TDP (Honours) 1st Semester Exam., 2017

STATISTICS

(Honours)

SECOND PAPER (CC-2)

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Section - A

1. Answer any *six* questions : 2×6=12

(a) Obtain the value of $0.3+0.03+0.003+.....$

(b) Solve $\log_{10} x+\log_{10}(x-3)=1$

(c) Show that $\Gamma 7/2=\frac{15}{8}\sqrt{\pi}$

(d) Show that $\frac{d}{dx}x^n = nx^{n-1}$

[Turn Over]

(e) Define "Orthogonal Vectors".

(f) What is meant by "Triangular Matrix" ?

(g) Show that $\Delta \equiv E-1$.

(h) Show that $\Delta^n C^x = \left(C^h-1\right)^n C^x$ where 'C' is a constant and '*h*' is the interval of differencing.

Section - B

(There are four questions from Question No. 2 to Question No. 5. Answer either (a) or (b) of each question.)

Answer the following questions. 12×4=48

2. (a) (i) Let $1 \le r \le n$, then prove that

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

(ii) How many terms of the progression 3, 6, 9, 12... must be taken at the least to have a sum not less than 2000 ?

(iii) The arithmetic mean between two numbers is 34 and their geometric mean is 16. Find the numbers. 4+4+4=12

Or

(b) (i) Obtain the value of

$${}^nC_1 + 2^2\,{}^nC_2 + 3^2\,{}^nC_3 + ... + n^2\,{}^nC_n$$

(ii) Show that $1+\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+.....=2$

(iii) Using the principle of mathematical induction, prove that

$$1^2+2^2+.....+n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n \in N$. 5+4+3=12

3. (a) (i) Show that $\int_0^\infty e^{-x} x^n dx = \overline{|(n+1)}$. Hence of otherwise obtain the value of $\int_0^\infty e^{-ax^2} x^n dx$.

(ii) Explain how will you evaluate $\int_1^2 x^2 dx$ with the help of the fundamental theorem of integral calculus. 7+5=12

[Turn Over]

Or

(b) (i) Evaluate $\int_0^\infty \int_0^\infty 4xye^{-(x^2+y^2)}dxdy$.

(ii) Examine whether $x^{\frac{1}{x}}$ possesses a maximum or minimum and determine the same.

7+5=12

4. (a) (i) Show that

$$\begin{vmatrix} 1 & x & x & \cdots & x \\ x & 1 & x & \cdots & x \\ x & x & 1 & \cdots & x \\ \vdots & \vdots & \vdots & & \vdots \\ x & x & x & \cdots & 1 \end{vmatrix}_{n\times n} = \{1+(n-1)x\}(1-x)^{n-1}$$

(ii) Show that "Any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix". 8+4=12

Or

(b) Obtain the inverse of the matrix given below :

$$\begin{pmatrix} 1+a & 1 & 1 & \cdots\cdots & 1 \\ 1 & 1+a & 1 & \cdots\cdots & 1 \\ 1 & 1 & 1+a & \cdots\cdots & 1 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots\cdots & 1+a \end{pmatrix}_{n\times n}$$

12

5. (a) (i) Write a short note on interpolation.

(ii) State and prove the Lagrange's interpolation formula. 4+8=12

Or

(b) Describe the "General Quadrature Formula" for numerical integration. Hence or otherwise obtain the formula of numerical integration under "Simpson's one-third rule". 7+5=12

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