

3rd Semester Mathematics (Honours)  
Assignment, Paper-CC-5

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1) Let  $f(x, y) = x \sin(4 + \tan^{-1} \frac{y}{x})$  for  $x > 0$  and  $f(0, y) = 0$  for all  $y$ . verify the following properties of  $f$  at the point  $(0, 0)$ :

(i)  $\frac{\partial}{\partial x} f(0, y)$  is continuous w.r.t.  $y$ .

(ii)  $\frac{\partial}{\partial y} f(x, 0)$  is discontinuous w.r.t.  $x$ .

(iii)  $f$  is not differentiable at  $(0, 0)$ . [6]

2) Expand  $F(x, y) = \sin(e^y + x^2 - 2)$  according to Taylor's series in powers of  $x-1$  and  $y$ , going far enough to include all terms of degree 2. [4]

3) Write a short note on uniform convergence and pointwise convergence in your own words. [5]

4) Elaborate the idea of radius of convergence of a power series. Prove that

$$1 + \frac{a \cdot b}{1 \cdot c} x + \frac{a(a+1)b(b+1)}{1 \cdot 2 \cdot c(c+1)} x^2 + \dots$$

has unit radius of convergence.

Submit latest by: 23rd July '2021