# Questions on Sampling Distribution

① Let $X_1, X_2, X_3, X_4$ be a random sample of size 4 from a standard normal distribution. If the statistic W is given by

$$W = \frac{X_1 - X_2 + X_3}{\sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2}}$$

then what is the expected value of W?

② If $X \sim N(0,1)$ and $X_1, X_2$ is random sample of size two from the population X, then what is the 75th percentile of the statistic $W = \frac{X_1}{\sqrt{X_2^2}}$ ?

③ Suppose $X_1, X_2, \ldots X_n$ is a random sample from a normal distribution with mean '$\mu$' and variance $\sigma^2$. If $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ and

$$V^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$$ and $X_{n+1}$ is an additional observation, what is the value of '$m$' so that the statistics $\frac{m(\bar{X} - X_{n+1})}{V}$ has a 't'-distribution.

④ Let, $X_1, X_2, X_3, X_4$ and $Y_1, Y_2, Y_3, Y_4, Y_5$ be two random samples of size 4 and 5 respectively, from a standard normal population. What is the variance of the statistic $T = \dfrac{\sum_{i=1}^{4} X_i^2/4}{\sum_{j=1}^{5} Y_j^2/5}$

⑤ Let, X, Y and Z be independent uniform random variables on the interval $(0,a)$. Let $W = \min\{X, Y, Z\}$ What is the expected value of $(1 - \frac{W}{a})^2$?

⑥ Suppose $X_1, X_2, \ldots X_{2n+1}$ is a random sample from the uniform distribution on $(0,1)$. What is the probability density function of sample median $X_{(n+1)}$?

⑦ Suppose $X_1, X_2, \ldots X_6$ and $Y_1, Y_2 \ldots Y_9$ are independent, ideantically distributed normal random variables, each with mean zero and variance $\alpha^2 > 0$. What is the 95th percentile of the statistics

$$W = \left[\sum_{i=1}^{6} X_i^2\right] \Big/ \left[\sum_{j=1}^{9} Y_j^2\right] ?$$

(8) Let, $X_1, X_2 \ldots X_6$ and $Y_1, Y_2 \ldots Y_8$ be independent random samples from a normal distribution with mean '0' and variance 1. ~~and~~ Then find the distribution of.

$$4 \sum_{i=1}^{6} X_i^2 \Big/ 3 \sum_{j=1} Y_j^2 .$$

(9) Let, $X_1, X_2, \ldots X_n$ and $Y_1, Y_2, \ldots Y_n$ be two random sample from the independent normal distributions with $V(X_i) = \alpha^2$ and $V(Y_i) = 2\alpha^2$, for $i = 1, 2, \ldots n$ and $\alpha^2 > 0$.

If $U = \sum_{i=1}^{n} (X_i - \bar{X})^2$ and $V = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$

then what is the sampling distribution of the statistic $\frac{2U+V}{2\alpha^2}$ ?