Semester-IV, Mathematics (Honours)

Skill Enhancement Course-II

Assignment

Unit-I

1. Let S be the set of all positive integral divisors of 30. Define the binary operations $(+)$, $(\cdot)$ and $(')$ on S by

$$a+b = \text{LCM of } a,b, \forall a,b \in S$$
$$a\cdot b = \text{GCD of } a,b, \forall a,b \in S$$
and $$a' = \frac{30}{a}, \forall a \in S$$

Show that $(S, +, \cdot, ')$ is a Boolean algebra.

2. In a Boolean algebra B, $a,b,c \in B$, show that
(i) $ab+a'b'+ab' = a+b'$, (ii) $a(a'+b+c) = ab+ac$
(iii) $a'(a+b'+c') = a'b'+a'c'$, (iv) $ab+a'b'+bc = ab+a'b'+a'c$
(v) $(a+b)(a'+b') = ab'+a'b$

3. Show that
(i) $[(abc+a'b')'+bc]' = a'b'$
(ii) $a(a+b)+b(b+c)+b = b$
(iii) $a\{b+c(ab+ac)'\} = ab$
(iv) $(ab'+a'b)'(ab+a'b')' = 0$
(v) $(ab+ab'+a'b)(a+b+c+a'b'c') = a+b$

4. In a Boolean algebra $(B, +, \cdot, ')$ show that the complement of
(i) $(ab'+c)$ is $(a'+b)c'$
(ii) $(abc'+b')(a+b'c)$ is $(a'+b'+c)b+a'(b+c')$
(iii) $(a+b)(a+c')$ is $a'(b'+c)$
(iv) $(ab'+ac+b'c)$ is $(a'+b)(a'+c')(b'+c)$

where $a,b,c \in B$.

Unit - II

1. Construct the switching circuit representing by the followings

(i) $ab + a'b' + ab'$, (ii) $ab' + [(a+c')b]$

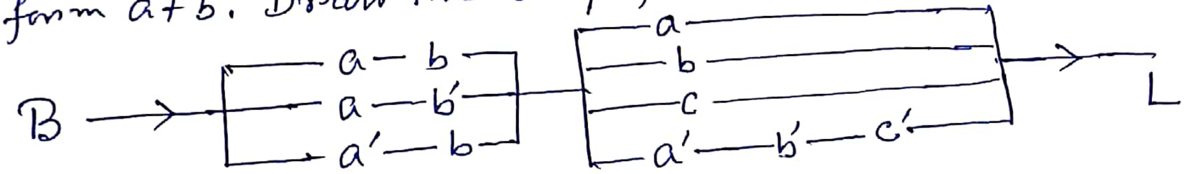
(iii) $(xy + xy' + x'y)(xy + xy')y$

(iv) $(a+c)b + (bd+d)a$, (V) $bc' + a(c+b')$

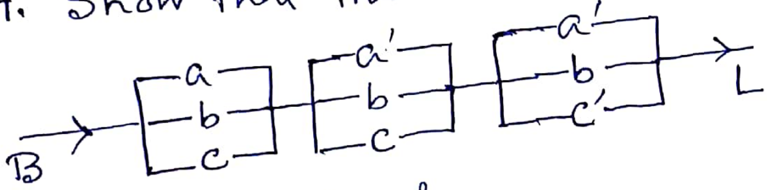
2. Draw the circuit represented by the Boolean function $a(a'+b) + b(b+c) + b$

Simplify the function and draw the simplified circuit.

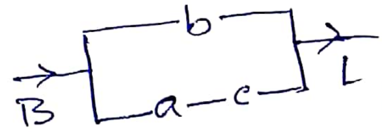
3. Find the Boolean function of the switching circuit given below and show that it can be simplified to the form $a+b$. Draw the simplified circuit.



4. Show that the circuits



and



are equivalent.

Unit - III

1. If $x \equiv a \pmod{16}$, $x \equiv b \pmod 5$ and $x \equiv c \pmod{11}$ then show that $x \equiv 385a + 176b - 560c \pmod{880}$

2. For a prime p, $(p-1)! \equiv (-1)^{\frac{p-1}{2}} \left(1 \cdot 2 \cdots \frac{p-1}{2}\right)^2 \pmod p$

Show that the integer $\left(\frac{p-1}{2}\right)!$ satisfies the Congruence $x^2 \pm 1 \equiv 0 \pmod p$ according as $p = 4k+1$ and $p = (4k+3)$

3. Solve $8x - 27y = 125$

4. Find the G.C.D. of 792 and 385 and express it in the form $792l + 385m$.

Unit - IV

1. Find the path on which a particle in the absence of friction will slide from one point to another in the shortest time under the action of gravity.

2. Solve the variational problem
$$\delta \int_1^2 [x^2 (y')^2 + 2y(x+y)]\, dx = 0$$
given $y(1) = y(2) = 0$

3. Show that the functional $\int_0^{\pi/2} \left[2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] dt$ such that $x(0) = 0$, $x(\pi/2) = -1$, $y(0) = 0$, $y(\pi/2) = 1$ is stationary for $x = -\sin t$, $y = \sin t$.

4. Show that the curve which extremizes the functional $I = \int_0^{\pi/4} (y''^2 - y^2 + x^2)\, dx$ under the conditions $y(0) = 0$, $y'(0) = 1$, $y(\pi/4) = y'(\pi/4) = \frac{1}{\sqrt{2}}$ is $y = \sin x$.

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