0.73368	0.75337	0.77831	0.80357	0.83567
65	0.53354	0.55148	0.56956	0.58762	0.60985	0.63853	0.67131	0.70915
70+[a]	0.23576	0.24644	0.25753	0.26896	0.28273	0.30008	0.31980	0.34230

[a] Value listed for age 70+ is $T(80)/T(70)$.

TABLE 27A. FEMALE 10-YEAR SURVIVORSHIP PROBABILITIES $_{10}S_{x,x+n}$, WEST MODEL

Age x	Survivorship probability $_{10}S_{x,x+n}$ for mortality level:							
	1	2	3	4	5	6	7	8
0	0.73701	0.76223	0.78468	0.80487	0.82317	0.83989	0.85524	0.86943
5	0.87354	0.88458	0.89459	0.90373	0.91213	0.91989	0.92710	0.93383
10	0.85757	0.86974	0.88079	0.89087	0.90016	0.90875	0.91674	0.92419
15	0.82858	0.84293	0.85595	0.86792	0.87892	0.88911	0.89860	0.90745
20	0.80498	0.82108	0.83574	0.84918	0.86158	0.87307	0.88377	0.89377
25	0.78413	0.80173	0.81775	0.83247	0.84606	0.85865	0.87039	0.88138
30	0.76600	0.78472	0.80177	0.81744	0.83189	0.84530	0.85782	0.86953
35	0.75174	0.77092	0.78842	0.80448	0.81930	0.83306	0.84590	0.85792
40	0.72673	0.74670	0.76493	0.78172	0.79723	0.81166	0.82531	0.83772
45	0.67504	0.69729	0.71769	0.73648	0.75392	0.77016	0.78534	0.79959
50	0.59196	0.61775	0.64152	0.66351	0.68401	0.70317	0.72117	0.73811
55	0.48197	0.51126	0.53842	0.56375	0.58747	0.60977	0.63082	0.65071
60	0.36171	0.39190	0.42025	0.44691	0.47209	0.49591	0.51854	0.54002
65	0.23628	0.26411	0.29072	0.31598	0.34011	0.36313	0.38517	0.40621
70+[a]	0.10087	0.11236	0.12402	0.13299	0.14248	0.15156	0.16046	0.16919

Age x	9	10	11	12	13	14	15	16
0	0.88258	0.89483	0.90626	0.91701	0.92732	0.93824	0.94783	0.95677
5	0.94012	0.94603	0.95159	0.95684	0.96180	0.96683	0.97147	0.97581
10	0.93117	0.93773	0.94392	0.94976	0.95523	0.96016	0.96550	0.97050
15	0.91575	0.92355	0.93091	0.93787	0.94436	0.94969	0.95618	0.96229
20	0.90314	0.91197	0.92029	0.92817	0.93552	0.94148	0.94875	0.95564
25	0.89169	0.90140	0.91056	0.91923	0.92731	0.93410	0.94188	0.94929
30	0.88054	0.89090	0.90066	0.90992	0.91857	0.92604	0.93403	0.94174
35	0.86920	0.87984	0.88988	0.89934	0.90826	0.91600	0.92386	0.93158
40	0.84957	0.86074	0.87130	0.88128	0.89064	0.89873	0.90671	0.91466
45	0.81301	0.82567	0.83766	0.84902	0.85966	0.86882	0.87767	0.88662
50	0.75410	0.76924	0.78360	0.79727	0.81007	0.82102	0.83154	0.84224
55	0.66955	0.68747	0.70451	0.72077	0.73606	0.74893	0.76137	0.77411
60	0.56051	0.58004	0.59871	0.61658	0.63345	0.64728	0.66090	0.67498
65	0.42644	0.44579	0.46439	0.48228	0.49922	0.51295	0.52649	0.54085
70+[a]	0.17790	0.18656	0.19528	0.20406	0.21281	0.22061	0.22861	0.23717

Age x	17	18	19	20	21	22	23	24
0	0.96490	0.97232	0.97911	0.98539	0.99065	0.99406	0.99671	0.99848
5	0.97985	0.98362	0.98716	0.99047	0.99342	0.99565	0.99742	0.99871
10	0.97518	0.97955	0.98366	0.98749	0.99119	0.99412	0.99644	0.99816
15	0.96802	0.97338	0.97842	0.98314	0.98783	0.99188	0.99499	0.99734
20	0.96213	0.96825	0.97400	0.97943	0.98483	0.98966	0.99347	0.99643
25	0.95635	0.96306	0.96941	0.97544	0.98145	0.98682	0.99137	0.99507
30	0.94917	0.95630	0.96313	0.96965	0.97615	0.98220	0.98773	0.99248
35	0.93912	0.94645	0.95357	0.96043	0.96725	0.97425	0.98105	0.98732
40	0.92255	0.93052	0.93786	0.94539	0.95285	0.96125	0.96983	0.97832
45	0.89537	0.90449	0.91333	0.92201	0.93082	0.94136	0.95255	0.96413
50	0.85305	0.86391	0.87474	0.88547	0.89642	0.91015	0.92514	0.94119
55	0.78709	0.80023	0.81343	0.82660	0.84018	0.85797	0.87792	0.90007
60	0.68943	0.70420	0.71917	0.73423	0.75006	0.77183	0.79604	0.82600
65	0.55531	0.57042	0.58589	0.60160	0.61835	0.64264	0.67130	0.70391
70+[a]	0.24628	0.25591	0.26603	0.27657	0.28790	0.30322	0.32100	0.34195

[a] Value listed for age 70+ is $T(80)/T(70)$.

TABLE 275. MALE 10-YEAR SURVIVORSHIP PROBABILITIES $_{10}S_{x,x+10}$, NORTH MODEL

Age x	Survivorship probability, $_{10}P_{x,x+10}$, for mortality level:							
	1	2	3	4	5	6	7	8
0	0.65634	0.68748	0.71539	0.74062	0.76363	0.78473	0.80420	0.82224
5	0.84223	0.85375	0.86793	0.87902	0.88917	0.89850	0.90717	0.91521
10	0.85955	0.87082	0.88098	0.89026	0.89877	0.90661	0.91388	0.92066
15	0.83144	0.84458	0.85648	0.86732	0.87728	0.88649	0.89502	0.90298
20	0.81099	0.82567	0.83898	0.85113	0.86229	0.87261	0.88219	0.89112
25	0.79587	0.81169	0.82606	0.83918	0.85126	0.86242	0.87279	0.88249
30	0.76937	0.78715	0.80330	0.81806	0.83168	0.84429	0.85602	0.86698
35	0.73122	0.75163	0.77015	0.78716	0.80286	0.81742	0.83099	0.84367
40	0.68660	0.70947	0.73032	0.74950	0.76723	0.78370	0.79908	0.81347
45	0.63032	0.65572	0.67902	0.70048	0.72036	0.73890	0.75624	0.77251
50	0.55070	0.57934	0.60572	0.63017	0.65292	0.67421	0.69420	0.71304
55	0.44525	0.47678	0.50611	0.53353	0.55920	0.58337	0.60620	0.62778
60	0.31157	0.34451	0.37570	0.40524	0.43331	0.45998	0.48539	0.50962
65	0.16853	0.19911	0.22903	0.25806	0.28626	0.31350	0.33982	0.36526
70+[a]	0.06188	0.07761	0.09389	0.10513	0.11745	0.12904	0.14014	0.15086

Age x	9	10	11	12	13	14	15	16
0	0.83904	0.85472	0.86941	0.88319	0.89741	0.91033	0.92232	0.93368
5	0.92273	0.92977	0.93636	0.94258	0.94849	0.95403	0.95932	0.96436
10	0.92700	0.93294	0.93852	0.94376	0.94830	0.95278	0.95719	0.96149
15	0.91042	0.91741	0.92399	0.93016	0.93532	0.94058	0.94578	0.95090
20	0.89947	0.90732	0.91471	0.92166	0.92742	0.93334	0.93918	0.94492
25	0.89154	0.90005	0.90805	0.91561	0.92189	0.92823	0.93449	0.94066
30	0.87724	0.88688	0.89597	0.90454	0.91180	0.91884	0.92582	0.93273
35	0.85958	0.86978	0.87735	0.88733	0.89400	0.90396	0.91190	0.91978
40	0.82699	0.83976	0.85178	0.86320	0.87321	0.88196	0.89076	0.89957
45	0.78781	0.80228	0.81594	0.82922	0.84041	0.85004	0.85984	0.86974
50	0.73080	0.74762	0.76336	0.77941	0.79216	0.80321	0.81444	0.82603
55	0.64824	0.66749	0.68621	0.70465	0.71923	0.73195	0.74498	0.75853
60	0.53280	0.55497	0.57620	0.59734	0.61392	0.62853	0.64379	0.65965
65	0.38989	0.41366	0.43661	0.45977	0.47769	0.49363	0.51041	0.52781
70+[a]	0.16134	0.17169	0.18196	0.19267	0.20150	0.20978	0.21872	0.22834

Age x	17	18	19	20	21	22	23	24
0	0.94424	0.95404	0.96315	0.97163	0.97957	0.98706	0.99158	0.99521
5	0.96913	0.97367	0.97796	0.98202	0.98586	0.98947	0.99290	0.99509
10	0.96566	0.96970	0.97358	0.97730	0.98085	0.98428	0.98840	0.99175
15	0.95589	0.96075	0.96543	0.96994	0.97425	0.97846	0.98411	0.98853
20	0.95054	0.95599	0.96126	0.96634	0.97118	0.97591	0.98215	0.98712
25	0.94671	0.95259	0.95828	0.96376	0.96901	0.97414	0.98042	0.98581
30	0.93951	0.94613	0.95255	0.95876	0.96472	0.97052	0.97709	0.98321
35	0.92757	0.93522	0.94267	0.94989	0.95687	0.96337	0.97067	0.97813
40	0.90835	0.91704	0.92557	0.93390	0.94198	0.94986	0.95786	0.96729
45	0.87968	0.88960	0.89943	0.90911	0.91856	0.92658	0.93757	0.94976
50	0.83770	0.84942	0.86111	0.87268	0.88407	0.89372	0.90767	0.92356
55	0.77227	0.78618	0.80015	0.81408	0.82785	0.83984	0.85774	0.87867
60	0.67561	0.69212	0.70883	0.72563	0.74235	0.75734	0.78031	0.80788
65	0.54591	0.56455	0.58341	0.60290	0.62228	0.64009	0.66803	0.70242
70+[a]	0.23462	0.24955	0.26506	0.27308	0.28552	0.29749	0.31507	0.33632

[a] Value listed for age 70+ = $T(80)/T(70)$.

TABLE 27B. MALE 10-YEAR SURVIVORSHIP PROBABILITIES, ${}_{10}S_{x,x+t}$, SOUTH MODEL

Age x	Survivorship probability, ${}_{10}S_{x,x+t}$, for mortality level:							
	1	2	3	4	5	6	7	8
0	0.70010	0.72846	0.75370	0.77636	0.79689	0.81561	0.83280	0.84864
5	0.89526	0.90426	0.91241	0.91986	0.92670	0.93303	0.93897	0.94460
10	0.88458	0.89414	0.90280	0.91073	0.91805	0.92481	0.93113	0.93700
15	0.84887	0.86123	0.87248	0.88278	0.89230	0.90111	0.90933	0.91700
20	0.83585	0.84911	0.86117	0.87224	0.88244	0.89191	0.90071	0.90896
25	0.83356	0.84663	0.85848	0.86937	0.87941	0.88873	0.89741	0.90552
30	0.82002	0.83363	0.84599	0.85734	0.86783	0.87755	0.88663	0.89510
35	0.79470	0.80949	0.82299	0.83536	0.84681	0.85743	0.86734	0.87661
40	0.75972	0.77588	0.79064	0.80422	0.81675	0.82842	0.83931	0.84952
45	0.70773	0.72576	0.74228	0.75751	0.77161	0.78475	0.79701	0.80854
50	0.62557	0.64664	0.66603	0.68396	0.70067	0.71624	0.73083	0.74457
55	0.50701	0.53197	0.55507	0.57658	0.59673	0.61563	0.63345	0.65029
60	0.36229	0.38954	0.41509	0.43909	0.46179	0.48328	0.50370	0.52311
65	0.21273	0.23805	0.26221	0.28528	0.30735	0.32850	0.34876	0.36820
70+[1]	0.08307	0.09453	0.10502	0.11466	0.12409	0.13295	0.14144	0.14971

Age x	9	10	11	12	13	14	15	16
0	0.86336	0.87674	0.89045	0.90309	0.91476	0.92557	0.93560	0.94494
5	0.94953	0.95443	0.95912	0.96350	0.96763	0.97151	0.97516	0.97859
10	0.94252	0.94752	0.95233	0.95716	0.96174	0.96605	0.97012	0.97397
15	0.92421	0.93034	0.93688	0.94332	0.94938	0.95508	0.96048	0.96556
20	0.91668	0.92325	0.93016	0.93701	0.94348	0.94961	0.95542	0.96089
25	0.91313	0.91959	0.92609	0.93278	0.93916	0.94523	0.95100	0.95648
30	0.90008	0.90963	0.91635	0.92330	0.92998	0.93635	0.94245	0.94830
35	0.88532	0.89273	0.89979	0.90722	0.91441	0.92136	0.92805	0.93450
40	0.86910	0.86733	0.87510	0.88303	0.89078	0.89836	0.90570	0.91285
45	0.81939	0.82883	0.83758	0.84631	0.85492	0.86338	0.87167	0.87977
50	0.75756	0.76888	0.77926	0.78960	0.79986	0.80999	0.82000	0.82983
55	0.66624	0.68009	0.69286	0.70563	0.71836	0.73103	0.74362	0.75604
60	0.54161	0.55762	0.57241	0.58738	0.60242	0.61751	0.63257	0.64754
65	0.38691	0.40512	0.41797	0.43524	0.44871	0.46436	0.48012	0.49591
70+[1]	0.15782	0.16506	0.17195	0.17921	0.18676	0.19465	0.20283	0.21131

Age x	17	18	19	20	21	22	23	24
0	0.95365	0.96199	0.97008	0.97735	0.98299	0.98778	0.99173	0.99483
5	0.98184	0.98493	0.98791	0.99037	0.99235	0.99446	0.99610	0.99745
10	0.97761	0.98107	0.98487	0.98785	0.99052	0.99289	0.99494	0.99666
15	0.97036	0.97493	0.98017	0.98413	0.98767	0.99080	0.99348	0.99571
20	0.96608	0.97104	0.97669	0.98115	0.98515	0.98872	0.99186	0.99453
25	0.96170	0.96670	0.97265	0.97745	0.98184	0.98587	0.98951	0.99269
30	0.95386	0.95925	0.96584	0.97129	0.97640	0.98120	0.98563	0.98962
35	0.94067	0.94666	0.95402	0.96041	0.96660	0.97257	0.97826	0.98359
40	0.91972	0.92643	0.93458	0.94220	0.94978	0.95734	0.96482	0.97213
45	0.88769	0.89542	0.90462	0.91387	0.92333	0.93301	0.94290	0.95295
50	0.83949	0.84900	0.86020	0.87200	0.88429	0.89710	0.91047	0.92442
55	0.76828	0.78040	0.79498	0.81051	0.82687	0.84415	0.86247	0.88193
60	0.66238	0.67716	0.69559	0.71535	0.73640	0.75903	0.78347	0.81003
65	0.51170	0.52734	0.54790	0.57023	0.59444	0.62104	0.65056	0.68364
70+[1]	0.32004	0.32909	0.34092	0.35387	0.36814	0.38403	0.40179	0.42175

[1] Value listed for age 70+ is $T(80)/T(70)$.

TABLE 277. MALE 10-YEAR SURVIVORSHIP PROBABILITIES, ${}_{10}S_{x,x+n}$, EAST MODEL

x	Survivorship probability, ${}_{10}S_{x,x+n}$, for mortality level							
	1	2	3	4	5	6	7	8
0	0.74414	0.76915	0.79119	0.81086	0.82863	0.84479	0.85960	0.87325
5	0.91040	0.91758	0.92415	0.93019	0.93577	0.94096	0.94581	0.95036
10	0.90296	0.91027	0.91697	0.92316	0.92887	0.93419	0.93917	0.94383
15	0.87343	0.88290	0.89153	0.89952	0.90692	0.91381	0.92026	0.92630
20	0.85726	0.86791	0.87768	0.88667	0.89503	0.90286	0.91015	0.91700
25	0.84027	0.85215	0.86307	0.87313	0.88249	0.89122	0.89940	0.90708
30	0.81048	0.82429	0.83704	0.84881	0.85976	0.86998	0.87958	0.88860
35	0.77344	0.78926	0.80384	0.81733	0.82991	0.84166	0.85269	0.86306
40	0.73224	0.74952	0.76538	0.78013	0.79387	0.80674	0.81882	0.83020
45	0.68145	0.69971	0.71652	0.73218	0.74675	0.76045	0.77331	0.78543
50	0.60999	0.62944	0.64752	0.66434	0.68005	0.69483	0.70875	0.72191
55	0.50703	0.52831	0.54822	0.56681	0.58430	0.60079	0.61638	0.63113
60	0.37574	0.39852	0.41955	0.43955	0.45846	0.47639	0.49345	0.50971
65	0.23828	0.25935	0.27945	0.29856	0.31677	0.33411	0.35093	0.36696
70+[a]	0.09812	0.10714	0.11558	0.12349	0.13100	0.13826	0.14531	0.15220

x	9	10	11	12	13	14	15	16
0	0.88591	0.89767	0.90970	0.92036	0.93022	0.93937	0.94787	0.95579
5	0.95463	0.95863	0.96243	0.96592	0.96928	0.97250	0.97559	0.97855
10	0.94822	0.95236	0.95597	0.95946	0.96289	0.96622	0.96946	0.97260
15	0.93200	0.93739	0.94214	0.94665	0.95107	0.95539	0.95960	0.96368
20	0.92345	0.92955	0.93501	0.94010	0.94507	0.94995	0.95469	0.95931
25	0.91432	0.92116	0.92739	0.93306	0.93861	0.94406	0.94936	0.95453
30	0.89710	0.90516	0.91259	0.91917	0.92565	0.93200	0.93823	0.94430
35	0.87289	0.88217	0.89076	0.89817	0.90551	0.91277	0.91993	0.92697
40	0.84097	0.85115	0.86052	0.86836	0.87622	0.88408	0.89191	0.89968
45	0.79692	0.80780	0.81772	0.82580	0.83398	0.84226	0.85057	0.85890
50	0.73436	0.74619	0.75689	0.76548	0.77423	0.78314	0.79219	0.80132
55	0.64520	0.65859	0.67050	0.68011	0.68998	0.70008	0.71039	0.72087
60	0.52520	0.54004	0.55304	0.56367	0.57465	0.58596	0.59756	0.60940
65	0.38235	0.39715	0.41000	0.42059	0.43161	0.44302	0.45481	0.46693
70+[a]	0.15896	0.16568	0.17183	0.17717	0.18285	0.18889	0.19529	0.20205

x	17	18	19	20	21	22	23	24
0	0.96318	0.97039	0.97739	0.98385	0.98836	0.99222	0.99523	0.99740
5	0.98138	0.98407	0.98664	0.98909	0.99128	0.99339	0.99530	0.99694
10	0.97565	0.97858	0.98139	0.98410	0.98700	0.98978	0.99240	0.99479
15	0.96764	0.97147	0.97516	0.97869	0.98251	0.98619	0.98968	0.99288
20	0.96378	0.96810	0.97227	0.97626	0.98039	0.98452	0.98845	0.99204
25	0.95955	0.96440	0.96907	0.97356	0.97799	0.98261	0.98700	0.99105
30	0.95022	0.95596	0.96150	0.96684	0.97202	0.97765	0.98305	0.98808
35	0.93286	0.94059	0.94712	0.95344	0.95973	0.96688	0.97296	0.98081
40	0.90733	0.91490	0.92229	0.92949	0.93705	0.94608	0.95343	0.96499
45	0.86723	0.87547	0.88362	0.89161	0.90047	0.91159	0.92358	0.93631
50	0.81052	0.81970	0.82882	0.83783	0.84825	0.86177	0.87662	0.89368
55	0.73146	0.74212	0.75276	0.76333	0.77598	0.79262	0.81155	0.83331
60	0.62145	0.63364	0.64587	0.65808	0.67320	0.69323	0.71643	0.74368
65	0.47953	0.49196	0.50470	0.51752	0.53389	0.55582	0.58174	0.61295
70+[a]	0.20917	0.21662	0.22436	0.23237	0.24231	0.25545	0.27101	0.28972

[a] Value listed for age 70+ is $T(80)/T(70)$.

TABLE 27B. MALE 10-YEAR SURVIVORSHIP PROBABILITIES $_{10}S_{x,x+9}$ WEST MODEL

Age x	Survivorship probability, $_{10}S_{x,x+9}$, for mortality level							
	1	2	3	4	5	6	7	8
0	0.74016	0.76542	0.78774	0.80768	0.82567	0.84202	0.85699	0.87076
5	0.88844	0.89789	0.90643	0.91422	0.92137	0.92799	0.93412	0.93986
10	0.84966	0.88036	0.89008	0.89894	0.90710	0.91465	0.92166	0.92820
15	0.83180	0.84544	0.85784	0.86919	0.87963	0.88932	0.89833	0.90673
20	0.80210	0.81813	0.83271	0.84610	0.85842	0.86986	0.88051	0.89046
25	0.77641	0.79254	0.80907	0.82426	0.83827	0.85129	0.86342	0.87477
30	0.73923	0.75973	0.77843	0.79567	0.81161	0.82642	0.84024	0.85320
35	0.69930	0.72202	0.74279	0.76193	0.77969	0.79619	0.81164	0.82611
40	0.65343	0.67801	0.70053	0.72131	0.74062	0.75863	0.77550	0.79133
45	0.59612	0.62232	0.64646	0.66877	0.68956	0.70893	0.72717	0.74431
50	0.52275	0.55025	0.57571	0.59935	0.62143	0.64213	0.66162	0.67998
55	0.42659	0.45518	0.48175	0.50661	0.52993	0.55190	0.57265	0.59229
60	0.31766	0.34568	0.37200	0.39678	0.42018	0.44236	0.46343	0.48348
65	0.20513	0.22993	0.25356	0.27614	0.29781	0.31862	0.33781	0.35663
70+ *	0.08772	0.09860	0.10842	0.11769	0.12630	0.13446	0.14235	0.15002

Age x	9	10	11	12	13	14	15	16
0	0.88350	0.89536	0.90637	0.91670	0.92814	0.93788	0.94618	0.95401
5	0.94520	0.95022	0.95494	0.95940	0.96416	0.96820	0.97175	0.97520
10	0.93432	0.94006	0.94549	0.95060	0.95562	0.96024	0.96436	0.96838
15	0.91461	0.92201	0.92900	0.93559	0.94179	0.94778	0.95317	0.95843
20	0.89980	0.90858	0.91686	0.92470	0.93202	0.93916	0.94562	0.95190
25	0.88543	0.89547	0.90494	0.91391	0.92229	0.93046	0.93785	0.94502
30	0.86538	0.87687	0.88773	0.89800	0.90766	0.91690	0.92528	0.93346
35	0.83974	0.85261	0.86479	0.87633	0.88732	0.89742	0.90661	0.91569
40	0.80626	0.82037	0.83373	0.84640	0.85874	0.86945	0.87926	0.88911
45	0.76048	0.77579	0.79029	0.80408	0.81777	0.82901	0.83942	0.85000
50	0.69736	0.71384	0.72949	0.74439	0.75923	0.77108	0.78219	0.79358
55	0.61095	0.62867	0.64557	0.66168	0.67762	0.69027	0.70222	0.71458
60	0.50258	0.52080	0.53825	0.55493	0.57131	0.58428	0.59663	0.60950
65	0.37466	0.39197	0.40863	0.42463	0.44023	0.45260	0.46446	0.47690
70+ *	0.15754	0.16497	0.17237	0.17978	0.18741	0.19395	0.20040	0.20738

Age x	17	18	19	20	21	22	23	24
0	0.96172	0.96889	0.97554	0.98182	0.98783	0.99123	0.99460	0.99711
5	0.97853	0.98172	0.98479	0.98771	0.99040	0.99288	0.99514	0.99705
10	0.97229	0.97606	0.97970	0.98320	0.98652	0.98976	0.99274	0.99543
15	0.96354	0.96848	0.97324	0.97781	0.98221	0.98648	0.99049	0.99403
20	0.95796	0.96384	0.96949	0.97490	0.98000	0.98495	0.98954	0.99353
25	0.95199	0.95871	0.96519	0.97140	0.97714	0.98280	0.98805	0.99262
30	0.94146	0.94921	0.95671	0.96394	0.97057	0.97742	0.98392	0.98972
35	0.92465	0.93343	0.94198	0.95029	0.95793	0.96645	0.97488	0.98281
40	0.89893	0.90866	0.91826	0.92767	0.93647	0.94709	0.95813	0.96922
45	0.86068	0.87138	0.88203	0.89261	0.90275	0.91578	0.92993	0.94300
50	0.80519	0.81694	0.82877	0.84057	0.85234	0.86808	0.88584	0.90569
55	0.72727	0.74022	0.75336	0.76656	0.78013	0.79885	0.82058	0.84382
60	0.62279	0.63647	0.65043	0.66457	0.67942	0.70054	0.72566	0.75582
65	0.48906	0.50329	0.51709	0.53118	0.54623	0.56833	0.59521	0.62830
70+ *	0.21487	0.22286	0.23137	0.24053	0.24997	0.26357	0.28011	0.30041

* Value listed for age 70+ is $T(80)/T(70)$.

Annex XI

LOGIT VALUES BY SINGLE YEAR OF AGE FOR COALE-DEMENY FEMALE LEVEL 16 MODEL LIFE TABLES

TABLE 279. LOGIT TRANSFORMATION OF THE COMPLEMENT OF THE PROBABILITY OF SURVIVING, $1 - l(x)$, NORTH MODEL

Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$
1	−1.2556	34	−0.6434	67	−0.0533
2	−1.1332	35	−0.6312	68	−0.0184
3	−1.0653	36	−0.6189	69	0.0187
4	−1.0177	37	−0.6065	70	0.0582
5	−0.9826	38	−0.5939	71	0.1009
6	−0.9601	39	−0.5812	72	0.1450
7	−0.9406	40	−0.5684	73	0.1927
8	−0.9237	41	−0.5554	74	0.2436
9	−0.9093	42	−0.5423	75	0.2979
10	−0.8968	43	−0.5291	76	0.3559
11	−0.8859	44	−0.5157	77	0.4180
12	−0.8763	45	−0.5023	78	0.4845
13	−0.8677	46	−0.4886	79	0.5554
14	−0.8596	47	−0.4746	80	0.6325
15	−0.8517	48	−0.4605	81	0.7132
16	−0.8424	49	−0.4459	82	0.8044
17	−0.8329	50	−0.4310	83	0.9011
18	−0.8233	51	−0.4157	84	1.0060
19	−0.8134	52	−0.3999	85	1.1202
20	−0.8033	53	−0.3836	86	1.2451
21	−0.7928	54	−0.3666	87	1.3819
22	−0.7821	55	−0.3490	88	1.5324
23	−0.7712	56	−0.3306	89	1.6984
24	−0.7601	57	−0.3114	90	1.8821
25	−0.7489	58	−0.2913	91	2.0859
26	−0.7376	59	−0.2703	92	2.3125
27	−0.7262	60	−0.2481	93	2.5650
28	−0.7146	61	−0.2247	94	2.8469
29	−0.7030	62	−0.2000	95	3.1619
30	−0.6912	63	−0.1740	96	3.5143
31	−0.6794	64	−0.1464	97	3.9087
32	−0.6676	65	−0.1172	98	4.3503
33	−0.6556	66	−0.0862	99	4.8450

TABLE 280. LOGIT TRANSFORMATION OF THE COMPLEMENT OF THE PROBABILITY OF SURVIVING, $1 - l(x)$, SOUTH MODEL

Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$
1	−1.0807	34	−0.6252	67	−0.1163
2	−0.9376	35	−0.6165	68	−0.0806
3	−0.8807	36	−0.6077	69	−0.0423
4	−0.8524	37	−0.5988	70	−0.0013
5	−0.8369	38	−0.5898	71	0.0427
6	−0.8265	39	−0.5807	72	0.0898
7	−0.8176	40	−0.5715	73	0.1404
8	−0.8100	41	−0.5621	74	0.1946
9	−0.8034	42	−0.5526	75	0.2527
10	−0.7976	43	−0.5429	76	0.3153
11	−0.7923	44	−0.5331	77	0.3824
12	−0.7879	45	−0.5229	78	0.4547
13	−0.7833	46	−0.5126	79	0.5327
14	−0.7789	47	−0.5018	80	0.6169
15	−0.7743	48	−0.4908	81	0.7081
16	−0.7685	49	−0.4793	82	0.8071
17	−0.7623	50	−0.4674	83	0.9150
18	−0.7558	51	−0.4549	84	1.0329
19	−0.7489	52	−0.4419	85	1.1623
20	−0.7418	53	−0.4282	86	1.3047
21	−0.7342	54	−0.4138	87	1.4622
22	−0.7264	55	−0.3984	88	1.6369
23	−0.7183	56	−0.3825	89	1.8315
24	−0.7101	57	−0.3654	90	2.0489
25	−0.7017	58	−0.3473	91	2.2926
26	−0.6933	59	−0.3279	92	2.5662
27	−0.6849	60	−0.3073	93	2.8742
28	−0.6764	61	−0.2853	94	3.2213
29	−0.6678	62	−0.2617	95	3.6128
30	−0.6592	63	−0.2365	96	4.0547
31	−0.6508	64	−0.2095	97	4.5537
32	−0.6423	65	−0.1806	98	5.1174
33	−0.6337	66	−0.1495	99	5.7541

TABLE 281. LOGIT TRANSFORMATION OF THE COMPLEMENT OF THE PROBABILITY OF SURVIVING, $1 - l(x)$. EAST MODEL

Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$
1	−1.0827	34	−0.6757	67	−0.0603
2	−0.9899	35	−0.6651	68	−0.0199
3	−0.9540	36	−0.6543	69	0.0230
4	−0.9315	37	−0.6434	70	0.0686
5	−0.9163	38	−0.6322	71	0.1171
6	−0.9049	39	−0.6210	72	0.1686
7	−0.8950	40	−0.6096	73	0.2235
8	−0.8864	41	−0.5983	74	0.2819
9	−0.8790	42	−0.5868	75	0.3442
10	−0.8725	43	−0.5750	76	0.4107
11	−0.8668	44	−0.5629	77	0.4818
12	−0.8615	45	−0.5505	78	0.5579
13	−0.8564	46	−0.5377	79	0.6396
14	−0.8515	47	−0.5244	80	0.7276
15	−0.8463	48	−0.5107	81	0.8224
16	−0.8399	49	−0.4964	82	0.9250
17	−0.8331	50	−0.4815	83	1.0363
18	−0.8259	51	−0.4659	84	1.1575
19	−0.8138	52	−0.4496	85	1.2900
20	−0.8104	53	−0.4326	86	1.4352
21	−0.8020	54	−0.4146	87	1.5949
22	−0.7932	55	−0.3957	88	1.7711
23	−0.7842	56	−0.3758	89	1.9661
24	−0.7749	57	−0.3547	90	2.1825
25	−0.7655	58	−0.3324	91	2.4232
26	−0.7560	59	−0.3088	92	2.6914
27	−0.7463	60	−0.2836	93	2.9906
28	−0.7366	61	−0.2575	94	3.3249
29	−0.7267	62	−0.2292	95	3.6986
30	−0.7166	63	−0.1994	96	4.1165
31	−0.7066	64	−0.1677	97	4.5841
32	−0.6964	65	−0.1340	98	5.1073
33	−0.6861	66	−0.0983	99	5.6929

TABLE 282. LOGIT TRANSFORMATION OF THE COMPLEMENT OF THE PROBABILITY OF SURVIVING, $1 - l(x)$. WEST MODEL

Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$	Age x	Logit of $1-l(x)$
1	−1.2093	34	−0.6793	67	−0.0225
2	−1.0951	35	−0.6661	68	0.0154
3	−1.0488	36	−0.6525	69	0.0552
4	−1.0198	37	−0.6393	70	0.0973
5	−0.9982	38	−0.6258	71	0.1416
6	−0.9850	39	−0.6122	72	0.1884
7	−0.9733	40	−0.5985	73	0.2378
8	−0.9628	41	−0.5850	74	0.2901
9	−0.9535	42	−0.5712	75	0.3454
10	−0.9449	43	−0.5573	76	0.4039
11	−0.9370	44	−0.5430	77	0.4660
12	−0.9294	45	−0.5285	78	0.5320
13	−0.9220	46	−0.5136	79	0.6022
14	−0.9146	47	−0.4983	80	0.6770
15	−0.9068	48	−0.4825	81	0.7570
16	−0.8975	49	−0.4663	82	0.8426
17	−0.8878	50	−0.4495	83	0.9344
18	−0.8775	51	−0.4321	84	1.0332
19	−0.8669	52	−0.4141	85	1.1398
20	−0.8558	53	−0.3954	86	1.2551
21	−0.8441	54	−0.3760	87	1.3801
22	−0.8321	55	−0.3557	88	1.5161
23	−0.8198	56	−0.3346	89	1.6644
24	−0.8073	57	−0.3125	90	1.8265
25	−0.7947	58	−0.2894	91	2.0040
26	−0.7822	59	−0.2652	92	2.1989
27	−0.7696	60	−0.2398	93	2.4133
28	−0.7596	61	−0.2132	94	2.6494
29	−0.7442	62	−0.1852	95	2.9097
30	−0.7313	63	−0.1559	96	3.1971
31	−0.7184	64	−0.1250	97	3.5144
32	−0.7055	65	−0.0926	98	3.8651
33	−0.6924	66	−0.0584	99	4.2528

Annex XII

GLOSSARY*

Age-heaping: tendency for enumerators or respondents to report certain ages instead of others; also known as age preference or "digit preference". Preference for ages ending in zero or five is widespread.

Age pattern of fertility: relative distribution of a set of "age-specific fertility rates". It expresses the relative contribution of each age group to "total fertility".

Age ratio: ratio of the population in a given age group to the average of the populations in the two neighbouring age groups, times 100.

Age-specific fertility rate: number of births occurring during a specified period to women of a specified age or age group, divided by the number of person-years lived during that period by women of that age or age group. When an age-specific fertility rate is calculated for a calendar year, the number of births to women of the specified age is usually divided by the mid-year population of women that age.

Age-specific mortality rate: number of deaths occurring during a specified period to persons (usually specified by sex) of a specified age or age group, divided by the number of person-years lived during that period by the persons of that age or age group. When an age-specific mortality rate is calculated for a calendar year, the number of deaths to persons of the specified age is usually divided by the mid-year population of persons of that age. Age-specific mortality rates are generally denoted by ${}_nM_x$, the annual death rate of persons aged from x to $x+n$.

Age standardization: a procedure of adjustment of crude rates (birth, death or other rates) designed to reduce the effect of differences in age structure when comparing rates for different populations.

Birth history: report of the number and dates of all live births experienced by a particular woman; see also "Pregnancy history". The sex of each child, the survival of each child to the date of the interview, and, where pertinent, the date of death are also generally recorded.

Birth order: the ordinal number of a given live birth in relation to all previous live births of the same woman (e.g., 5 is the birth order of the fifth live birth occurring to the same woman).

Birth rate: see "Crude birth rate".

Chandrasekaran-Deming technique: procedure to estimate the coverage of two independent systems collecting information about demographic or other events, based on the assumption that the probability of an event being recorded by one system is the same regardless of whether the event is recorded by the other system. The events from both systems are matched to establish M, the number of events recorded by both systems; U_1, the number recorded only by system 1; and U_2, the number recorded only by system 2. The Chandrasekaran-Deming formula then estimates total events, N, as

$$N = M + U_1 + U_2 + U_1U_2/M$$

Childbearing ages: the span within which women are capable of bearing children, generally taken to be from age 15 to age 49 or, sometimes, to age 44.

Children ever borne(e): number of children ever borne alive by a particular woman; synonymous with "parity". In demographic usage, stillbirths are specifically excluded.

Cohort: group of persons who experienced the same class of events in the same period. Thus, an age cohort is a group of people born during a particular period, and a marriage cohort is a group of people who married during a particular period. The effects of a given set of mortality or fertility rates are often illustrated by applying them to hypothetical cohorts.

Cohort fertility: the fertility experienced over time by a group of women or men who form a birth or marriage cohort. The analysis of cohort fertility is contrasted with that of "period fertility".

Crude birth rate: number of births in a population during a specified period divided by the number of person-years lived by the population during the same period. It is frequently expressed as births per 1,000 population. The crude birth rate for a single year is usually calculated as the number of births during the year divided by the mid-year population.

Crude death rate: number of deaths in a population during a specified period divided by the number of person-years lived by the population during the same period. It is frequently expressed as deaths per 1,000 population. The crude death rate for a single year is usually calculated as the number of deaths during the year divided by the mid-year population.

Cumulated fertility: an estimate of the average number of children ever borne by women of some age x, obtained by cumulating "age-specific fertility rates" up to age x; also often calculated for age groups.

Death rate: see "Crude death rate".

De facto *population:* population enumerated on the basis of those present in a particular time, including temporary visitors and excluding residents temporarily absent. See "*De jure* population".

De jure *population:* population enumerated on the basis of normal residence, excluding temporary visitors and including residents temporarily absent. See "*De facto* population".

Digit preference: see "Age-heaping".

Dual record system: see "Chandrasekaran-Deming technique".

Expectation of life at birth: average number of years that a member of a "cohort" of births would be expected to live if the cohort were subject to the mortality conditions expressed by a particular set of "age-specific mortality rates". Denoted by the symbol e_0 in "life table" notation.

Fertility history: either a "birth history" or a "pregnancy history".

Forward survival: procedure for estimating the age distribution at some later date by projecting forward an observed age distribution. The procedure uses "survival ratios", often obtained from model "life tables". The procedure is basically a form of population projection without the introduction of new entrants (births) to the population.

General fertility rate: ratio of the number of live births in a period to the number of person-years lived by women of "childbearing ages" during the period. The general fertility rate for a year is usually calculated as the number of births divided by the number of women of childbearing age at mid-year.

Gross reproduction rate: average number of female children a woman would have if she survived to the end of her childbearing years and if, throughout, she were subject to a given set of "age-specific fertility rates" and a given "sex ratio at birth". This number provides a measure of replacement fertility in the absence of mortality.

Growth rate: the increase or decrease of a population in a period divided by the number of person-years lived by the population during the same period. The increase in a population is the result of a surplus (or deficit) of births over deaths and of immigrants over emigrants. (The annual increase is often expressed as a fraction of the total population at the beginning of the year, but this convention has the inconvenient characteristic of not being readily defined for a five-year interval and of being unequal to the difference between the birth rate and the death rate even in the absence of migration.) See also "Rate of natural increase".

Infant mortality rate: number of deaths of children under one year of age occurring in the same year, also used in a more rigorous sense to mean the number of deaths that would occur under one year of age

*Terms in quotation marks are listed in the glossary.

in a "life table" with a "radix" of 1,000, in which sense it is denoted by the symbol q_0.

Life table: listing of the number of survivors at different ages (up to the highest age attained) in a hypothetical "cohort" subject from birth to a particular set of "age-specific mortality rates". The rates are usually those observed in a given population during a particular period of time. The survivors of the "radix" to age x are generally denoted by $l(x)$. The tabulations commonly accompanying a life table include other features of the cohort's experience: its expectation of life at each age x, denoted by e_x; the probability of dying from each age x to age $x+n$, denoted by ${}_nq_x$; the person-years lived by the hypothetical cohort as it ages from age x to age $x+n$, denoted by ${}_nL_x$ (also equivalent to the population aged $x, x+n$ in a "stationary population" experiencing a number of births each year equal to the radix of the life table); and the person-years lived by the hypothetical cohort from age x onward, denoted by $T(x)$.

Logit: The logit of a proportion p is $\frac{1}{2}\ln[p/(1-p)]$. As a linearizing transformation, the logit has been proposed as the basis of a model life-table system in which the logit of a probability of dying by age $x({}_xq_0)$ is related linearly to the logit of a standard probability of dying by age $x({}_xq_0^s)$ so that

$$\text{logit}\,({}_xq_0) = \alpha + \beta[\text{logit}({}_xq_0^s)]$$

where α is a measure of mortality level relative to the standard and β is a parameter that alters the shape of the standard mortality function.

Marital fertility: any measure of fertility in which the births (in the numerator) are births to married women and in which the number of person-years lived (in the denominator) also pertains to married women. In some instances, the category of married includes persons in consensual unions.

Mean age of childbearing: average age at which a mortality-free "cohort" of women bear their children according to a set of "age-specific fertility rates".

Mean age of childbearing in the population: average age of the mothers of the children born in a population during a year. This measure incorporates the effects of both mortality and the age distribution.

Median: the value associated with the central member of a set that is ordered by size or some other characteristic expressed in numbers.

Migration rate: number of migrants during a specified period divided by the person-years lived of the population exposed to migration. Also see "Population change due to migration".

Model life table: expression of typical mortality experience derived from a group of observed "life tables".

Moving average: the successive averaging of two or more adjacent values of a series in order to remove sharp fluctuations.

Myers index: an index of digit preference that essentially rates in turn the population ending in each digit over some age range, often 10-89, expressing the total as a percentage of the total population, and which avoids the bias introduced by the fact that the population is not evenly distributed among all ages by repeating the calculations 10 times, once for each beginning digit, and averaging the results. The difference between the average percentage for each digit and the expected value of 10 per cent provides a measure of the preference for or avoidance of the digit over the age range considered.

Natural fertility: age pattern of "marital fertility" observed in non-contraceptive populations where reproductive behaviour is not affected by the number of children already born.

Net migration: the difference between gross immigration and gross emigration.

Net reproduction rate: the average number of female children born per woman in a "cohort" subject to a given set of "age-specific fertility rates", a given set of "age-specific mortality rates" and a given "sex ratio at birth". This rate measures replacement fertility under given conditions of fertility and mortality: it is the ratio of daughters to mothers, assuming continuation of the specified conditions of fertility and mortality.

Own-children method: a refinement of the "reverse-survival" procedure for fertility estimation, whereby estimates of "age-specific fertility rates" for the recent past are obtained by relating mothers to their own children, using information on relationship and other characteristics available from a census or survey.

Parity: see "Children ever born".

Partial birth rate: the proportion of the population that enters (that is, is born into) a given age category in a year. The age categories used are normally open-ended; thus, the partial birth rate $x+$ designates the proportion of the population becoming x years and older.

Partial death rate: the proportion of the population that leaves (that is, dies out of) a given age category in a year. See "Partial birth rate".

Period fertility: fertility experienced during a particular period of time by women from all relevant birth or marriage "cohorts"; see also "Cohort fertility".

P/F ratio method: a consistency check for survey information on fertility. Information on recent fertility is extrapolated to obtain measures that are equivalent to average parities. Lifetime fertility in the form of reported average parities by age group, P, can then be compared for consistency with the parity equivalents, F, by calculating the ratio P/F for successive age groups. If certain assumptions about error patterns are met, an improved estimate of fertility can sometimes be obtained by correcting the age pattern of current fertility to agree with the level of lifetime fertility reported by younger women.

Population change due to migration: the sum of in-migrants minus out-migrants during a specified period of time. The change may also be expressed as a rate by dividing the change by person-years lived in the population during the same period.

Pregnancy history: a report of the number and dates of occurrence of all the pregnancies experienced by a particular woman. The outcome of the pregnancy—live birth, stillbirth, foetal death—is also recorded.

Radix: the hypothetical birth "cohort" of a "life table". Common values are 1, 1,000 and 100,000.

Rate of natural increase: the difference between the births and deaths occurring during a given period divided by the number of person-years lived by the population during the same period. This rate, which specifically excludes changes resulting from migration, is the difference between the "crude birth rate" and the "crude death rate".

Retrospective survey: survey that obtains information about demographic events that occurred in a given past period, generally terminating at the time of the survey.

Reverse projection: see "Reverse survival".

Reverse survival: a technique to estimate an earlier population from an observed population, allowing for those members of the population who would have died according to observed or assumed mortality conditions. It is used as a method of estimating fertility by calculating from the observed number of survivors of a given age x the expected number of births that occurred x years earlier. (In situations for which both fertility and mortality are known or can be reliably estimated, reverse survival can be used to estimate migration.)

Robustness: a characteristic of estimates that are not greatly affected by deviations from the assumptions on which the estimation procedure is based.

Sex ratio at birth: number of male births for each female birth or male births per 100 female births.

Singulate mean age at marriage (SMAM): a measure of the mean age at first marriage, derived from a set of proportions of people single at different ages or in different age groups, usually calculated separately for males and females.

Stable population: a population exposed for a long time to constant fertility and mortality rates, and closed to migration, establishes a fixed age distribution and constant growth rate characteristic of the vital rates. Such a population, with a constant age structure and constant rate of growth, is called a "stable population".

Stationary population: a "stable population" that has a zero growth rate, with constant numbers of births and deaths per year. Its age structure is determined by the mortality rates and is equivalent to the person-years lived ($_nL_x$) column of a conventional "life table".

Survival ratio: the probability of surviving between one age and another; often computed for age groups, in which case the ratios correspond to those of the person-years lived function, $_nL_x$, of a "life table". Also called "survivorship probabilities".

Survivorship probabilities: see "Survival ratio".

Synthetic parity: the average parity calculated for a hypothetical cohort exposed indefinitely to a set of period "age-specific fertility rates".

Total fertility rate (TFR): the average number of children that would be born per woman if all women lived to the end of their childbear-

ing years and bore children according to a given set of "age-specific fertility rates"; also referred to as total fertility. It is frequently used to compare the consequence of childbearing at the rates currently observed.

United Nations age/sex accuracy index: an index of age-reporting accuracy that is based on deviations from the expected regularity of population size and sex ratio, age group by age group. The index is calculated as the sum of: (*a*) the mean absolute deviation from 100 of the age ratios for males; (*b*) the mean absolute deviation from 100 of the age ratios for females; and (*c*) three times the mean of the absolute difference in reported sex ratios from one age group to the next. The United Nations defines age/sex data as "accurate", "inaccurate", or "highly inaccurate", depending upon whether the index is less than 20, from 20 to 40, or greater than 40.

Whipple's index: a measure of the quality of age-reporting based on the extent of preference for a particular target digit or digits. The index essentially compares the reported population at ages ending in the target digit or digits with the population expected on the assumption that population is a linear function of age. For a particular age range, often 23-62, the population with ages ending in the target digits is divided by one tenth of the total population; the result is then multiplied by 100 and divided by the number of different target digits. A value of 100 indicates no preference for those digits, whereas values over 100 indicate positive preference for them.

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